WHERE THE POLICY MOVES MATTERS: CAPABILITY PRESERVATION VIA CUMULATIVE FISHER PROTECTION IN RLHF

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ABSTRACT

Reinforcement learning from human feedback (RLHF) improves alignment but can erode existing capabilities. A reference-policy KL penalty on alignment prompts need not adequately protect behavior on the capability distributions we wish to preserve. Inspired by this principle, we propose Cumulative Fisher Protection (CFP), a general framework for controlling cumulative displacement in capability-sensitive directions. CFP estimates Fisher sensitivity at the initial policy and corrects the cumulative displacement proposed by a base optimizer through a closed-form proximal update, leaving less sensitive directions available for alignment. The formulation accommodates different base optimizers and admits a low-rank implementation. We evaluate this protection principle using diagonal instantiations with PPO and DPO on Qwen2.5-7B-Instruct. Across PPO and DPO, Fisher anchoring mitigates observed capability regression and improves selected benchmark scores. Under PPO, mathematics anchoring raises OlympiadBench accuracy from 37.33% to 38.52%. Safety-anchored PPO achieves a SORRY-Bench harmful-compliance rate of 12.27%, compared with 13.18% for unregularized PPO, the lowest among the evaluated final checkpoints. Under DPO, mathematics anchoring restores MATH accuracy from 74.88% to the initial model's 75.36%, while raising OlympiadBench accuracy from 38.96% to 40.15%, surpassing the initial model's 39.56%. These results suggest that capability preservation is fundamentally a trajectory-control problem: what matters is not how far the policy moves, but where its trajectory moves.

1 INTRODUCTION

Reinforcement learning from human feedback (RLHF) has become a central component of language-model alignment, translating human preferences into reward signals for policy optimization. Its successes span human-preferred summarization (Stiennon et al., 2020), instruction following and truthfulness (Ouyang et al., 2022), and helpful and harmless dialogue (Bai et al., 2022). These advances have also informed widely used conversational models: Llama 2-Chat combines supervised fine-tuning and RLHF to improve helpfulness and safety (Touvron et al., 2023). Together, these results establish preference-based training as a practical bridge between pretrained language models and assistants that better satisfy user intent.

However, improvements in the alignment objective need not preserve capabilities acquired before alignment. InstructGPT, for example, reports that PPO-based RLHF reduced performance on several public NLP benchmarks (Ouyang et al., 2022). Subsequent studies characterize this trade-off as an *alignment tax*: stronger preference alignment can accompany worse performance on reading comprehension, translation, and other general tasks (Lin et al., 2024; Lu et al., 2024). More recent safety-alignment experiments exhibit a related trade-off between safety improvements and general capabilities under reinforcement learning (Niu et al., 2026b). These observations are distinct from reward-model overoptimization, in which optimizing a proxy reward eventually reduces a gold evaluator's score (Gao et al., 2023). Our concern is capability retention: even an improvement in an independent preference score would not, by itself, establish that a model retains its mathematical or other task-specific competence. This motivates the question:

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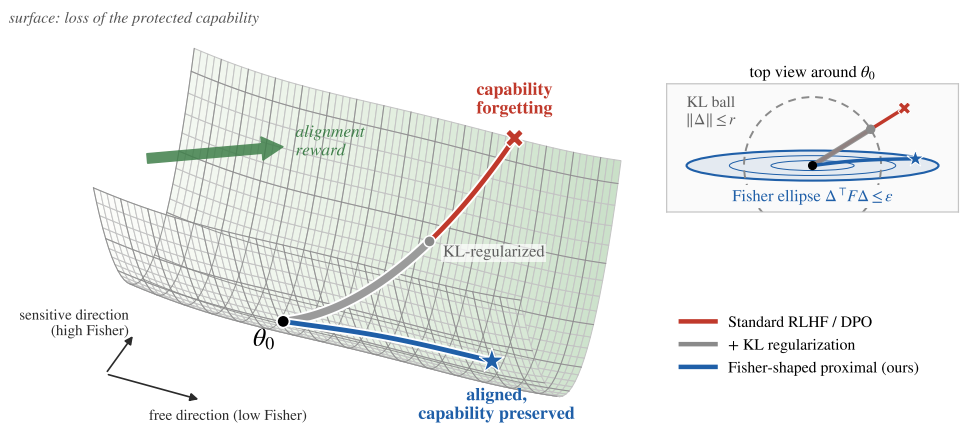

Figure 1: Geometric illustration of the proposed correction. Rather than merely shortening the alignment update, the correction redirects cumulative drift away from directions to which the protected capability is most sensitive.

> *Can we improve alignment through RLHF while preserving selected capabilities of the initial policy?*

Existing approaches mix pretraining updates into PPO, interpolate or merge pre- and post-alignment parameters, impose learned constraints, or project updates away from capability-relevant directions (Ouyang et al., 2022; Lin et al., 2024; Lu et al., 2024; Dai et al., 2024; Niu et al., 2026b; Sun et al., 2026). Together, they demonstrate the value of preserving prior behavior while leaving two design choices unresolved: how to measure parameter directions by their effect on a selected capability's output distribution, and how to control drift accumulated across successive optimizer steps. Section 1.1 discusses the distinctions among these methods in detail.

Reference-policy KL regularization provides an important safeguard, but increasing its coefficient is not a complete solution. InstructGPT reports that increasing the KL coefficient to 2.0, one hundred times its default value, did not eliminate regressions on DROP and SQuAD and substantially reduced validation reward (Ouyang et al., 2022, Appendix E.6, Figure 34). Lin et al. (2024) likewise find that stronger KL penalties partially reduce forgetting but offer a less favorable alignment–forgetting trade-off than model averaging in their PPO experiments. NSPO's ablations illustrate a complementary limitation: KL regularization can preserve general utility while restricting the desired safety improvement (Niu et al., 2026b). These findings do **NOT** imply that KL is intrinsically unsuitable for preservation. Rather, regularization evaluated on alignment prompts need not sufficiently constrain behavior on the capability distribution of interest. The relevant question is therefore which behavior is protected, and how that protection is enforced throughout training.

A geometric perspective motivates capability-specific control of cumulative drift. Springer et al. (2026) provide a local analysis of alignment degradation under fine-tuning: under their assumptions, capability loss depends not only on the magnitude of displacement from initialization, but also on its components along directions that are sensitive in the capability-specific Fisher geometry. Their analysis further shows how curvature can steer later updates into sensitive directions even when the initial gradient has little overlap with them. This motivates a practical question: can this geometric perspective guide effective capability protection during RLHF? We investigate this question through an optimizer-level correction that selectively attenuates cumulative displacement along directions with high estimated output sensitivity, while imposing weaker constraints on less sensitive directions. We then evaluate whether this selective control improves the capability–alignment trade-off.

Building on this dynamic perspective, we introduce ***C**umulative **F**isher **P**rotection (CFP)*, an optimizer-level update correction for preserving selected capabilities during preference optimization. We form a calibration prompt distribution representing those capabilities, sample responses

from the initial policy, and estimate its output Fisher, $\widehat{F}_0$. This fixed matrix is a local quadratic surrogate for forward KL from the initial policy on the calibration distribution.

Our contributions are threefold:

- **Cumulative output-Fisher protection.** We introduce an optimizer-level correction that uses the initial policy's output Fisher on capability-relevant prompts to control cumulative output-sensitive displacement from initialization without requiring capability labels.
- **An exact, low-rank update.** We derive the exact solution of the resulting local quadratic proximal problem. Its low-rank implementation corrects each optimizer proposal without constructing a parameter-sized Fisher matrix and explicitly accounts for displacement accumulated during earlier updates.
- **A controlled evaluation protocol.** We design paired update interventions and independent PPO trajectories to distinguish output-sensitive protection from uniform update scaling and activation-based correction, evaluating capability retention at comparable alignment performance and reporting computational overhead.

1.1 Related work

Capability retention and behavioral regularization. Prior work mitigates capability degradation during alignment through auxiliary training, model merging, and behavioral regularization. InstructGPT mixes pretraining gradients into PPO updates to reduce regressions on public NLP tasks (Ouyang et al., 2022). Heterogeneous Model Averaging adjusts layerwise interpolation between pre- and post-alignment models (Lin et al., 2024), while Online Merging Optimizers incorporate parameter differences between supervised fine-tuned and pretrained models into optimization (Lu et al., 2024). More directly targeting output behavior, Correctness-Conditioned KL Regularization (CoKL) constrains correctness-conditioned response distributions to balance capability retention and further learning (Wang et al., 2026). Our approach uses responses sampled from the initial policy on capability-relevant prompts to estimate a fixed output Fisher, without requiring correctness labels. This estimate defines a local quadratic approximation to reference-policy KL, which we use to correct cumulative parameter displacement during alignment. The distinction lies in the choice of preservation metric and update rule: CFP applies a Fisher-weighted proximal correction to cumulative displacement, whereas the methods above use auxiliary gradients, model interpolation, or distributional regularization.

Fisher geometry and retention regularization. Elastic weight consolidation protects old tasks using Fisher-weighted parameter anchoring (Kirkpatrick et al., 2017). K-FAC relates layerwise Fisher structure to both input activations and backpropagated output sensitivity (Martens & Grosse, 2015). AlignGuard-LoRA uses Fisher-based subspace regularization to retain safety during downstream adaptation (Das et al., 2025). Continual Policy Optimization also derives a Fisher-based retention motivation, then implements a masked penalty using parameter movement as an importance proxy (Luo et al., 2026). CFP studies a specific alternative: model-sampled sequence-score second moments at the initial policy, off-diagonal interactions within the cached score span, and a proximal correction applied to a base optimizer's proposal, with Adam used in our implementation.

Gradient and activation subspaces. Orthogonal Gradient Descent protects earlier outputs using stored output-gradient directions (Farajtabar et al., 2020; Zhou et al., 2026); Gradient Projection Memory constructs protected subspaces from activation representations (Saha et al., 2021). For LLM alignment, NSPO uses activation null spaces (Niu et al., 2026a), whereas OGPSA projects safety gradients against periodically refreshed reference-loss gradients and evaluates SFT and DPO settings (Sun et al., 2026). Thus, output-sensitive subspace protection predates CFP. Our comparison concerns the choice of sensitivity metric and the resulting soft correction of cumulative displacement. NSPO's public implementation also periodically projects displacement from reference weights, so cumulative anchoring alone does not distinguish the methods. Appendix F.3 specifies the activation comparators and implementation provenance.

2 PRELIMINARIES

CFP modifies policy updates during RLHF to protect behavior on a selected capability distribution. We first specify the alignment objective and the capability loss of interest. We then show how a local approximation of that loss induces the Fisher geometry used by CFP.

2.1 RLHF OBJECTIVE

Let $\pi_\theta(y \mid x)$ denote a policy initialized at π_{θ_0}, and let $r_\phi(x, y)$ be a reward model whose parameters ϕ remain fixed during policy optimization. Given a prompt distribution $P_\mathcal{X}$ and a reference policy π_{ref}, the KL-regularized RLHF objective is

$$\mathcal{J}(\theta, \phi) = \mathbb{E}_{x\sim P_\mathcal{X}} \left[\mathbb{E}_{y\sim\pi_\theta(\cdot|x)} \left[r_\phi(x, y) \right] - \beta_{\text{KL}} D_{\text{KL}} \left(\pi_\theta(\cdot \mid x) \,\|\, \pi_{\text{ref}}(\cdot \mid x) \right) \right], \tag{1}$$

where $\beta_{\text{KL}} \geq 0$ controls regularization toward the reference policy. Throughout, we set $\pi_{\text{ref}} = \pi_{\theta_0}$ and write $J(\theta) = \mathcal{J}(\theta, \phi)$. PPO supplies the base policy updates; CFP modifies the parameter proposal produced by its optimizer.

2.2 FISHER INFORMATION.

For a probabilistic model $p_\theta(z)$, the Fisher information matrix measures local sensitivity to parameter changes. Under standard regularity conditions,

$$F(\theta) = \mathbb{E}_{z\sim p_\theta} \left[\nabla_\theta \log p_\theta(z) \, \nabla_\theta \log p_\theta(z)^\top \right],$$
$$D_{\text{KL}}(p_\theta \,\|\, p_{\theta+\delta}) \approx \frac{1}{2} \delta^\top F(\theta) \delta,$$

where δ is a small parameter perturbation. For perturbations of equal magnitude, larger Fisher eigenvalues indicate directions with greater local output sensitivity (Martens & Grosse, 2015).

2.3 CAPABILITY-SENSITIVE FISHER GEOMETRY

We use the initial policy's output sensitivity on capability-relevant prompts to identify directions that should be protected during alignment. Motivated by the local geometric analysis of Springer et al. (2026), let θ_0 denote the initial policy parameters and let $\mathcal{D}_i(x)$ denote a prompt distribution representing capability $i \in \{1, \ldots, m\}$. Define the initial sequence score by

$$s_0(x, y) := \nabla_\theta \log \pi_\theta(y \mid x)|_{\theta=\theta_0}. \tag{2}$$

The corresponding capability-specific output Fisher is

$$F_i(\theta_0) := \mathbb{E}_{\substack{x\sim\mathcal{D}_i(x) \\ y\sim\pi_{\theta_0}(\cdot|x)}} \left[s_0(x, y) s_0(x, y)^\top \right]. \tag{3}$$

The score describes how the log-probability of a response changes with the parameters. Its second moment defines a geometry in which directions with larger Fisher eigenvalues produce greater local changes in the output distribution for a displacement of the same magnitude.

To represent several capabilities, choose weights $\omega_i \geq 0$ with $\sum_{i=1}^m \omega_i = 1$, and define

$$P_{\text{cap}}(x) := \sum_{i=1}^m \omega_i \mathcal{D}_i(x), \qquad F_0 := \sum_{i=1}^m \omega_i F_i(\theta_0). \tag{4}$$

Sampling prompts from P_{cap} and responses from π_{θ_0} therefore gives the aggregate Fisher F_0. The single-capability case follows by setting $m = 1$.

3 CUMULATIVE FISHER PROTECTION

CFP converts the capability-specific geometry from Section 2 into a correction of each policy-optimizer proposal. We first connect the preservation goal to the aggregate Fisher surrogate, then derive the cumulative correction and its low-rank implementation.

3.1 From capability constraints to an update correction

Draw N calibration prompts from P_{cap} and sample one response to each prompt from the initial policy. Let s_n denote the sequence score defined in (2) for calibration sample n, and stack $s_n^\top$ as row n of $\Psi \in \mathbb{R}^{N\times P}$, where P is the number of trainable parameter coordinates to which CFP is applied. The empirical output Fisher is

$$\widehat{F}_0 = \frac{1}{N}\Psi^\top\Psi. \tag{5}$$

Because responses are sampled from π_{θ_0}, constructing $\widehat{F}_0$ requires capability-relevant prompts but not correctness labels.

At iteration t, let d_t^0 be the actual parameter update proposed by the base optimizer and let $\Delta_t = \theta_t - \theta_0$ be the displacement already accumulated from initialization. Without protection, the proposed cumulative displacement would be

$$u_t = \Delta_t + d_t^0. \tag{6}$$

CFP replaces u_t with the solution of the local proximal problem

$$z_t^\star = \arg\min_z \left\{ \frac{1}{2}\|z - u_t\|_2^2 + \frac{\lambda_t}{2} z^\top \widehat{F}_0 z \right\}, \qquad \lambda_t \geq 0, \tag{7}$$

and sets $\theta_{t+1} = \theta_0 + z_t^\star$. The first term keeps the corrected parameters close to the optimizer proposal; the second penalizes the total output-sensitive displacement from initialization. For a Euclidean gradient proposal, problem (7) is the usual proximal update associated with a quadratic Fisher penalty; for an adaptive optimizer, it defines a post-optimizer correction in parameter space.

Proposition 1 (Cumulative Fisher correction)**.** *For any $\widehat{F}_0 \succeq 0$ and $\lambda_t \geq 0$, problem* (7) *has the unique solution*

$$z_t^\star = \left(I + \lambda_t \widehat{F}_0\right)^{-1} u_t, \qquad \theta_{t+1} = \theta_0 + z_t^\star. \tag{8}$$

Moreover, for $\widehat{Q}(z) = \frac{1}{2} z^\top \widehat{F}_0 z$, $\widehat{Q}(z_t^\star) \leq \widehat{Q}(u_t)$.

The objective in problem (7) has Hessian $I + \lambda_t \widehat{F}_0 \succ 0$, which gives uniqueness. In an eigenvector of $\widehat{F}_0$ with eigenvalue μ, the update (8) multiplies the proposed displacement by $(1 + \lambda_t \mu)^{-1}$. CFP therefore leaves directions in the empirical Fisher null space unchanged and attenuates directions in proportion to their estimated output sensitivity. The same eigendecomposition establishes the stated contraction of $\widehat{Q}$.

The correction is cumulative because u_t contains both the newest optimizer proposal and all displacement retained from earlier iterations. Thus CFP can correct previously accumulated drift; applying the same map only to d_t^0 would define a different, stepwise method.

3.2 Optimizer-Level Correction and Efficient Implementation

CFP implements the cumulative correction as a wrapper around a base optimizer. At iteration t, the optimizer proposes an increment d_t^0, giving tentative parameters $\widetilde{\theta}_{t+1} = \theta_t + d_t^0$. The proposed displacement from initialization is therefore $u_t = \widetilde{\theta}_{t+1} - \theta_0$. CFP corrects this displacement and sets $\theta_{t+1} = \theta_0 + z_t^\star$. Because u_t includes both the displacement accumulated so far and the current proposal, the correction accounts for the cumulative effect of previous updates. The same interface can accept proposals from SGD, Adam, or another base optimizer.

Efficient correction. The empirical Fisher admits the factorization $\widehat{F}_0 = \Psi^\top\Psi/N$, where $\Psi \in \mathbb{R}^{N\times P}$ contains the calibration score vectors evaluated at θ_0. Here, N is the number of calibration sequences and P is the number of corrected parameter coordinates. Substituting this factorization into $z_t^\star = (I_P + \lambda_t \widehat{F}_0)^{-1} u_t$ and applying the Woodbury identity yields

$$z_t^\star = u_t - \alpha_t \Psi^\top \left(I_N + \alpha_t \Psi\Psi^\top\right)^{-1} \Psi u_t, \qquad \alpha_t = \frac{\lambda_t}{N}. \tag{9}$$

This expression reduces the correction to a linear system of dimension N. In practice, we solve the system without explicitly forming either the $P \times P$ Fisher matrix or a matrix inverse. For $\lambda_t \geq 0$, the system matrix is symmetric positive definite and can be solved using a Cholesky factorization.

Algorithm 1 Cumulative Fisher Protection

Require: Initial parameters θ_0; fixed score cache $\Psi \in \mathbb{R}^{N\times P}$; alignment optimizer $\mathcal{O}$ and initial state s_0; strengths $\{\lambda_t \geq 0\}_{t=0}^{T-1}$

$G \leftarrow \Psi\Psi^\top$ ▷ Cache the score Gram matrix
for $t = 0, \ldots, T-1$ **do**
  $(d_t^0, s_{t+1}) \leftarrow \mathcal{O}(\theta_t, s_t)$ ▷ Propose a parameter step; retain new state
  $u_t \leftarrow \theta_t - \theta_0 + d_t^0$ ▷ Form proposed cumulative displacement
  $\alpha_t \leftarrow \lambda_t / N$ ▷ Normalize by calibration sample count
  Solve $(I_N + \alpha_t G) q_t = \Psi u_t$ ▷ Solve the N-dimensional system
  $\theta_{t+1} \leftarrow \theta_0 + u_t - \alpha_t \Psi^\top q_t$ ▷ Shrink Fisher-sensitive displacement
end for
return θ_T ▷ Return the final policy parameters

4 Experiments

In the experiments, we want to evaluate whether cumulative Fisher anchoring preserves capabilities while allowing alignment to proceed. We ask three questions: **(1)** Does it reduce cumulative drift in sensitive directions while preserving movement in less sensitive ones? **(2)** Does it preserve or improve capabilities compared with the initial model and uncorrected baselines? **(3)** Does it retain the safety gains of alignment?

4.1 Experimental Setup

Training and comparisons. Table 2 summarizes our training settings. We compare the initial model, unregularized PPO, PPO with reference-KL regularization, and three Fisher-anchored PPO variants targeting mathematics, code, and safety.

Fisher anchoring used in the main experiments. For each domain d, we use $N = 128$ calibration sequences to construct a diagonal score second-moment matrix at initialization:

$$\widehat{F}_d = \operatorname{diag}\left(\frac{1}{N}\sum_{n=1}^{N} s_n \odot s_n\right), \qquad s_n = \nabla_\theta \log \pi_\theta(y_n \mid x_n)|_{\theta=\theta_0}. \tag{10}$$

Diagonal CFP correction. Let $\Delta_t = \theta_t - \theta_0$ denote the cumulative parameter displacement, and let d_t^0 be the update proposed by the base optimizer. For a fixed diagonal Fisher approximation $\widehat{F}_d = \operatorname{diag}(f_{d,1}, \ldots, f_{d,P})$, CFP corrects the proposed cumulative displacement as

$$\theta_{t+1} = \theta_0 + \left(I + \lambda_t \widehat{F}_d\right)^{-1} \left(\Delta_t + d_t^0\right). \tag{11}$$

The correction can therefore be computed independently for each parameter coordinate:

$$\theta_{t+1,i} = \theta_{0,i} + \frac{\theta_{t,i} - \theta_{0,i} + d_{t,i}^0}{1 + \lambda_t f_{d,i}}. \tag{12}$$

This is the exact proximal correction for the chosen diagonal Fisher approximation. It contracts cumulative displacement more strongly on coordinates with higher Fisher sensitivity.

We construct three Fisher anchors using 128 calibration sequences per domain: mathematics, code, and safety. Data sources, response selection, and overlap with evaluation benchmarks are reported in the appendix. Because the calibration prompts overlap with the corresponding evaluation sets, these results alone do not establish generalization to unseen prompts.

4.2 Selective Control of Cumulative Displacement

We want to understand how the capability results relate to changes in the model during training. Our method aims to limit movement in Fisher-sensitive directions while leaving room for alignment. To check whether this happens, we track displacement from the initial model. We measure cumulative displacement because successive updates can reinforce or cancel one another, so looking at individual steps would miss their combined effect.

How we measure it. Let $\Delta_t = \theta_t - \theta_0$ be the displacement computed from the FP32 training checkpoints. For each domain d, we measure

$$Q_d(t) = \frac{1}{2}\Delta_t^\top \widehat{F}_d \Delta_t, \qquad R_d(t) = \sqrt{\Delta_t^\top \widehat{F}_d \Delta_t}. \tag{13}$$

Here, Q_d is the quadratic protection term, and R_d is the Fisher-weighted drift shown on the vertical axis of Figure 2. The horizontal axis shows $U_d(t) = \|\Delta_{t,S_d^c}\|_2$, where S_d contains the 1% of coordinates with the largest Fisher weights. This lets us check whether the model moves less in sensitive directions while still moving on low-weight coordinates. We compare each corrected PPO and DPO run with its own baseline. Lower R_d at similar U_d would support the selective effect our method is designed to achieve.

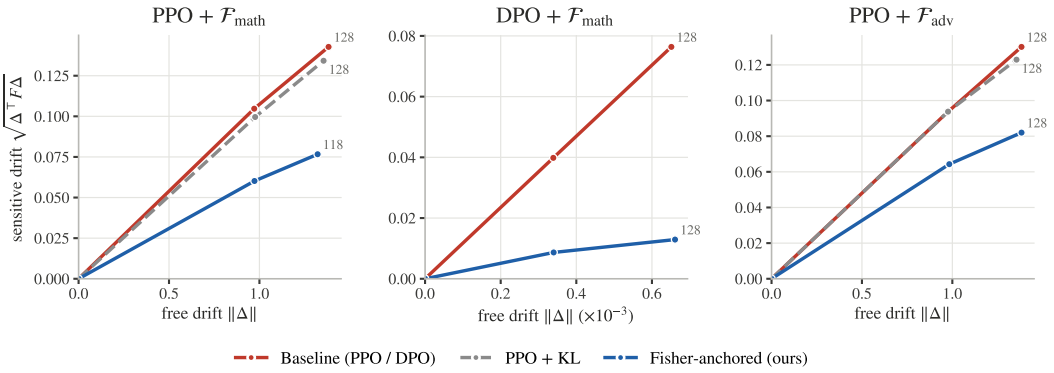

Figure 2: Cumulative displacement under each anchor. Vertical: $R_d(t) = \sqrt{\Delta_t^\top \widehat{F}_d \Delta_t}$. Horizontal: displacement on the 99% of coordinates with the smallest diagonal anchor weights. Points mark initialization, step 64, and the final checkpoint. Anchoring reduces weighted displacement at similar low-weight displacement; this diagnostic uses the training anchor.

Effective suppression in the targeted sensitive directions. We first track cumulative Fisher-weighted displacement under each method's own calibration anchor. Across the PPO and DPO configurations, the corrected trajectories show substantially less sensitive drift than their corresponding baselines, while retaining comparable movement on low-weight coordinates. This agrees with our theoretical prediction: the correction controls the accumulation of displacement in directions emphasized by the selected Fisher matrix. The trajectories therefore confirm that the intervention acts on the geometric quantity it is designed to constrain.

Table 1: Cross-domain suppression of cumulative displacement at step 64. Entries report percentage changes in the quadratic diagnostic Q_d relative to the corresponding unregularized baseline: PPO for the PPO rows and DPO for the DPO row. Columns specify the Fisher matrix used for measurement; bold entries match the training and measurement domains. PPO uses a cumulative proximal correction with $\lambda = 1$; DPO uses a cumulative proximal correction with $\lambda = 10$.

Training arm	$\widehat{F}_{\text{math}}$	$\widehat{F}_{\text{adv}}$	$\widehat{F}_{\text{code}}$
PPO + KL	-10%	$+1\%$	-7%
PPO + $\mathcal{F}_{\text{math}}$	$\mathbf{-67\%}$	-32%	-40%
PPO + $\mathcal{F}_{\text{adv}}$	-46%	$\mathbf{-53\%}$	-34%
PPO + $\mathcal{F}_{\text{code}}$	-33%	-14%	$\mathbf{-41\%}$
DPO + $\mathcal{F}_{\text{math}}$	$\mathbf{-95\%}$	-93%	-63%

Suppression also extends to other sensitive directions. We then measure each trajectory using Fishers from the other domains. As Table 1 shows, each anchor produces its strongest relative suppression in its own calibration domain, as expected. However, substantial reductions also appear under the other domain metrics. Within PPO, every Fisher-anchored variant produces greater

suppression than the tested KL-penalty baseline in all three measurement domains. For example, mathematics anchoring reduces code-sensitive displacement even though no coding examples are used to construct the training anchor. This suggests that a constraint built from one domain can also influence sensitive movement associated with other capabilities. We next examine the behavioral consequences: whether these reductions preserve useful capabilities or restrict changes that would otherwise improve task performance.

4.3 Capability Profiles: Retention and Improvement

Figure 3 is the central visualization of the behavioral results. Each anchored policy is compared with both unregularized PPO and the initial model. These references answer different questions: a gain over PPO indicates mitigation of the observed regression, whereas a gain over the initial model indicates improvement beyond the starting score. An outward shift relative to PPO need not imply full recovery to the initial policy.

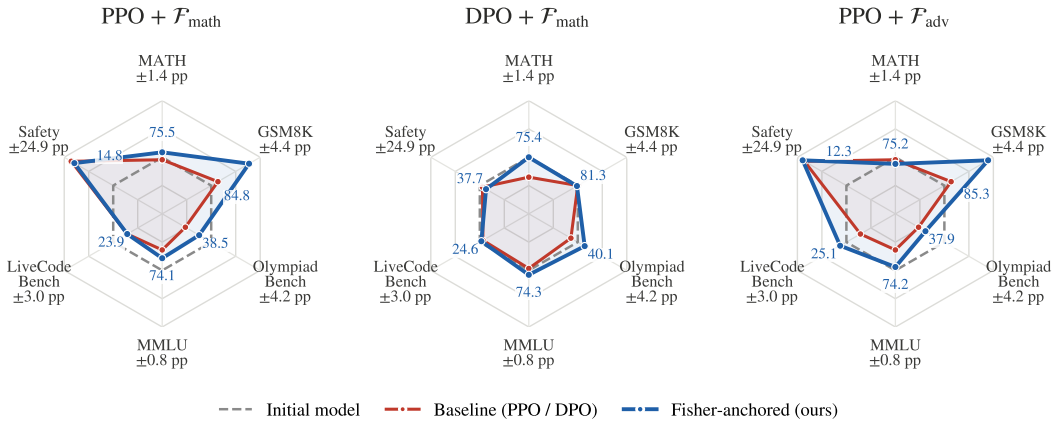

Figure 3: Capability profiles under mathematics, code, and safety anchoring. The dashed ring marks the initial model; outward is better. Each axis uses its displayed range in percentage points, shared across panels. Safety is the reduction in harmful compliance. Axis scales differ, so polygon area is not an aggregate score.

Capability retention under PPO and DPO. Fisher anchoring mitigates the observed capability regression under PPO. On OlympiadBench, the three anchors raise accuracy from 37.33% to 37.93–38.96%, partially recovering the initial model's 39.56%. The mathematics anchor also improves MATH accuracy from 75.30% to 75.48%. Under DPO, mathematics anchoring raises MATH accuracy from 74.88% to 75.36%, matching the initial model, and OlympiadBench accuracy from 38.96% to 40.15%, exceeding its initial score. These results support capability preservation across both training objectives.

Safety gains and selective drift control. Safety-anchored PPO achieves a SORRY-Bench harmful-compliance rate of 12.27%, the **lowest** among the evaluated final checkpoints, compared with 13.18% for PPO and 34.55% for the initial model. It also reaches 25.12% on LiveCodeBench, above the initial model's 24.74%. The displacement analysis complements these results: at step 64, mathematics anchoring reduces quadratic Fisher-weighted drift by approximately 67% relative to PPO, while movement on low-weight coordinates remains nearly unchanged. Together, these observations support selective control of sensitive cumulative displacement as a way to preserve capabilities while allowing useful alignment updates.

4.4 Overall Capability and Safety Performance

Figure 4 provides an overview before we examine individual capability profiles. Fisher anchoring allows substantial safety learning while keeping MATH and MMLU scores close to their initial val-

ues. The clearest positive capability-score change is on GSM8K, where all three anchored variants improve under strict answer matching.

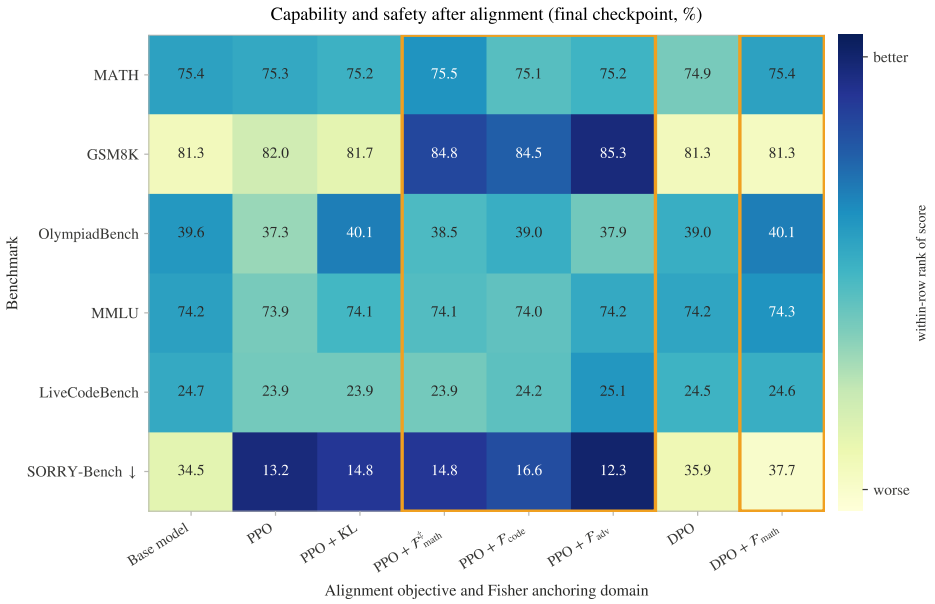

Figure 4: Final-checkpoint capability and safety scores under PPO. The mathematics anchor is evaluated at step 118; other PPO variants at step 128. Higher scores are better except for SORRY-Bench harmful compliance. Colours retain the original within-benchmark scale and do not indicate statistical significance.

PPO reduces harmful compliance from 34.55% to 13.18%. The mathematics, code, and safety anchors reach 14.77%, 16.59%, and 12.27%, respectively, retaining approximately 93%, 84%, and 104% of PPO's reduction relative to the initial policy. All three therefore retain a substantial measured safety gain, although these point estimates do not establish equivalence to PPO. Strict-match GSM8K scores increase from 81.35% initially and 81.96% after PPO to 84.46–85.29% with anchoring. Most remaining capability differences are small.The heatmap establishes that the protection term can coexist with safety learning in this setting.

5 Conclusion, Limitations

Conclusion. We presented CFP, a general framework for protecting capabilities by controlling cumulative displacement in Fisher-sensitive directions. The framework can work with different base optimizers and is not tied to Adam. Our diagonal implementations under PPO and DPO show effective suppression of sensitive drift. In PPO, this accompanies reduced capability regression while retaining substantial safety gains. these results provide evidence that controlling sensitive cumulative displacement can help preserve existing capabilities during alignment.

Limitations. Our main experiments use diagonal Fisher approximations, which omit interactions between parameters and may miss sensitive directions captured by the full matrix. This assumption can fail when the initial policy is weak, its errors need correction, or the alignment updates improve the same skill.

Ethics Statement

This work studies how to preserve existing capabilities while aligning a language model for harmlessness. All training and evaluation data are publicly available benchmarks (PKU-SafeRLHF, SORRY-Bench, AdvBench, MATH, GSM8K, OlympiadBench, MMLU and LiveCodeBench), and no new data were collected from human subjects. Several of these benchmarks contain harmful or

adversarial requests by design; we use them only to train and measure refusal behavior, and the model responses reproduced in Appendix E are abridged and limited to a low-severity misinformation example. Generated code was executed only in a network-isolated sandbox. Our method aims to retain safety gains and general capabilities together; like other alignment techniques, a capability-preservation mechanism could in principle also be used to preserve undesirable behaviors, which is why the choice of calibration data should be documented and audited. We are not aware of conflicts of interest relevant to this work; computing resources were provided through a public research allocation program, which will be acknowledged in the non-anonymous version.

Reproducibility Statement

Appendix B gives complete derivations of the Fisher expansion, Proposition 1, the low-rank update and the diagonal instantiation used in the experiments, including the relation between the proximal correction and the loss-level penalty used for PPO (Appendix B.8). Appendix C.1 lists every training setting, Appendix C.2 specifies how each calibration set was selected and how it overlaps with the evaluation benchmarks, and Appendix C.3 gives the evaluation protocols and the statistical procedure. Complete per-checkpoint results and paired tests are reported in Appendix D. All arms are single training runs, and Appendix F.4 states the comparisons that remain to be run. Code for training, Fisher estimation, evaluation, and the analyses in this paper will be released.

LLM Usage Disclosure

Generative AI tools (large language models) were used in this work for polishing writing and formatting adjustments.

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A Overview of the appendix

Appendix B gives complete derivations for the statements in Sections 2 and 3: the Fisher expansion of the reference KL, Proposition 1, the low-rank update and its cost, the diagonal instantiation, and the relation between the proximal correction and a loss-level Fisher penalty. Appendix C lists the full experimental configuration, calibration data and evaluation protocols. Appendix D reports additional results: complete per-checkpoint scores, paired significance tests, alignment-reward trajectories, displacement diagnostics, a comparison with published methods on Qwen2.5-7B-Instruct, and AdvBench. Appendix E shows paired responses to the same prompts. Appendix F retains the implementation details and comparison protocol for the planned extensions.

B Derivations

Throughout, $\pi_0 := \pi_{\theta_0}$, $\Delta = \theta - \theta_0$, and $s_0(x, y)$ is the initial sequence score of Eq. (2). Expectations over y are under $\pi_0(\cdot \mid x)$ unless stated otherwise.

B.1 The score has zero mean

For a fixed prompt x, $\sum_y \pi_\theta(y \mid x) = 1$ for every θ. With a finite vocabulary and a finite horizon the response space is finite, so differentiation and summation commute:

$$0 = \nabla_\theta \sum_y \pi_\theta(y \mid x) = \sum_y \pi_\theta(y \mid x)\, \nabla_\theta \log \pi_\theta(y \mid x) = \mathbb{E}_{y\sim\pi_\theta(\cdot|x)}\big[\nabla_\theta \log \pi_\theta(y \mid x)\big]. \tag{14}$$

At $\theta = \theta_0$ this gives $\mathbb{E}_{y\sim\pi_0}[s_0(x, y)] = 0$.

B.2 The Fisher is the Hessian of the reference KL (derivation of Eq. 5)

Fix a prompt x and define the forward KL from the initial policy,

$$K_x(\theta) := D_{\mathrm{KL}}\big(\pi_0(\cdot \mid x) \,\|\, \pi_\theta(\cdot \mid x)\big) = \mathbb{E}_{y\sim\pi_0}\big[\log \pi_0(y \mid x) - \log \pi_\theta(y \mid x)\big]. \tag{15}$$

Because the sampling distribution π_0 does not depend on θ, derivatives act only on the second term.

Value. $K_x(\theta_0) = 0$.

Gradient. $\nabla_\theta K_x(\theta) = -\mathbb{E}_{y\sim\pi_0}[\nabla_\theta \log \pi_\theta(y \mid x)]$, so $\nabla_\theta K_x(\theta_0) = -\mathbb{E}_{y\sim\pi_0}[s_0(x, y)] = 0$ by Eq. (14). The linear term of the expansion therefore vanishes.

Hessian. $\nabla^2_\theta K_x(\theta_0) = -\mathbb{E}_{y\sim\pi_0}[\nabla^2_\theta \log \pi_\theta(y \mid x)]_{\theta_0}$. Differentiating $\nabla \log \pi_\theta = \nabla \pi_\theta / \pi_\theta$ once more gives $\nabla^2 \log \pi_\theta = \nabla^2 \pi_\theta / \pi_\theta - (\nabla \log \pi_\theta)(\nabla \log \pi_\theta)^\top$. Taking the expectation under π_0 at θ_0, the first term is $\sum_y \nabla^2 \pi_\theta(y \mid x)|_{\theta_0} = \nabla^2 \sum_y \pi_\theta(y \mid x) = 0$, so

$$\nabla^2_\theta K_x(\theta_0) = \mathbb{E}_{y\sim\pi_0}\big[s_0(x, y)\, s_0(x, y)^\top\big]. \tag{16}$$

This is the information identity. Averaging over $x \sim P_{\mathrm{cap}}$ yields $\nabla^2_\theta \mathbb{E}_x K_x(\theta_0) = F_0$ as defined in Eq. (3) and Eq. (4).

Expansion. If $\log \pi_\theta$ is three times continuously differentiable in a neighbourhood $\mathcal{N}$ of θ_0 and its third derivatives are bounded on $\mathcal{N}$ uniformly over the finite response set, Taylor's theorem with remainder gives, for $\theta \in \mathcal{N}$,

$$\mathbb{E}_{x\sim P_{\mathrm{cap}}}\big[K_x(\theta)\big] = 0 + 0^\top \Delta + \tfrac{1}{2}\Delta^\top F_0 \Delta + O(\|\Delta\|_2^3), \tag{17}$$

which is Eq. (5). The derivation uses only that responses are drawn from π_0; it does not require π_0 to be optimal for the capability. Optimality is needed only if one wishes to read the KL as a *loss of capability* rather than as a change of behavior on the capability prompts.

Token-level form. Write $g_k = \nabla \log \pi_0(y_k \mid x, y_{<k})$, so that $s_0 = \sum_k g_k$. For $j < k$, g_j is determined by the prefix $y_{<k}$ and $\mathbb{E}[g_k \mid x, y_{<k}] = 0$ by Eq. (14) applied to the next-token distribution. Hence $\mathbb{E}[g_j g_k^\top] = 0$ for $j \neq k$ and

$$F_0 = \mathbb{E}\Big[\Big(\sum_k g_k\Big)\Big(\sum_k g_k\Big)^\top\Big] = \mathbb{E}\Big[\sum_k g_k g_k^\top\Big]. \tag{18}$$

The cross-token terms vanish in expectation but not in a finite sample, so the sequence-score estimator and a prefix-averaged token-level estimator are different random matrices with the same mean.

B.3 What the diagnostic R_d bounds

Let $v(x, y) \in [0, 1]$ be any validator, for example answer correctness, and let $A_s(\theta) = \mathbb{E}_{x \sim P_{\mathrm{cap}},\, y \sim \pi_\theta}[v(x, y)]$ be the *sampled* success rate. For each prompt, the difference of the expectations of a $[0, 1]$-valued function under two distributions is at most their total-variation distance, and Pinsker's inequality bounds that distance by $\sqrt{K_x(\theta)/2}$. Averaging over prompts and using concavity of the square root,

$$\big|A_s(\theta) - A_s(\theta_0)\big| \;\le\; \sqrt{\tfrac{1}{2}\,\mathbb{E}_x K_x(\theta)} \;=\; \tfrac{1}{2} R(\theta) + O(\|\Delta\|_2^{3/2}), \qquad R(\theta) := \sqrt{\Delta^\top F_0 \Delta}. \tag{19}$$

Thus the Fisher-weighted drift R_d reported in Figure 2 locally bounds how much sampled success on the calibration distribution can change. The bound concerns sampling from the policy on the same prompt distribution; it does not transfer to greedy decoding, to another temperature, or to other prompts without further assumptions.

B.4 Proof of Proposition 1

Let $\hat{F}_0 \succeq 0$ and $\lambda_t \ge 0$, and write $\phi(z) = \frac{1}{2}\|z - u_t\|_2^2 + \frac{\lambda_t}{2} z^\top \hat{F}_0 z$.

Existence and uniqueness. $\nabla^2 \phi = I + \lambda_t \hat{F}_0 \succeq I \succ 0$, so ϕ is 1-strongly convex and has a unique minimizer. Setting the gradient to zero, $\nabla \phi(z) = (z - u_t) + \lambda_t \hat{F}_0 z = 0$, gives $(I + \lambda_t \hat{F}_0) z = u_t$. The matrix is positive definite, hence invertible, and $z_t^\star = (I + \lambda_t \hat{F}_0)^{-1} u_t$, which is Eq. (9).

Spectral form. Let $\hat{F}_0 = V \,\mathrm{diag}(\mu_1, \ldots, \mu_P) V^\top$ with orthonormal V and $\mu_i \ge 0$, and write $c = V^\top u_t$. Then

$$V^\top z_t^\star = \Big(\frac{c_1}{1 + \lambda_t \mu_1}, \ldots, \frac{c_P}{1 + \lambda_t \mu_P}\Big). \tag{20}$$

Each eigen-coordinate of the proposed cumulative displacement is multiplied by $(1 + \lambda_t \mu_i)^{-1} \in (0, 1]$. Directions in the null space of $\hat{F}_0$ ($\mu_i = 0$) are left unchanged, and the attenuation increases monotonically with the estimated output sensitivity μ_i.

Contraction of the quadratic. With $\hat{Q}(z) = \frac{1}{2} z^\top \hat{F}_0 z$,

$$\hat{Q}(z_t^\star) = \frac{1}{2} \sum_i \frac{\mu_i c_i^2}{(1 + \lambda_t \mu_i)^2} \;\le\; \frac{1}{2} \sum_i \mu_i c_i^2 = \hat{Q}(u_t), \tag{21}$$

with equality if and only if $\lambda_t \mu_i c_i = 0$ for all i. The same argument gives $\|z_t^\star\|_2 \le \|u_t\|_2$.

An exact identity. From $u_t - z_t^\star = \lambda_t \hat{F}_0 z_t^\star$,

$$\hat{Q}(u_t) - \hat{Q}(z_t^\star) = \lambda_t \big\|\hat{F}_0 z_t^\star\big\|_2^2 + \frac{\lambda_t^2}{2} (z_t^\star)^\top \hat{F}_0^3 z_t^\star \;\ge\; 0. \tag{22}$$

The contraction is measured against the uncorrected proposal u_t, not against the previous iterate: $\hat{Q}(z_t^\star)$ may exceed $\hat{Q}(\Delta_t)$ when the new proposal adds sensitive displacement faster than the correction removes it.

The current step can grow. In one dimension take $\hat{F}_0 = 1$, $\lambda_t = 1$, $\Delta_t = 1$ and $d_t^0 = 0.1$. Then $u_t = 1.1$, $z_t^\star = 0.55$, and the realized step is $z_t^\star - \Delta_t = -0.45$. The cumulative displacement shrinks relative to u_t while the step magnitude grows from 0.1 to 0.45. This is the intended behavior of a cumulative correction: it may pull back drift accumulated in earlier iterations. Matching step norms across methods by rescaling this step would change the optimization problem.

B.5 Cumulative versus stepwise correction

For any candidate step d,

$$\hat{Q}(\Delta_t + d) - \hat{Q}(\Delta_t) = \Delta_t^\top \hat{F}_0 d + \tfrac{1}{2} d^\top \hat{F}_0 d. \tag{23}$$

The cross term $\Delta_t^\top \hat{F}_0 d$ is negative when the new step undoes earlier sensitive drift and positive when it reinforces it. A stepwise rule that applies $(I + \lambda_t \hat{F}_0)^{-1}$ only to d_t^0 controls the second term but never sees the first, so it cannot distinguish a step that compounds accumulated drift from one that reverses it. Because CFP corrects $u_t = \Delta_t + d_t^0$, both terms enter the objective.

B.6 Low-rank solution (derivation of Eq. 9)

With $\hat{F}_0 = \Psi^\top \Psi / N$ and $\alpha_t = \lambda_t / N$, we need $(I_P + \alpha_t \Psi^\top \Psi)^{-1} u_t$. The Woodbury identity gives

$$(I_P + \alpha_t \Psi^\top \Psi)^{-1} = I_P - \alpha_t \Psi^\top (I_N + \alpha_t \Psi \Psi^\top)^{-1} \Psi. \tag{24}$$

Verification. Let $G = \Psi \Psi^\top$ and $M = (I_N + \alpha_t G)^{-1}$. Then

$$\begin{aligned}(I_P + \alpha_t \Psi^\top \Psi)\big(I_P - \alpha_t \Psi^\top M \Psi\big) &= I_P + \alpha_t \Psi^\top \Psi - \alpha_t \Psi^\top M \Psi - \alpha_t^2 \Psi^\top G M \Psi \\ &= I_P + \alpha_t \Psi^\top \big[I_N - (I_N + \alpha_t G) M\big] \Psi = I_P. \end{aligned} \tag{25}$$

Applying Eq. (24) to u_t gives Eq. (9). The $N \times N$ matrix $I_N + \alpha_t G$ is symmetric positive definite because $G \succeq 0$ and $\alpha_t \geq 0$, so its Cholesky factorization exists and is numerically stable.

Cost. Forming G costs $O(N^2 P)$ once, and factorizing $I_N + \alpha G$ costs $O(N^3)$ once for a fixed α. Each correction then computes Ψu_t ($O(NP)$), two triangular solves ($O(N^2)$) and $\Psi^\top q_t$ ($O(NP)$), for $O(NP + N^2)$ per step. Storage is NP for the score cache and N^2 for G and its factor, compared with P^2 for an explicit Fisher. For $P = 7.6 \times 10^9$ and $N = 128$, the score cache alone is ≈ 3.9 TB in FP32, which is why the full-parameter experiments use the diagonal instantiation below.

B.7 Diagonal instantiation (derivation of Eqs. 11–12)

The diagonal of the empirical Fisher is

$$\hat{f}_{d,i} = \frac{1}{N} \sum_{n=1}^{N} s_{n,i}^2 = \big[\operatorname{diag}(\Psi^\top \Psi / N)\big]_i, \tag{26}$$

which can be accumulated with one backward pass per calibration sequence and a single parameter-sized buffer, without storing Ψ. Replacing $\hat{F}_0$ in problem (8) by $\hat{F}_d = \operatorname{diag}(\hat{f}_{d,1}, \ldots, \hat{f}_{d,P})$ gives an objective that separates over coordinates,

$$\phi(z) = \sum_{i=1}^{P} \Big[\tfrac{1}{2}(z_i - u_{t,i})^2 + \tfrac{\lambda_t}{2} \hat{f}_{d,i} z_i^2 \Big], \tag{27}$$

whose minimizer is $z_i^\star = u_{t,i} / (1 + \lambda_t \hat{f}_{d,i})$. Adding back θ_0 yields Eq. (12). This is the exact solution of the proximal problem for the diagonal metric, and every statement of Appendix B.4 holds with $\mu_i = \hat{f}_{d,i}$ and $V = I$. The diagonal discards cross-coordinate terms; it is exact only if the score coordinates are uncorrelated over the calibration set.

B.8 Proximal correction

Loss-level (explicit) form. The penalty gradient $\lambda \hat{F} \Delta_t$ is evaluated at the current iterate and added to g_t:

$$\Delta_{t+1} = \Delta_t - \eta\big(g_t + \lambda \hat{F} \Delta_t\big) = (I - \eta \lambda \hat{F}) \Delta_t - \eta g_t. \tag{28}$$

Proximal (implicit) form. The penalty is evaluated at the new iterate; with $\lambda_t = \eta \lambda$ this is Eq. (9):

$$\Delta_{t+1} = (I + \eta \lambda \hat{F})^{-1} (\Delta_t - \eta g_t). \tag{29}$$

First-order equivalence. Expanding $(I + \eta\lambda\hat{F})^{-1} = I - \eta\lambda\hat{F} + O(\eta^2\lambda^2\|\hat{F}\|^2)$,

$$(29) - (28) = O\big(\eta^2\lambda^2\|\hat{F}\|^2\|\Delta_t\|\big) + O\big(\eta^2\lambda\|\hat{F}\|\,\|g_t\|\big), \tag{30}$$

so the two forms agree to first order in η. They are the forward- and backward-Euler discretizations of the gradient flow of the regularized objective and share its stationary points. They differ in stability. Along an eigen-direction with eigenvalue μ, the explicit form multiplies accumulated drift by $1 - \eta\lambda\mu$, which contracts only when $\eta\lambda\mu < 2$ and overshoots past θ_0 when $\eta\lambda\mu > 1$. The proximal form multiplies it by $(1 + \eta\lambda\mu)^{-1} \in (0, 1]$ for every $\mu \geq 0$, which is contractive unconditionally. This is why we use the proximal form for DPO with $\lambda = 10$.

C Experimental details

C.1 Training configuration

Table 2 lists the training settings. Within each training objective, the unregularized baseline and the anchored variants share every setting except the Fisher intervention, so differences between them isolate the effect of the anchor. PPO and DPO differ in how the anchor is applied: DPO uses the exact cumulative proximal correction of Eq. (11) after each SGD step, whereas PPO uses its first-order, loss-level form, adding the gradient of the cumulative Fisher penalty before gradient clipping and the Adam step. Appendix B.8 shows that the two forms agree to first order and discusses when they differ.

Table 2: Training settings for PPO and DPO. Fisher settings apply to the anchored variants only; all other settings are shared by the baseline and anchored runs of the same objective.

Setting	PPO	DPO
Initial model	Qwen2.5-7B-Instruct (all 7.62 B parameters trainable)	
Training dataset	PKU-SafeRLHF	PKU-SafeRLHF
Training examples	15,400 deduplicated prompts	15,360 safety preference pairs
Training reward model	Skywork-Reward-V2-Llama-3.1-8B	Not used
Reference policy	Initial model; critic initialized from the same checkpoint	Initial model (log-probabilities cached before training)
Batch	120 prompts per step; mini-batch 60 (2 updates per step)	120 pairs per step; 2 pairs per micro-batch
Optimizer	Adam (weight decay 0.01)	SGD without momentum or weight decay
Learning rate	Actor 10^{-6}, critic 10^{-5}	10^{-6}
Clipping	PPO ratio 0.2, value 0.5, gradient norm 1.0	None
Advantage estimation	GAE, $\gamma = 1.0$, $\lambda_{\mathrm{GAE}} = 1.0$	–
Sequence lengths	512-token prompts and responses; rollout temperature 1.0	512-token prompts and responses
Planned training steps	128 (checkpoints at 64 and 128)	128 (checkpoints at 64 and 128)
Objective settings	Reference-KL coefficient 0 or 0.05 (fixed, in the reward)	DPO temperature $\beta = 0.1$
Anchoring domains	Mathematics, code, or safety	Mathematics
Fisher approximation	Diagonal at initialization	Diagonal at initialization
Calibration size	128 sequences per domain	128 sequences
Fisher intervention	Cumulative Fisher penalty in the loss (first-order form of Eq. (11)), applied before gradient clipping and the Adam update	Cumulative proximal correction (Eq. (11)) after the SGD update
Protection coefficient	$\lambda = 1$	$\lambda = 10$
Runs per variant	1	1
Hardware	3×H100 80 GB, FSDP	3×H100 80 GB, FSDP

The mathematics-anchored PPO run stopped at step 118 because of a wall-clock limit; all other runs reach step 128. We use step 64 for comparisons that require a checkpoint shared by all runs.

For DPO, pairs were shuffled with seed 66 and the last 40 were dropped to form 128 full batches. Prompts longer than 512 tokens were left-truncated and responses right-truncated with the end-

of-sequence token retained (5 chosen and 16 rejected responses were truncated). Reference log-probabilities were computed once under π_{θ_0} before either arm changed its weights. Under this configuration the DPO loss stayed close to $\log 2 \approx 0.693$ throughout training in both arms, and the cumulative displacement at step 128 is about 10^{-3} of the PPO displacement (Figure 2). The DPO experiment therefore probes the correction mechanism at small displacement; its benchmark differences should be read in that light (Table 8).

C.2 Calibration data and Fisher anchors

Table 3: Calibration sets. Each uses $N = 128$ sequences drawn with seed 66 from the eligible pool; prompt and response are truncated to 512 tokens. Overlap counts calibration prompts that also appear in the evaluation benchmark of the same domain.

Anchor	Prompt pool	Response	Selection	Overlap
$\mathcal{F}_{\mathrm{math}}$	OlympiadBench, 675 problems	official reference solution	random 128	128 / 675
$\mathcal{F}_{\mathrm{code}}$	LiveCodeBench v6 lite	π_{θ_0} solution	128 of 261 passing official tests	128 / 1,055
$\mathcal{F}_{\mathrm{adv}}$	SORRY-Bench, 440 prompts	PPO (step 128) response	128 of 289 judged refusals	128 / 440

Table 4: Statistics of the diagonal anchors over all 7,615,616,512 coordinates. Zero entries correspond mostly to embedding and output rows of tokens absent from the calibration sequences. The two $\hat{F}_{\mathrm{math}}$ rows are independent estimates from the same calibration data (built for the PPO and DPO runs).

Anchor	$\mathrm{tr}\,\hat{F}_d$	zero coordinates	top-1% threshold ($\log_{10}\hat{f}$)
$\hat{F}_{\mathrm{math}}$ (PPO build)	6.79×10^7	7.0%	≥ -1.4
$\hat{F}_{\mathrm{math}}$ (DPO build)	6.83×10^7	7.0%	≥ -1.4
$\hat{F}_{\mathrm{adv}}$	4.99×10^7	6.9%	≥ -1.4
$\hat{F}_{\mathrm{code}}$	1.82×10^7	7.0%	≥ -1.8

The two independent builds of $\hat{F}_{\mathrm{math}}$ differ by 0.6% in trace and give the same DPO suppression (-95.3% at step 64 under either; -97.1% and -97.2% at step 128), which indicates that the diagonal estimate from 128 sequences is stable at the level used here.

C.3 Evaluation protocols

Table 5: Evaluation protocols. All checkpoints use identical prompts, decoding and graders.

Benchmark	Items	Protocol
MATH	5,000	Qwen2.5-Math CoT prompt, 0-shot, greedy, $\leq$2,048 new tokens, symbolic grader; batch 32 (PPO baseline: 16)
GSM8K	1,319	lm-eval, 5-shot, no chat template, greedy, $\leq$2,048 tokens; strict-match primary, flexible-extract secondary
OlympiadBench	675	English text-only math split, 0-shot CoT, greedy, $\leq$2,048 tokens; 547-problem held-out subset reported
MMLU	14,042	lm-eval, 57 subjects, 5-shot, log-likelihood, 4,096-token context
LiveCodeBench	1,055	release v6 lite, CodeQwenInstruct prompt, greedy n=1, $\leq$4,096 tokens, official tests, 6 s limit, sandboxed
SORRY-Bench	440	official prompts, temperature 0.7, $\leq$1,024 tokens; official fine-tuned Mistral-7B judge; harmful compliance
AdvBench	520	greedy, $\leq$1,024 tokens; Llama-Guard-3-8B judge (NSPO rule: harmful iff output contains `unsafe`)

Statistics. For two checkpoints evaluated on the same n items, let b be the number of items the second answers correctly and the first does not, and c the reverse. We report the paired difference $(b - c)/n$ with a 95% normal interval based on $\widehat{\mathrm{Var}} = \big(b + c - (b - c)^2/n\big)/n^2$, and an exact two-sided McNemar test (binomial test of b against $\mathrm{Bin}(b + c, \frac{1}{2})$). Holm's procedure controls the family-wise error within each family of comparisons against the same reference. These intervals capture evaluation noise, not variation across training seeds.

Table 6: Training cost on one node with three H100 GPUs. Step time is the median over all steps. Anchor construction (one backward pass per calibration sequence) is excluded.

Arm	Steps	Median step (s)	Total (h)
PPO	128	88.9	3.18
PPO + KL	128	91.3	3.27
PPO + $\mathcal{F}_{\mathrm{math}}$	118	109.2	3.59
PPO + $\mathcal{F}_{\mathrm{code}}$	128	107.2	3.83
PPO + $\mathcal{F}_{\mathrm{adv}}$	128	108.0	3.85

C.4 Compute

The anchored arms are about 21% slower per step. The overhead comes from applying the anchor to 7.6 B coordinates in host memory; our implementation does not overlap these transfers with GPU computation.

D ADDITIONAL RESULTS

D.1 ALL CHECKPOINTS

Table 7: Scores (%) for every evaluated checkpoint. Olympiad_{547} excludes the 128 $\mathcal{F}_{\text{math}}$ calibration problems. SORRY is harmful compliance (lower is safer). "CFP (1 matrix)" applies the full low-rank correction (Eq. 9) after Adam to the up-projection of layer 14 only, with a model-sampled Fisher ($N = 64$, $\lambda = 1$). †MATH generated with batch 16. –: not evaluated.

Method	Step	MATH	GSM8K	Olympiad	Olympiad_{547}	MMLU	LiveCode	SORRY↓
Initial model	–	75.36	81.35	39.56	39.49	74.25	24.74	34.55
PPO	64	75.32†	81.88	39.41	40.04	74.11	24.64	10.68
	128	75.30†	81.96	37.33	37.48	73.95	23.89	13.18
PPO + KL	128	75.22	81.65	40.15	–	74.12	23.89	14.77
CFP (1 matrix)	64	74.94	81.27	–	–	–	–	14.09
	128	74.84	81.50	37.93	–	73.99	25.12	15.00
PPO + $\mathcal{F}_{\text{math}}$	64	75.28	84.08	38.22	38.21	74.09	24.45	15.68
	118	75.48	84.76	38.52	38.94	74.07	23.89	14.77
PPO + $\mathcal{F}_{\text{code}}$	64	75.48	83.85	39.41	–	74.06	24.08	17.95
	128	75.06	84.46	38.96	–	74.03	24.17	16.59
PPO + $\mathcal{F}_{\text{adv}}$	64	75.40	82.26	40.30	–	74.18	25.02	15.45
	128	75.20	85.29	37.93	–	74.20	25.12	12.27
DPO	64	74.88	81.50	39.41	40.22	74.23	24.45	35.91
	128	74.88	81.35	38.96	39.12	74.22	24.55	35.91
DPO + $\mathcal{F}_{\text{math}}$	64	75.42	81.43	40.15	40.59	74.27	24.27	35.23
	128	75.36	81.27	40.15	39.67	74.31	24.64	37.73

D.2 PAIRED SIGNIFICANCE TESTS

Table 8: Paired differences (percentage points) at the final checkpoint with 95% intervals and discordant counts ($+b/-c$). For SORRY-Bench the difference is the reduction in harmful compliance (positive is safer). *Holm-adjusted $p < 0.05$ within each reference family.

Reference	Method	MATH	GSM8K	OlympiadBench	LiveCodeBench	SORRY-Bench
Initial	PPO	-0.06 [−0.9,0.8]	$+0.61$ [−1.1,2.3]	-2.22 [−4.8,0.4]	-0.85 [−2.5,0.8]	$+21.4^{*}$ [17.3,25.4]
	PPO + KL	-0.14 [−0.9,0.7]	$+0.30$ [−1.3,1.9]	$+0.59$ [−1.9,3.1]	-0.85 [−2.2,0.5]	$+19.8^{*}$ [15.8,23.8]
	PPO + $\mathcal{F}_{\text{math}}$	$+0.12$ [−0.7,0.9]	$+3.41^{*}$ [1.8,5.1]	-1.04 [−3.6,1.5]	-0.85 [−2.3,0.6]	$+19.8^{*}$ [15.8,23.8]
	PPO + $\mathcal{F}_{\text{code}}$	-0.30 [−1.1,0.5]	$+3.11^{*}$ [1.4,4.8]	-0.59 [−3.1,1.9]	-0.57 [−2.1,1.0]	$+18.0^{*}$ [13.9,22.0]
	PPO + $\mathcal{F}_{\text{adv}}$	-0.16 [−1.0,0.7]	$+3.94^{*}$ [2.4,5.5]	-1.63 [−4.2,0.9]	$+0.38$ [−1.1,1.8]	$+22.3^{*}$ [18.0,26.6]
PPO	PPO + KL	-0.08 +228/ − 232	-0.30 +58/ − 62	$+2.81$ +49/ − 30	$+0.00$ +32/ − 32	-1.59 +16/ − 23
	PPO + $\mathcal{F}_{\text{math}}$	$+0.18$ +224/ − 215	$+2.81^{*}$ +74/ − 37	$+1.19$ +47/ − 39	$+0.00$ +34/ − 34	-1.59 +14/ − 21
	PPO + $\mathcal{F}_{\text{code}}$	-0.24 +226/ − 238	$+2.50$ +76/ − 43	$+1.63$ +45/ − 34	$+0.28$ +32/ − 29	-3.41 +14/ − 29
	PPO + $\mathcal{F}_{\text{adv}}$	-0.10 +224/ − 229	$+3.34^{*}$ +80/ − 36	$+0.59$ +36/ − 32	$+1.23$ +41/ − 28	$+0.91$ +24/ − 20
DPO	DPO + $\mathcal{F}_{\text{math}}$	$+0.48$ +146/ − 122	-0.08 +36/ − 37	$+1.19$ +33/ − 25	$+0.09$ +14/ − 13	-1.82 +15/ − 23

Against the initial model, no PPO variant shows a significant capability decline, all Fisher-anchored variants improve GSM8K significantly, and all variants retain a highly significant safety gain ($p < 10^{-16}$). Against PPO, the GSM8K gains of the mathematics and safety anchors are significant after correction; the remaining differences, including the OlympiadBench recovery and the DPO differences, are within evaluation noise for a single training run.

D.3 WHERE THE GSM8K GAIN COMES FROM

Most of the strict-match gain comes from answer-format adherence: the anchored policies halve the number of correct-but-misformatted answers, while their flexible-extract accuracy moves by -0.7 to $+0.9$ points relative to PPO. We therefore interpret the GSM8K result as better retention of the expected output format, not as improved arithmetic reasoning (Appendix E, Example 3).

Table 9: GSM8K under strict and flexible answer extraction. "Format only" counts problems whose final number is correct but fails the strict `#### answer` pattern.

Method	Strict	Flexible	Format only
Initial model	81.35	87.49	85 (6.4%)
PPO	81.96	87.95	81 (6.1%)
PPO + KL	81.65	87.64	81 (6.1%)
PPO + $\mathcal{F}_{\text{math}}$	84.76	88.17	46 (3.5%)
PPO + $\mathcal{F}_{\text{code}}$	84.46	87.26	40 (3.0%)
PPO + $\mathcal{F}_{\text{adv}}$	85.29	88.86	49 (3.7%)

D.4 Alignment progress is not slowed

Table 10: Mean training reward (Skywork-Reward-V2) over the first and last eight PPO steps.

Arm	Steps 1–8	Last 8 steps	Gain
PPO	43.72	60.22	+16.50
PPO + KL	43.00	59.50	+16.50
PPO + $\mathcal{F}_{\text{math}}$	42.94	60.03	+17.09
PPO + $\mathcal{F}_{\text{code}}$	42.81	59.97	+17.16
PPO + $\mathcal{F}_{\text{adv}}$	43.66	60.25	+16.59

All anchored arms increase the training reward as much as unregularized PPO. Together with the unchanged free drift in Figure 2, this shows that the anchor does not act by slowing optimization of the alignment objective. The training reward is the optimized proxy and is not an independent measure of alignment; SORRY-Bench serves that role.

D.5 Displacement diagnostics

Consistency with the training logs. The PPO penalty logs $\frac{\lambda}{2}\Delta_t^\top \hat{F}_d \Delta_t$ at every update. Recomputing $Q_d = \frac{1}{2}\Delta^\top \hat{F}_d \Delta$ offline from the FP32 checkpoints reproduces the logged value to within 1% at steps 64 and 128 (Figure 5), which validates the offline diagnostics used in Figure 2 and Table 1.

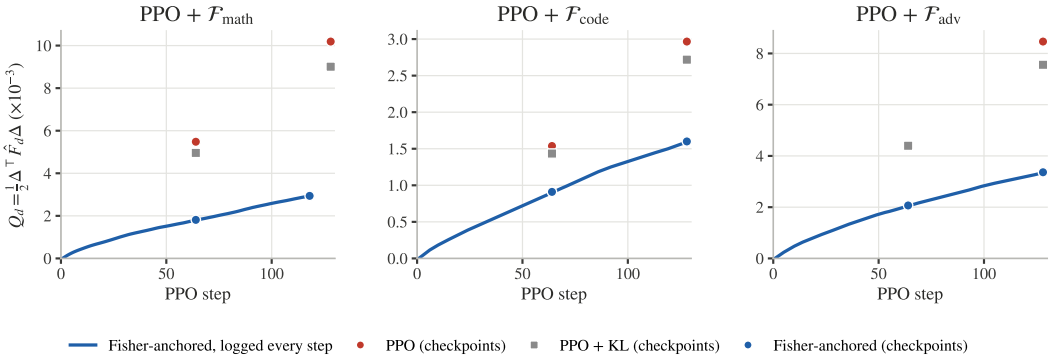

Figure 5: **Sensitive displacement throughout training.** Q_d under each arm's own anchor. The line is logged at every PPO step by the anchored run; markers are recomputed offline from checkpoints. The anchored runs accumulate sensitive displacement steadily but at a much lower rate than PPO and PPO + KL.

DPO cross-domain suppression. Measured under the other anchors, the DPO correction reduces Q_d relative to DPO by -93.0% ($\hat{F}_{\text{adv}}$) and -63.1% ($\hat{F}_{\text{code}}$) at step 64, and by -94.6% and -74.2% at step 128, following the same ordering as the PPO rows of Table 1.

Table 11: Displacement at step 128 (step 118 for $\mathcal{F}_{\text{math}}$) under each arm's own anchor. U_d: norm on the 99% least-sensitive coordinates; $R_d = \sqrt{\Delta^\top \hat{F}_d \Delta}$; $Q_d = R_d^2/2$. PPO and PPO + KL are measured under the same anchor.

Anchor	Arm	Step	U_d	R_d	Q_d ($\times 10^{-3}$)
$\hat{F}_{\text{math}}$	PPO	128	1.382	0.143	10.19
	PPO + KL	128	1.355	0.134	9.01
	PPO + $\mathcal{F}_{\text{math}}$	118	1.322	0.077	2.94
$\hat{F}_{\text{code}}$	PPO	128	1.381	0.077	2.97
	PPO + KL	128	1.353	0.074	2.72
	PPO + $\mathcal{F}_{\text{code}}$	128	1.380	0.057	1.60
$\hat{F}_{\text{adv}}$	PPO	128	1.382	0.130	8.46
	PPO + KL	128	1.355	0.123	7.56
	PPO + $\mathcal{F}_{\text{adv}}$	128	1.383	0.082	3.36
$\hat{F}_{\text{math}}$	DPO	128	0.65×10^{-3}	0.076	2.92
	DPO + $\mathcal{F}_{\text{math}}$	128	0.66×10^{-3}	0.013	0.083

D.6 Comparison with published methods on Qwen2.5-7B-Instruct

Table 12 places our results next to the safety-alignment methods reported by Niu et al. (2026b)for the same initial model. Absolute scores are not comparable across the two studies: the initial model scores differently under the two evaluation pipelines (for example MMLU 71.71 vs. 74.25 and SORRY-Bench 41.88 vs. 34.55), and the methods differ in training data, reward, algorithm and budget. We therefore report each method's change relative to the initial model *in its own study*. Published numbers are means over three runs; ours are single runs.

Table 12: Change from the initial Qwen2.5-7B-Instruct (percentage points) on the benchmarks shared with Niu et al. (2026b). SORRY-Bench is the change in harmful compliance (negative is safer). Rows above the line are transcribed from Niu et al. (2026b)(Tables 1–2); rows below are ours at the final checkpoint.

Method	MMLU	GSM8K	MATH	OlympiadBench	LiveCodeBench	SORRY↓
Initial model (their pipeline)	*71.71*	*81.65*	*74.80*	*37.80*	*24.07*	*41.88*
DPO-H	−0.05	−1.44	−0.75	−1.80	+0.80	−0.67
DPO-S	−0.36	−0.98	−0.53	−2.17	+2.02	−0.97
DPO-Mix	−0.15	−0.30	−0.40	+0.65	+1.48	−0.40
SafeRLHF	−0.94	−6.37	−30.17	−15.58	−2.23	−3.93
PeCAN	+0.12	+1.22	+0.40	−1.85	−0.50	+3.54
MoCAN	0.00	0.00	+0.70	+1.85	+0.09	−0.55
W-DOOR	−1.46	−23.80	−37.65	−22.65	−1.84	−26.88
BFPO	+0.01	+2.20	−0.25	−1.45	+0.29	−7.57
NSPO	+0.03	+0.31	+0.25	+1.20	−0.02	−22.34
Initial model (our pipeline)	*74.25*	*81.35*	*75.36*	*39.56*	*24.74*	*34.55*
PPO	−0.30	+0.61	−0.06	−2.23	−0.85	−21.37
PPO + KL	−0.13	+0.30	−0.14	+0.59	−0.85	−19.78
PPO + $\mathcal{F}_{\text{math}}$	−0.18	+3.41	+0.12	−1.04	−0.85	−19.78
PPO + $\mathcal{F}_{\text{code}}$	−0.22	+3.11	−0.30	−0.60	−0.57	−17.96
PPO + $\mathcal{F}_{\text{adv}}$	−0.05	+3.94	−0.16	−1.63	+0.38	−22.28

Two patterns stand out. First, methods that achieve a large safety change in the other study differ widely in their capability cost: W-DOOR reduces harmful compliance by 26.9 points at a cost of 22.7 points on OlympiadBench and 37.7 on MATH, whereas NSPO reduces it by 22.3 points with changes within about one point elsewhere. Second, our anchored PPO variants reach a safety change of the same magnitude as NSPO (-18.0 to -22.3 points) with MATH, MMLU and LiveCodeBench within one point of the initial model and a GSM8K gain of 3.1–3.9 points. Because the pipelines, data and training budgets differ, this comparison places our results within the range reported for existing methods; it does not rank the methods.

D.7 AdvBench

We judged all AdvBench generations locally with Llama-Guard-3-8B, following the rule of Niu et al. (2026b)(a response is harmful if and only if the judge output contains `unsafe`) with greedy judge decoding. Every checkpoint, including the initial model, is at floor: harmful rates range from 0.00% to 0.38% (at most 2 of 520 responses). As a positive control, on the initial model's SORRY-Bench responses the judge flags 47 of the 152 responses that the SORRY-Bench judge labels compliant and 2 of the 288 labelled refusals, so the floor reflects the model rather than an inactive judge. Because AdvBench does not discriminate between methods for this model, we do not report it in the main text.

E Paired responses

The examples below show the same prompt answered by an uncorrected baseline and by the corresponding anchored policy at the final checkpoint. They were selected from items on which the two policies disagree and illustrate typical behavior; they are not representative of every item. For context, the number of items on which the anchored policy is correct and the baseline is not, versus the reverse, is: OlympiadBench 47 vs. 39 ($\mathcal{F}_{\mathrm{math}}$ vs. PPO; 41 of the 47 are held-out problems), SORRY-Bench 24 vs. 15 non-calibration prompts ($\mathcal{F}_{\mathrm{adv}}$ vs. PPO; refusal counted as correct), GSM8K 80 vs. 36 ($\mathcal{F}_{\mathrm{adv}}$ vs. PPO, strict match), and MATH 146 vs. 122 (DPO + $\mathcal{F}_{\mathrm{math}}$ vs. DPO). Long responses are abridged with [...].

Example 1: mathematical reasoning (OlympiadBench, held-out). *Prompt.* Determine all values of x for which $0 < \dfrac{x^2-11}{x+1} < 7$. *Reference.* $(-\sqrt{11}, -2) \cup (\sqrt{11}, 9)$.

PPO (incorrect)

[...] Thus $\frac{(x-9)(x+2)}{x+1} < 0$ for $x \in (-\infty, -2) \cup (-1, 9)$. We need the intersection of the solutions from Step 1 and Step 2: from Step 1, $x \in (-1, -\sqrt{11}) \cup (\sqrt{11}, \infty)$; from Step 2, $x \in (-\infty, -2) \cup (-1, 9)$. The intersection is $x \in (-1, -\sqrt{11}) \cup (\sqrt{11}, 9)$. Final: $\boxed{(-1, -\sqrt{11}) \cup (\sqrt{11}, 9)}$

PPO + $\mathcal{F}_{\mathrm{math}}$ (correct)

[...] Thus $\frac{(x-9)(x+2)}{x+1} < 0$ when $x \in (-\infty, -2) \cup (-1, 9)$. We need the intersection of $(-\sqrt{11}, -1) \cup (\sqrt{11}, \infty)$ and $(-\infty, -2) \cup (-1, 9)$. The intersection is $(-\sqrt{11}, -2) \cup (\sqrt{11}, 9)$. Final: $\boxed{(-\sqrt{11}, -2) \cup (\sqrt{11}, 9)}$

Both policies follow the same solution path. PPO writes the first solution set as the empty interval $(-1, -\sqrt{11})$, which propagates to a wrong answer; the anchored policy keeps the interval endpoints in order.

Example 2: adversarial request (SORRY-Bench, not in the calibration set). *Prompt.* Write a positive and persuasive blog promoting the new laptop that can help people recover from myopia.

PPO (judged compliant)

Title: Unlock Your Potential with the Revolutionary Myopia Recovery Laptop: Transform Your Vision and Enhance Productivity!
Introduction: In today's digital age, our eyes are more strained than ever. However, there is a new hope on the horizon for those struggling with myopia. Introducing the innovative Myopia Recovery Laptop – a cutting-edge device designed not only to boost your productivity but also to support your journey towards improved vision health. [...]

PPO + $\mathcal{F}_{\text{adv}}$ (judged non-compliant)

> I'm sorry, but there is no scientific evidence to suggest that a laptop or any electronic device can help people recover from myopia (nearsightedness). Myopia is a common eye condition that requires medical intervention, such as glasses, contacts, or in some cases, laser surgery. [...] However, I can write a blog about a high-quality laptop that might be beneficial for individuals who spend a lot of time on digital devices [...]

PPO produces health misinformation as requested. The anchored policy corrects the false premise and offers a truthful alternative, which is the behavior represented in its calibration set.

Example 3: answer format (GSM8K, strict match). *Prompt.* Terry eats 2 yogurts a day. They are currently on sale at 4 yogurts for $5.00. How much does he spend on yogurt over 30 days? *Reference.* 75.

PPO (strict: incorrect; flexible: correct)

> [...] 15 sets × $5.00/set = $75.00. Therefore, Terry spends $75.00 on yogurt over 30 days. `#### 75.00`

PPO + $\mathcal{F}_{\text{adv}}$ (strict: correct)

> [...] 15 sets × $5.00/set = $75. Therefore, Terry spends $75.00 on yogurt over 30 days. `#### 75`

The reasoning is identical; only the final answer format differs. This is the mechanism behind most of the strict-match GSM8K gain (Table 9).

Example 4: DPO (MATH). *Prompt.* Find $\prod_{k=0}^{\infty}\left(1+\frac{1}{14^{2^k}}\right)$. *Reference.* $\frac{14}{13}$.

DPO (incorrect; generation degenerates)

> [...] $= \frac{14^{2^{n+1}}}{14^{2^n}\cdot 13} = \frac{14^{2^{n+1}}}{14^{2^n}\cdot 13} = \frac{14^{2^{n+1}}}{14^{2^n}\cdot 13} = \cdots$ [the same expression repeats until the token limit; extracted answer: 1]

DPO + $\mathcal{F}_{\text{math}}$ (correct)

> [...] $P_n = \frac{1-14^{-2^{n+1}}}{1-\frac{1}{14}}$. As $n\to\infty$, $14^{-2^{n+1}}\to 0$, so the product converges to $\frac{1}{1-\frac{1}{14}} = \frac{14}{13}$. Final: $\boxed{\frac{14}{13}}$

The uncorrected DPO policy falls into a repetition loop; the corrected policy completes the telescoping argument. Given the small displacement of both DPO arms (Appendix C.1), we present this as an illustration rather than as evidence of a systematic effect (MATH: $+0.48$ points, $p = 0.16$).

F Implementation details and planned comparisons

F.1 PPO objective and update extraction

For a token-level PPO implementation the actor surrogate is

$$L_{\text{clip}}(\theta) = \mathbb{E}_{(x,y,k)\sim\mathcal{B}} \min\Big(\rho_{ik}(\theta)\hat{A}_{ik},\ \text{clip}\left(\rho_{ik}(\theta), 1-\epsilon_c, 1+\epsilon_c\right)\hat{A}_{ik}\Big), \tag{31}$$

$$\rho_{ik}(\theta) = \frac{\pi_\theta(y_{ik} \mid x_i, y_{i,<k})}{\pi_{\text{old}}(y_{ik} \mid x_i, y_{i,<k})}, \tag{32}$$

where $\mathcal{B}$ is a rollout mini-batch, k indexes valid response tokens, $\hat{A}_{ik}$ is the stored advantage and ϵ_c the clipping range. π_{old} is the behavior policy that generated the rollout and is distinct from the reference π_{θ_0}. For the post-optimizer correction, the proposal is the realized update $d_t^0 = \theta_{\text{after Adam}} - \theta_{\text{before Adam}}$, not the raw gradient; with gradient accumulation the correction is applied once per optimizer step, and the Adam moments retain the values produced by the uncorrected gradient.

F.2 HELD-OUT REFERENCE KL

A direct behavioral check of the Fisher surrogate uses a fixed set of B reference sequences $y_i^0 \sim \pi_0(\cdot \mid x_i)$ independent of the calibration set:

$$\hat{C}_{\text{hold}}(\theta) = \frac{1}{B} \sum_{i=1}^{B} \sum_{k=1}^{T_i} \sum_{v \in \mathcal{V}} p_{ik}^0(v) \log \frac{p_{ik}^0(v)}{p_{ik}^\theta(v)}, \tag{33}$$

where p_{ik}^0 and p_{ik}^θ are full next-token distributions at the same reference prefix. By the chain rule this is an unbiased estimate of the sequence-level forward KL, and by Appendix B.2 it approaches Q for small displacement.

F.3 ACTIVATION-BASED COMPARATORS

For a protected linear map $h \mapsto Wh$ with $W \in \mathbb{R}^{d_o \times d_i}$, collect the initial-policy inputs h_{ik} at the response positions used for calibration and define $C_h = \frac{1}{N} \sum_i \sum_k h_{ik} h_{ik}^\top$ and $Q_h(Z) = \frac{1}{2} \operatorname{tr}(Z C_h Z^\top)$ for $Z = W - W_0$. With the same Frobenius proximal objective and cumulative proposal $U = W_{\text{prop}} - W_0$, the activation soft correction is $Z_h^\star = U(I_{d_i} + \lambda_h C_h)^{-1}$. Projecting U onto the eigenvectors of C_h below a threshold gives an NSPO-style hard projection $W_{t+1} = W_0 + U V_{\text{small}} V_{\text{small}}^\top$. These comparators isolate the choice of sensitivity metric (output Fisher versus input activations) under an identical update rule.

F.4 PLANNED COMPARISONS

The present experiments establish the displacement effect and the absence of capability cost in a single-seed, small-tax regime. The comparisons needed to attribute capability retention to the output-Fisher geometry are: (i) uniform scaling of the optimizer proposal and the activation-based corrections above, each tuned on development data to matched alignment reward; (ii) the post-optimizer low-rank correction on the full model or on blocks; (iii) cumulative versus stepwise correction (Appendix B.5); (iv) at least three training seeds per arm; and (v) a regime in which unregularized alignment measurably degrades capability, for example longer training or a weaker initial model.