ADAPTIVE SEMI-SUPERVISED FEDERATED MULTI-VIEW CLUSTERING

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ABSTRACT

Federated multi-view clustering enables collaborative clustering of multi-view data distributed across isolated clients while preserving data privacy. However, existing methods largely rely on unsupervised learning, which may overlook available prior information and consequently limit the discriminability of local representations and the reliability of global consensus. To address this issue, we introduce supervisory information into clustering process and propose Adaptive Semi-Supervised Federated Multi-View Clustering (ASFMVC). Specifically, to reduce the dependence on intrinsic data structure alone, we introduce a label-guided representation generation mechanism that incorporates the observed labels into representation learning. Since different views may provide clustering structures of unequal quality, we further design a consensus-aware adaptive weighting strategy to adjust their contributions according to their discrepancies from the current consensus. Moreover, the same supervision is transformed into signed pairwise relations to guide consensus refinement through must-link and cannot-link constraints. These components are integrated into a unified alternating optimization framework with compact communication and of linear complexity. Experiments on six benchmark datasets demonstrate the effectiveness of the proposed method.

1 INTRODUCTION

Multi-view clustering (MVC) aims to exploit cross-view consistency and complementarity to obtain a unified clustering result (Liu et al., 2026b; Li et al., 2022). Representative MVC methods include matrix factorization, graph/subspace learning, anchor-based large-scale clustering, and contrastive kernel learning (Zhang et al., 2021; Liu et al., 2021; Wang et al., 2022; Wan et al., 2023; Liu et al., 2023). Despite their effectiveness, most conventional MVC methods require centralized access to raw multi-view data, which may be infeasible when different views are held by independent organizations or devices.

Federated multi-view clustering (FMVC) enables collaborative clustering without directly sharing raw data. Existing studies have explored global self-supervision (Chen et al., 2023), factorization-based learning (Feng et al., 2024b), and efficient communication(Liu et al., 2026a), while recent work further considers heterogeneous or incomplete views (Ren et al., 2024; Liu et al., 2025). However, existing methods largely rely on unsupervised or self-supervised information generated from the data themselves, leaving externally available supervisory information insufficiently exploited. For example, medical examinations of the same patients may be distributed across different healthcare institutions, while only a limited portion of cases have expert-provided annotations. Such supervision can provide valuable prior knowledge for clustering but is not explicitly utilized by existing FMVC paradigms.

Limited supervision has been shown to provide useful prior knowledge for clustering. In centralized settings, such information has been exploited through pairwise constraints and supervised multi-view learning (Jia et al., 2023; Chen et al., 2019; Wang et al., 2021; Zhang et al., 2024). However, such information has not been explicitly incorporated into existing FMVC methods. How to effectively exploit limited supervisory information to improve clustering reliability in federated settings without violating data privacy therefore remains a key challenge.

To address these issues, we propose a **Adaptive Semi-Supervised Federated Multi-View Clustering (ASFMVC)** framework. Observed labels are first incorporated into representation learning

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to improve the learned features. The resulting clustering structures are then adaptively weighted according to their discrepancies from the current consensus. Finally, the same supervision is transformed into signed pairwise relations to refine the consensus through must-link and cannot-link constraints. These components are integrated into a unified alternating optimization framework without introducing additional high-dimensional communication variables.

The main contributions of this paper are summarized as follows:

- We address the underexplored problem of exploiting limited external supervisory information in federated multi-view clustering and establish a semi-supervised FMVC framework while preserving local data privacy.
- We propose a dual-level supervision mechanism together with consensus-aware adaptive aggregation. This design improves representation quality and consensus reliability, while adaptively coordinating client contributions according to their consistency with the evolving consensus.
- An efficient alternating optimization procedure is developed with compact communication and linear computational complexity. Extensive experiments on six benchmark datasets demonstrate the effectiveness of the proposed method.

2 Related Work

2.1 Federated Multi-View Clustering

According to the distribution of multi-view data, existing FMVC methods can be broadly categorized into horizontal and vertical settings. For horizontal federated clustering, Corcuera Bárcena et al. (Bárcena et al., 2024) extended c-means and fuzzy c-means to distributed data, while Chen et al. (Chen et al., 2024) investigated deep multi-view fuzzy clustering under horizontal federation. For vertical FMVC, Huang et al. (Huang et al., 2022) developed an efficient federated multi-view learning framework, and Chen et al. (Chen et al., 2023) introduced global self-supervision for deep multi-view clustering. Feng et al. explored tensor and matrix factorization (Feng et al., 2024a), whereas Jiang et al. (Jiang et al., 2024b) considered heterogeneous feature representations. Recent studies further investigate graph consistency (Sun et al., 2025), incomplete or unaligned views (Ren et al., 2024; Chao et al., 2025; Liu et al., 2025), secure collaboration (Chen et al., 2025), and efficient communication(Liu et al., 2026a).

2.2 Semi-Supervised Multi-View Clustering

Semi-supervised multi-view clustering exploits limited supervisory information to guide clustering. Günnemann et al. (Günnemann et al., 2014) incorporated supervision into multi-view subspace projections, while Whang et al. (Whang et al., 2020) investigated semi-supervised clustering over multiple hypergraphs. Chen et al. (Chen et al., 2019) developed constrained spectral clustering, and Wang et al. (Wang et al., 2021) introduced weighted anchor graph embedding. Supervision has also been incorporated into structured subspace and tensor representation learning (Qin et al., 2022; Tang et al., 2022), deep multi-view clustering (Chen et al., 2022), and fuzzy or factorization-based models (P. Branco & de A. T. de Carvalho, 2023; Peng et al., 2023; Cui et al., 2024; Jiang et al., 2024a). More recently, side-constrained graph fusion (Zhang et al., 2024), active constraint selection (Zhang et al., 2025), pairwise-constrained scalable clustering (Yu et al., 2025), and adaptive semi-supervised matrix factorization (Mohammadi et al., 2025) have been investigated.

3 Method

Figure 1 illustrates the overall framework of ASFMVC. The proposed method proceeds in three stages. First, each client learns a local representation from its private view under the guidance of limited observed labels. The learned representation is then approximated by a pseudo label and a centroid matrix, which are transmitted to the server instead of the original representation. Second, the server adaptively coordinates the uploaded clustering structures according to their consistency

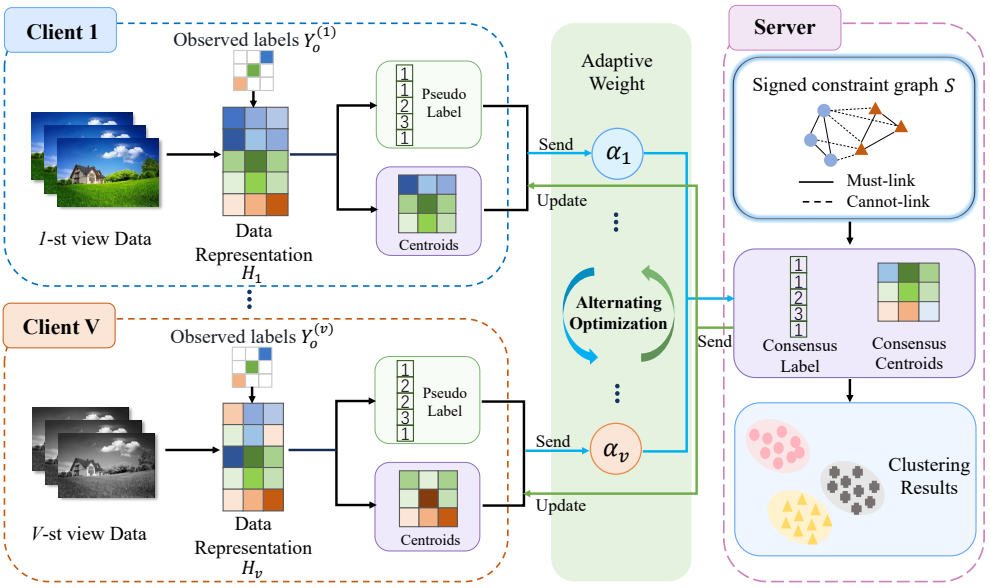


Figure 1: Framework of the proposed algorithm. It consists of V clients and a unique server, where each client privately holds one view of the multi-view data and exchanges only compact clustering information with the server. Limited supervision is incorporated into representation learning and consensus refinement, while adaptive weighting coordinates the contributions of different clients.

with the current consensus. Third, the available supervision is further transformed into signed pairwise constraints to refine the consensus labels through must-link and cannot-link relations, and the final clustering results are determined by the consensus label.

3.1 Formulation

Given the multi-view data $\{\mathbf{X}_v\}_{v=1}^V$ distributed across V isolated clients, we consider the vertical federated multi-view setting, where the v-th client privately holds the v-th view $\mathbf{X}_v \in \mathbb{R}^{N\times D_v}$ of samples. Let k denote the number of clusters. Each client learns a local representation $\mathbf{H}_v \in \mathbb{R}^{N\times k}$, which is approximated as $\mathbf{H}_v \approx \mathbf{Y}_v\mathbf{C}_v$, where $\mathbf{Y}_v \in \{0,1\}^{N\times k}$ is the discrete pseudo-label matrix and $\mathbf{C}_v \in \mathbb{R}^{k\times k}$ is the local centroid matrix. Each row of $\mathbf{Y}_v$ contains exactly one nonzero entry, and $\mathbf{C}_v$ is initialized before federated optimization and remains fixed thereafter. On the server, $\mathbf{Y} \in \{0,1\}^{N\times k}$ denotes the consensus label matrix and $\mathbf{C} \in \mathbb{R}^{k\times Vk}$ denotes the global consensus centroid matrix. We horizontally divide $\mathbf{C}$ into V blocks as

$$\mathbf{C} = [\bar{\mathbf{C}}_1, \bar{\mathbf{C}}_2, \ldots, \bar{\mathbf{C}}_V], \tag{1}$$

where $\bar{\mathbf{C}}_v \in \mathbb{R}^{k\times k}$ corresponds to the v-th client.

Label-Guided Local Representation. For the v-th client, the local representation $\mathbf{H}_v$ is first learned from the raw view $\mathbf{X}_v$. Specifically, the term $\mathrm{Tr}(\mathbf{H}_v^\top \mathbf{X}_v \mathbf{X}_v^\top \mathbf{H}_v)$ preserves the dominant structure of the local data under $\mathbf{H}_v^\top \mathbf{H}_v = \mathbf{I}_k$. To incorporate the available supervision into this representation learning process, let Ω_l denote the index set of labeled samples. We construct a sparse observed-label matrix $\mathbf{Y}_o \in \{0,1\}^{N\times k}$ as

$$(\mathbf{Y}_o)_{ir} = \begin{cases} 1, & i \in \Omega_l \text{ and sample } i \text{ belongs to class } r, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Based on $\mathbf{Y}_o$, we introduce the label-guided representation term

$$\mathcal{J}_{\mathrm{LGR}} = \sum_{v=1}^{V} \mathrm{Tr}(\mathbf{H}_v^\top \mathbf{Y}_o \mathbf{Y}_o^\top \mathbf{H}_v). \tag{3}$$

Since

$$\mathrm{Tr}(\mathbf{H}_v^\top \mathbf{Y}_o \mathbf{Y}_o^\top \mathbf{H}_v) = \|\mathbf{Y}_o^\top \mathbf{H}_v\|_F^2, \tag{4}$$

With a better representation $\mathbf{H}_v$, the local pseudo-label matrix $\mathbf{Y}_v$ is further coupled with the local centroid matrix $\mathbf{C}_v$ through $\mathrm{Tr}(\mathbf{H}_v^\top \mathbf{Y}_v \mathbf{C}_v)$, thereby forming the local clustering structure $\mathbf{Y}_v\mathbf{C}_v$ for subsequent federated aggregation.

Consensus-Aware Adaptive Weighting. Different clients may exhibit different degrees of agreement with the current global consensus. For the v-th client, we measure

$$\alpha_v = \frac{1}{2\sqrt{\|\mathbf{Y}_v\mathbf{C}_v - \mathbf{Y}\bar{\mathbf{C}}_v\|_F^2 + \epsilon}} \tag{5}$$

where ϵ is a small positive constant. The reconstructed representation concatenation of all clients on the server is defined as

$$\mathbf{H}_c^\alpha = [\sqrt{\alpha_1}\mathbf{Y}_1\mathbf{C}_1,\ \sqrt{\alpha_2}\mathbf{Y}_2\mathbf{C}_2,\ \ldots,\ \sqrt{\alpha_v}\mathbf{Y}_V\mathbf{C}_V]. \tag{6}$$

The consensus label matrix $\mathbf{Y}$ and the consensus centroid matrix$\mathbf{C}$ is then encouraged to align with $\mathbf{H}_c^\alpha$ through the weighted alignment term

$$\mathcal{J}_{\mathrm{con}} = \mathrm{Tr}\left[(\mathbf{H}_c^\alpha)^\top \mathbf{Y}\mathbf{C}\right] = \sum_{v=1}^{V} \sqrt{\alpha_v}\,\mathrm{Tr}\left[(\mathbf{Y}_v\mathbf{C}_v)^\top \mathbf{Y}\bar{\mathbf{C}}_v\right]. \tag{7}$$

Signed Constraint-Guided Consensus. The labeled sample set used here is exactly the same Ω_l employed to construct $\mathbf{Y}_o$. Let $\mathcal{E}$ denote the queried sample pairs among the labeled samples. The initial signed constraint graph $\mathbf{S}^{(0)} \in \mathbb{R}^{N\times N}$ is defined as

$$S_{ij}^{(0)} = \begin{cases} +1, & (i,j)\in\mathcal{E},\ l_i = l_j, \\ -1, & (i,j)\in\mathcal{E},\ l_i \neq l_j, \\ 0, & \text{otherwise}, \end{cases} \tag{8}$$

where l_i denotes the observed label of sample i. A positive value indicates a must-link relation and a negative value indicates a cannot-link relation.

Since the numbers of must-link and cannot-link relations may be imbalanced, directly using $\mathbf{S}^{(0)}$ may cause one type of constraint to dominate the other. Let m_+ and m_- denote the numbers of must-link and cannot-link pairs, respectively. When both types are available, their balancing coefficients are defined as

$$s_+ = \frac{m_+ + m_-}{2m_+}, \qquad s_- = \frac{m_+ + m_-}{2m_-}. \tag{9}$$

Accordingly, the balanced signed graph $\widetilde{\mathbf{S}}$ is given by

$$\widetilde{S}_{ij} = \begin{cases} +s_+, & S_{ij}^{(0)} > 0, \\ -s_-, & S_{ij}^{(0)} < 0, \\ 0, & \text{otherwise}. \end{cases} \tag{10}$$

We further apply symmetric absolute-degree normalization. Specifically, let $\mathbf{D}$ denote the diagonal absolute-degree matrix. Then,

$$D_{ii} = \sum_{j=1}^{N} |\widetilde{S}_{ij}|, \qquad \mathbf{S} = \mathbf{D}^{-\frac{1}{2}}\widetilde{\mathbf{S}}\mathbf{D}^{-\frac{1}{2}}. \tag{11}$$

The server-side semi-supervised objective is formulated as

$$\mathcal{J}_{\mathrm{sup}} = \mathrm{Tr}\left(\mathbf{Y}^\top \mathbf{S}\mathbf{Y}\right). \tag{12}$$

Since $\mathbf{Y}$ is a one-hot cluster indicator matrix,

$$\mathrm{Tr}\left(\mathbf{Y}^\top \mathbf{S}\mathbf{Y}\right) = \sum_{i,j} S_{ij}\,\mathbb{I}(y_i = y_j). \tag{13}$$

Unified Formulation. The proposed model is formulated as

$$\begin{aligned}\max_{\mathbf{H}_v,\mathbf{Y}_v,\mathbf{Y},\mathbf{C}} &\sum_{v=1}^{V}\mathrm{Tr}\big(\mathbf{H}_v^\top\mathbf{X}_v\mathbf{X}_v^\top\mathbf{H}_v\big)+\eta\sum_{v=1}^{V}\mathrm{Tr}\big(\mathbf{H}_v^\top\mathbf{Y}_o\mathbf{Y}_o^\top\mathbf{H}_v\big)\\ &+\lambda\sum_{v=1}^{V}\mathrm{Tr}\big(\mathbf{H}_v^\top\mathbf{Y}_v\mathbf{C}_v\big)+\beta\sum_{v=1}^{V}\sqrt{\alpha_v}\mathrm{Tr}\big[(\mathbf{Y}_v\mathbf{C}_v)^\top\mathbf{Y}\bar{\mathbf{C}}_v\big]+\gamma\mathrm{Tr}\big(\mathbf{Y}^\top\mathbf{S}\mathbf{Y}\big),\\ \text{s.t.}\quad &\mathbf{H}_v^\top\mathbf{H}_v=\mathbf{I}_k,\ \mathbf{C}\mathbf{C}^\top=\mathbf{I}_k,\ \mathbf{Y}_v\in\{0,1\}^{N\times k},\ \mathbf{Y}\in\{0,1\}^{N\times k}.\end{aligned}\tag{14}$$

in which η, λ, β, and γ are the trade-off parameters of label-guided representation ,local representation approximation, federated consensus learning and signed supervisory guidance.

3.2 Optimization

We solve Eq. (14) by alternating optimization. Initially, each $\mathbf{H}_v^{(0)}$ is obtained from the leading k left singular vectors of $\mathbf{X}_v$. The local representations are then horizontally concatenated as

$$\mathbf{H}_c^{(0)}=[\mathbf{H}_1^{(0)},\mathbf{H}_2^{(0)},\ldots,\mathbf{H}_V^{(0)}].\tag{15}$$

The server performs k-means on $\mathbf{H}_c^{(0)}$ to obtain the initial consensus labels $\mathbf{Y}^{(0)}$ and centroid matrix $\widehat{\mathbf{C}}^{(0)}$. Let $\mathbf{U}^{(0)}\mathbf{\Sigma}^{(0)}\mathbf{V}^{(0)\top}=\widehat{\mathbf{C}}^{(0)}$ be its SVD. The global centroid matrix is initialized by

$$\mathbf{C}^{(0)}=\mathbf{U}^{(0)}\mathbf{V}^{(0)\top}.\tag{16}$$

Meanwhile, $\widehat{\mathbf{C}}^{(0)}$ is horizontally partitioned into V blocks to initialize the local centroid matrices $\{\mathbf{C}_v\}_{v=1}^V$, and $\mathbf{Y}_v^{(0)}=\mathbf{Y}^{(0)}$ is set for all clients.

Client-Side Update. On each client, $\mathbf{H}_v$ and $\mathbf{Y}_v$ are updated alternately with the remaining variables fixed.

$\mathbf{H}_v$*-subproblem.* The terms related to $\mathbf{H}_v$ lead to the following fixed-point SVD update:

$$\begin{aligned}&\mathbf{M}_v=\mathbf{X}_v(\mathbf{X}_v^\top\mathbf{H}_v)+\eta\mathbf{Y}_o(\mathbf{Y}_o^\top\mathbf{H}_v)+\lambda\mathbf{Y}_v\mathbf{C}_v,\\ &\mathbf{M}_v=\mathbf{U}_v\mathbf{\Sigma}_v\mathbf{V}_v^\top,\qquad \mathbf{H}_v\leftarrow\mathbf{U}_v\mathbf{V}_v^\top.\end{aligned}\tag{17}$$

$\mathbf{Y}_v$*-subproblem.* Given $\mathbf{H}_v$, define

$$\mathbf{P}_v=\lambda\mathbf{H}_v\mathbf{C}_v^\top+\beta\sqrt{\alpha_v}\,\mathbf{Y}\bar{\mathbf{C}}_v\mathbf{C}_v^\top,\qquad y_{v,i}=\arg\max_r(\mathbf{P}_v)_{ir}.\tag{18}$$

Server-Side Update. The server first constructs the adaptively weighted representation

$$\mathbf{H}_c^{\alpha}=[\sqrt{\alpha_1}\mathbf{Y}_1\mathbf{C}_1,\ldots,\sqrt{\alpha_V}\mathbf{Y}_V\mathbf{C}_V].\tag{19}$$

$\mathbf{C}$*-subproblem.* For fixed $\mathbf{Y}$, let $(\mathbf{H}_c^{\alpha})^\top\mathbf{Y}=\mathbf{U}_c\mathbf{\Sigma}_c\mathbf{V}_c^\top$. The orthogonal Procrustes solution is

$$\mathbf{C}=\mathbf{V}_c\mathbf{U}_c^\top.\tag{20}$$

$\mathbf{Y}$*-subproblem.* For fixed $\mathbf{C}$, define $\mathbf{A}=\beta\mathbf{H}_c^{\alpha}\mathbf{C}^\top$. Since $\mathrm{Tr}(\mathbf{Y}^\top\mathbf{S}\mathbf{Y})=\sum_{i,j}S_{ij}\mathbb{I}(y_i=y_j)$, the consensus label is updated by

$$y_i\leftarrow\arg\max_r\left[A_{ir}+2\gamma\sum_{j=1}^{N}S_{ij}\mathbb{I}(y_j=r)\right].\tag{21}$$

Adaptive Weight Refreshing. After $\{\mathbf{Y}_v\}_{v=1}^V$, $\mathbf{C}$, and $\mathbf{Y}$ are updated, $\{\alpha_v\}_{v=1}^V$ are refreshed according to Eq. (5) and fixed for the next outer iteration.

3.3 Discussion

3.3.1 Computation Complexity

Let $d_\Sigma=\sum_{v=1}^V D_v$, and let I_H and I_S denote the numbers of local fixed-point updates and server coordinate sweeps. The main cost of one outer iteration is $O(I_HNkd_\Sigma+(I_H+1)VNk^2+I_S(Nk+m)+Vk^3)$, where m is the number of queried pairwise constraints. Since $m=O(N)$ under the sparse/capped constraint setting and V,k,I_H,I_S are fixed, the complexity is linear in N.

Algorithm 1 Optimization of ASFMVC

Input: Multi-view data $\{\mathbf{X}_v\}_{v=1}^V$, observed-label matrix $\mathbf{Y}_o$, signed constraint matrix $\mathbf{S}$, cluster number k, parameters $\lambda, \beta, \eta, \gamma$.
Output: Consensus clustering labels $\mathbf{Y}$.

v-th Client

Initialize $\mathbf{H}_v^{(0)}$ and send it to Server $\mathcal{S}$.
Receive $\mathbf{Y}^{(0)}$ and $\bar{\mathbf{C}}_v^{(0)}$; set $\mathbf{Y}_v^{(0)} = \mathbf{Y}^{(0)}$.
repeat
  Update $\mathbf{H}_v$ by Eq. (17).
  Update $\mathbf{Y}_v$ by Eq. (18).
  Send $\mathbf{Y}_v$ and $\mathrm{obj}_v^{(t)}$ to Server $\mathcal{S}$; receive $\mathbf{Y}$ and $\bar{\mathbf{C}}_v$.
  $t \leftarrow t + 1$.
until converged

Server $\mathcal{S}$

Receive $\{\mathbf{H}_v^{(0)}\}_{v=1}^V$ and construct $\mathbf{H}_c^{(0)}$ by Eq. (15).
Initialize $\mathbf{Y}^{(0)}$, $\widehat{\mathbf{C}}^{(0)}$ by k-means, $\mathbf{C}^{(0)}$ by Eq. (16), and $\alpha_v^{(0)} = 1$.
Send $\mathbf{Y}^{(0)}$ and $\{\bar{\mathbf{C}}_v^{(0)}\}_{v=1}^V$ to clients.
repeat
  Receive $\{\mathbf{Y}_v\}_{v=1}^V$ and $\{\mathrm{obj}_v^{(t)}\}_{v=1}^V$ from clients.
  Construct $\mathbf{H}_c^{\alpha}$ by Eq. (19); update $\mathbf{C}$ and $\mathbf{Y}$ by Eqs. (20) and (21).
  Refresh $\{\alpha_v\}_{v=1}^V$ by Eq. (5) and send $\mathbf{Y}$ and $\{\bar{\mathbf{C}}_v\}_{v=1}^V$ to clients.
  Compute $\mathrm{obj}^{(t)}$ by Eq. (14); $t \leftarrow t + 1$.
until $(\mathrm{obj}^{(t)} - \mathrm{obj}^{(t-1)})/\mathrm{obj}^{(t)} < \delta$
return $\mathbf{Y}$.

3.3.2 Communication Overhead

Assuming convergence after t-th iteration, the communication overhead of ASFMVC method is $Nk + (t+1)k^2 + t$ floating-point values and $(2t+1)N$ uints. The detailed derivation is provided in Appendix D.2. In comparison, the existing federated multi-view clustering methods directly transmit the data representations between client and server, requiring at least $2tkN$ floats.

3.3.3 Convergence

Let $\mathcal{L}_t$ denote the objective in Eq. equation 14. ASFMVC alternately optimizes one of the variables while keeping the others fixed at the t + 1 iteration, inducing to

$$\begin{aligned} &\mathcal{L}_t\left(\{\mathbf{H}_v^{(t)}\}_{v=1}^V, \{\mathbf{Y}_v^{(t)}\}_{v=1}^V, \mathbf{Y}^{(t)}, \mathbf{C}^{(t)}\right) \\ \leq\ &\mathcal{L}_t\left(\{\mathbf{H}_v^{(t+1)}\}_{v=1}^V, \{\mathbf{Y}_v^{(t)}\}_{v=1}^V, \mathbf{Y}^{(t)}, \mathbf{C}^{(t)}\right) \\ \leq\ &\cdots \\ \leq\ &\mathcal{L}_t\left(\{\mathbf{H}_v^{(t+1)}\}_{v=1}^V, \{\mathbf{Y}_v^{(t+1)}\}_{v=1}^V, \mathbf{Y}^{(t+1)}, \mathbf{C}^{(t+1)}\right). \end{aligned} \tag{22}$$

Since $\mathbf{H}_v^\top \mathbf{H}_v = \mathbf{I}_k$ and $\mathbf{Y}_v, \mathbf{Y}$ are one-hot matrices, we have $\|\mathbf{H}_v\|_F^2 = k$ and $\|\mathbf{Y}_v\|_F^2 = \|\mathbf{Y}\|_F^2 = N$. Moreover, $\mathbf{C}\mathbf{C}^\top = \mathbf{I}_k$ implies $\|\bar{\mathbf{C}}_v\|_2 \leq 1$. Let N_l be the number of labeled samples and $c_v = \|\mathbf{C}_v\|_2$. Besides, $\sqrt{\alpha_v} \leq \sqrt{\alpha_{\max}}$, where $\alpha_{\max} = 1/(2\sqrt{\epsilon})$. Using the Cauchy–Schwarz inequality, the objective is bounded by

$$\begin{aligned} \mathcal{L}_t \leq\ &\sum_{v=1}^V \|\mathbf{X}_v\|_F^2 + \eta V N_l + \lambda\sqrt{kN}\sum_{v=1}^V c_v \\ &+ \beta N\sqrt{\alpha_{\max}}\sum_{v=1}^V c_v + \gamma\sum_{i,j}|S_{ij}| \triangleq \mathcal{B} < \infty. \end{aligned} \tag{23}$$

The alternating updates generate a non-decreasing and upper-bounded objective sequence and converge to a stationary coordinate-wise solution. The detailed derivation is provided in Appendix D.1.

Table 1: Details of the used benchmark datasets.

Dataset	Samples	Views	Clusters
ORL	400	3	40
BBC	685	4	5
Reuters	1200	5	6
Flower17	1360	7	17
HW	2000	6	10
Scene-15	4485	3	15

Table 2: Performance comparison with unsupervised federated MVC methods under 20% label ratio on six datasets. The best and suboptimal results are highlighted in bold and underlined, respectively.

Dataset	FedCMv	FedFCMv	FedMVL	CeFMC	ASFMVC
ACC					
BBC	$40.26_{\pm 9.6}$	$63.08_{\pm 4.5}$	$27.46_{\pm 2.8}$	$\underline{77.24_{\pm 0.1}}$	$\mathbf{84.26}_{\pm 0.5}$
Reuters	$32.05_{\pm 5.0}$	$29.75_{\pm 1.6}$	$22.02_{\pm 1.6}$	$\underline{47.27_{\pm 0.3}}$	$\mathbf{53.00}_{\pm 2.3}$
HW	$24.32_{\pm 8.6}$	$50.50_{\pm 2.2}$	$13.16_{\pm 0.3}$	$\underline{94.17_{\pm 0.2}}$	$\mathbf{95.59}_{\pm 0.3}$
Scene-15	$25.81_{\pm 3.5}$	$16.70_{\pm 0.2}$	$9.69_{\pm 0.4}$	$\underline{30.31_{\pm 4.8}}$	$\mathbf{38.20}_{\pm 4.8}$
Flower17	$7.69_{\pm 1.7}$	$12.94_{\pm 0.0}$	$10.60_{\pm 0.3}$	$\underline{33.93_{\pm 2.0}}$	$\mathbf{40.90}_{\pm 2.1}$
ORL	$8.55_{\pm 0.9}$	$25.75_{\pm 0.0}$	$16.68_{\pm 0.8}$	$\underline{71.88_{\pm 2.6}}$	$\mathbf{73.72}_{\pm 2.6}$
NMI					
BBC	$11.68_{\pm 13.8}$	$39.26_{\pm 5.8}$	$3.58_{\pm 2.0}$	$\underline{56.42_{\pm 0.1}}$	$\mathbf{66.24}_{\pm 0.9}$
Reuters	$13.85_{\pm 3.9}$	$8.72_{\pm 1.9}$	$2.11_{\pm 1.2}$	$\underline{22.79_{\pm 0.2}}$	$\mathbf{29.82}_{\pm 2.1}$
HW	$31.79_{\pm 16.8}$	$54.36_{\pm 1.2}$	$0.99_{\pm 0.1}$	$\underline{87.44_{\pm 0.3}}$	$\mathbf{90.12}_{\pm 0.5}$
Scene-15	$20.73_{\pm 3.6}$	$11.22_{\pm 0.3}$	$1.18_{\pm 0.1}$	$\underline{26.94_{\pm 5.0}}$	$\mathbf{33.93}_{\pm 4.9}$
Flower17	$1.91_{\pm 1.7}$	$7.96_{\pm 0.0}$	$3.75_{\pm 0.4}$	$\underline{32.64_{\pm 1.6}}$	$\mathbf{39.30}_{\pm 1.2}$
ORL	$29.71_{\pm 1.4}$	$49.34_{\pm 0.0}$	$36.73_{\pm 1.2}$	$\underline{85.63_{\pm 1.6}}$	$\mathbf{86.13}_{\pm 1.8}$

4 EXPERIMENT

4.1 EXPERIMENT SETTING

Benchmark Datasets. The details of the six benchmark datasets used in our experiments are summarized in Table 1.

Compared methods. We compare ASFMVC with two categories of baseline methods. The first includes unsupervised federated multi-view clustering methods: FedCMv and FedFCMv(Bárcena et al., 2024), FedMVL (Huang et al., 2022), and CeFMC (Liu et al., 2026a). The second comprises centralized semi-supervised multi-view clustering methods: AMGL (Nie et al., 2016), MLAN (Nie et al., 2018), SMVC-WAGE(denoted as WAGE in the tables)(Wang et al., 2021), and SMGC (Zhang et al., 2024).

4.2 PERFORMANCE EVALUATION

As shown in Table 2, ASFMVC outperforms all compared unsupervised federated multi-view clustering methods. Moreover, although the centralized semi-supervised methods can directly access the raw multi-view data, ASFMVC achieves competitive performance against them and even obtains superior results on BBC dataset, as shown in Table 3. These results demonstrate that ASFMVC can still achieve strong clustering performance while preserving data privacy in the federated setting. For compactness, only ACC and NMI are reported here; the corresponding Purity results are provided in Appendix C.

Moreover, as shown in Fig. 2, our method achieves consistently low communication cost on all six datasets, remaining close to CeFMC and substantially lower than FedCMv, FedFCMv, and Fed-

Table 3: Performance comparison with centralized semi-supervised MVC methods under 20% label ratio on six datasets. The best and suboptimal results are highlighted in bold and underlined, respectively.

Dataset	AMGL	MLAN	WAGE	SMGC	ASFMVC
			ACC		
BBC	$36.28_{\pm 8.6}$	$23.47_{\pm 11.1}$	$\underline{77.45}_{\pm 6.9}$	$32.70_{\pm 0.0}$	$\mathbf{84.26}_{\pm 0.5}$
Reuters	$25.97_{\pm 1.7}$	$23.94_{\pm 2.0}$	$\mathbf{59.30}_{\pm 4.5}$	$18.25_{\pm 0.0}$	$\underline{53.00}_{\pm 2.3}$
HW	$86.13_{\pm 2.2}$	$\mathbf{97.30}_{\pm 0.3}$	$91.69_{\pm 0.4}$	$81.16_{\pm 0.2}$	$\underline{95.59}_{\pm 0.3}$
Scene-15	$40.21_{\pm 1.1}$	$33.51_{\pm 2.7}$	$\mathbf{50.54}_{\pm 1.6}$	$\underline{44.74}_{\pm 1.0}$	$38.20_{\pm 4.8}$
Flower17	$21.69_{\pm 2.3}$	$20.60_{\pm 2.2}$	$\mathbf{63.39}_{\pm 2.2}$	$18.84_{\pm 0.9}$	$\underline{40.90}_{\pm 2.1}$
ORL	$\mathbf{79.06}_{\pm 2.3}$	$69.17_{\pm 1.0}$	$53.16_{\pm 3.4}$	$\underline{75.78}_{\pm 2.3}$	$73.72_{\pm 2.6}$
			NMI		
BBC	$11.05_{\pm 2.5}$	$3.27_{\pm 0.8}$	$\underline{53.93}_{\pm 10.3}$	$4.18_{\pm 0.0}$	$\mathbf{66.24}_{\pm 0.9}$
Reuters	$11.95_{\pm 1.3}$	$10.45_{\pm 3.2}$	$\mathbf{37.89}_{\pm 3.3}$	$8.54_{\pm 0.9}$	$\underline{29.82}_{\pm 2.1}$
HW	$87.53_{\pm 1.7}$	$\mathbf{93.91}_{\pm 0.6}$	$83.60_{\pm 0.7}$	$79.21_{\pm 0.1}$	$\underline{90.12}_{\pm 0.5}$
Scene-15	$40.89_{\pm 1.0}$	$38.14_{\pm 2.3}$	$\underline{41.56}_{\pm 1.1}$	$\mathbf{47.42}_{\pm 0.4}$	$33.93_{\pm 4.9}$
Flower17	$25.74_{\pm 2.1}$	$24.99_{\pm 2.0}$	$\mathbf{53.99}_{\pm 2.5}$	$17.67_{\pm 0.8}$	$\underline{39.30}_{\pm 1.2}$
ORL	$\underline{89.57}_{\pm 0.8}$	$82.78_{\pm 0.6}$	$55.62_{\pm 4.2}$	$\mathbf{90.41}_{\pm 1.2}$	$86.13_{\pm 1.8}$

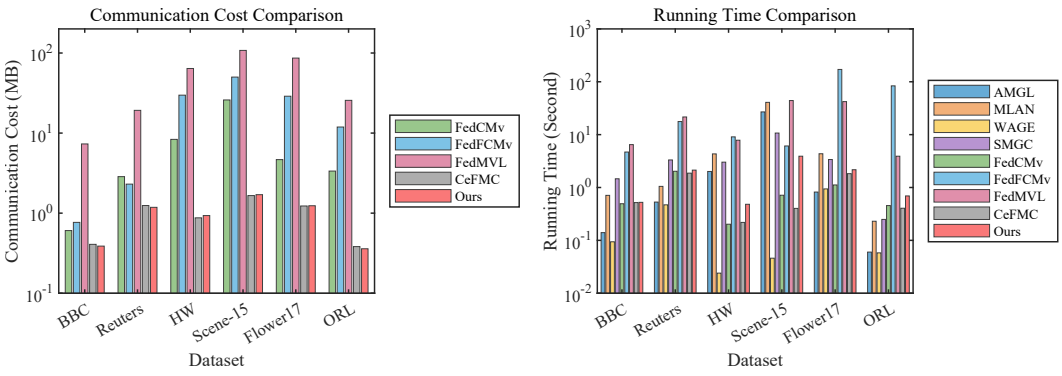


Figure 2: Comparison of communication cost and running time on different datasets.

MVL. Although ASFMVC does not always attain the lowest running time, its computational cost remains competitive with the compared methods.

4.3 Ablation Study

To assess the effectiveness of the three key components in ASFMVC, we conduct an ablation study by removing (1) the server-side signed supervision, denoted as w/o S, (2) the Label-Guided Representation module, denoted as w/o LGR, and (3) the adaptive client weighting strategy, denoted as w/o AW. As shown in Table 4, removing any component generally degrades the clustering performance.

To investigate the sensitivity of the proposed method to the amount of supervised information, we vary the label ratio from 1% to 30% and report ACC in Table 5. As shown, the performance consistently improves as the label ratio increases.

4.4 Convergence Analysis

We plot the objective values and ACC over iterations on Flower17 and Reuters in Fig. 3(a)–(b). It can be observed that the objective value changes rapidly in the first few iterations and then gradu-

Table 4: Ablation study on three components of the proposed method in terms of ACC.

Variant	BBC	Reuters	HW	Scene15	Flower17	ORL
w/o S	$81.36_{\pm 0.8}$	$48.18_{\pm 0.3}$	$95.47_{\pm 0.2}$	$30.25_{\pm 5.0}$	$33.97_{\pm 2.1}$	$73.35_{\pm 2.8}$
w/o LGR	$83.61_{\pm 0.5}$	$52.97_{\pm 2.2}$	$95.44_{\pm 0.2}$	$38.19_{\pm 4.8}$	$40.53_{\pm 1.8}$	$73.08_{\pm 2.5}$
w/o AW	$83.66_{\pm 1.0}$	$49.43_{\pm 0.4}$	$95.50_{\pm 0.2}$	$38.11_{\pm 4.9}$	$40.03_{\pm 2.0}$	$73.22_{\pm 2.6}$
ASFMVC	$\mathbf{84.26}_{\pm 0.5}$	$\mathbf{53.00}_{\pm 2.3}$	$\mathbf{95.59}_{\pm 0.3}$	$\mathbf{38.20}_{\pm 4.8}$	$\mathbf{40.90}_{\pm 2.1}$	$\mathbf{73.72}_{\pm 2.6}$

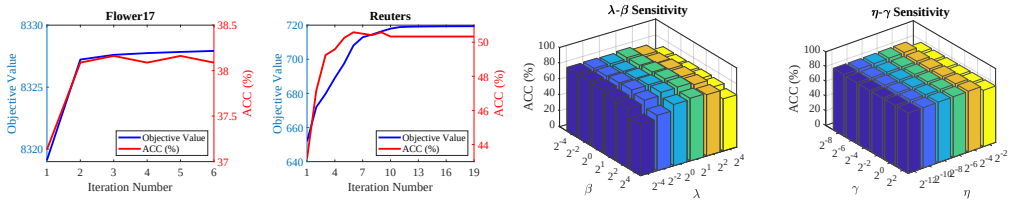
Table 5: ACC of the proposed method under different label ratios (1%, 5%, 10%, 20%, 30%) on three datasets.

Datasets	1%	5%	10%	20%	30%
BBC	$78.28_{\pm 1.1}$	$79.62_{\pm 1.1}$	$81.50_{\pm 0.4}$	$84.26_{\pm 0.5}$	$86.15_{\pm 0.8}$
Reuters	$48.04_{\pm 0.3}$	$48.70_{\pm 0.7}$	$50.05_{\pm 0.8}$	$53.00_{\pm 2.3}$	$55.47_{\pm 3.2}$
Flower17	$33.99_{\pm 2.1}$	$34.66_{\pm 2.0}$	$36.34_{\pm 2.1}$	$40.90_{\pm 2.1}$	$44.35_{\pm 3.7}$

ally becomes stable. Meanwhile, the clustering accuracy also stabilizes quickly, demonstrating the satisfactory convergence behavior of our method.

4.5 Parameter Sensitivity Analysis

To investigate the influence of hyperparameters, we conduct parameter sensitivity experiments on BBC with respect to λ, β, η, and γ. As shown in Fig. 3(c)–(d), the clustering performance remains relatively stable within moderate parameter ranges, while overly large values may lead to performance degradation.



(a) Flower17 (b) Reuters (c) λ-β (d) η-γ

Figure 3: Convergence and parameter sensitivity analysis of ASFMVC.

5 Conclusion

In this paper, we proposed Adaptive Semi-Supervised Federated Multi-View Clustering to exploit supervisory information in federated multi-view clustering. ASFMVC incorporates observed label information into local representation learning through the label-guided representation mechanism, adaptively adjusts the contribution of each client according to its consistency with the global consensus, and further introduces a signed constraint graph on the server to guide consensus clustering with must-link and cannot-link relations. These components are integrated into a unified federated clustering framework while retaining compact client–server communication and linear computational complexity with respect to the number of samples under the sparse-constraint setting. Experiments on six benchmark datasets demonstrate the effectiveness of our method. In future work, we plan to investigate more flexible supervision mechanisms and extend our method to more challenging federated multi-view scenarios, such as incomplete or noisy views.

AI USE STATEMENT

Generative AI tools were used to assist with language polishing and LaTeX formatting. We have reviewed all AI-assisted work.The authors take full responsibility for the methodology, implementation, experimental results, analysis and final manuscript.

REPRODUCIBILITY STATEMENT

We provide sufficient details to facilitate the reproduction of our method. The complete objective formulation and alternating optimization procedure are described in Sections 3.1 and 3.2, including the updates of local representations, pseudo-labels, global consensus variables, adaptive client weights, and server-side signed constraints. Dataset statistics, data preprocessing, compared methods are reported in Section 4.1. Ablation studies, parameter sensitivity analysis, convergence analysis, and computational and communication complexity are also provided to verify the behavior of the proposed method. The source code and additional implementation details will be included in the supplementary material for reproducibility.

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A TABLE OF NOTATIONS

Table 6: Table of notations.

Notation	Meaning
N, V, k	numbers of data samples, views and classes
$\mathbf{X}_v$	the v-th view data
$\mathbf{H}_v$	local data representation learned on the v-th client.
$\mathbf{Y}_o$	Sparse observed-label matrix constructed from labeled samples
$\mathbf{Y}_v$	one-hot pseudo-label matrix of the v-th client.
$\mathbf{C}_v$	the centroid matrix on the v-th client.
$\mathbf{Y}$	Global consensus label matrix on the server.
$\mathbf{C}$	Global consensus centroid matrix on the server.
$\bar{\mathbf{C}}_v$	The v-th block of the global consensus centroid matrix $\mathbf{C}$
$\mathbf{S}$	Balanced and normalized signed constraint matrix on the server
$\eta, \lambda, \beta, \gamma$	Trade-off parameters for label-guided representation, local representation approximation, global consensus and signed supervision

B ADDITIONAL IMPLEMENTATION DETAILS

Table 7: Parameter settings used in the main experiments.

Dataset	$\boldsymbol{\lambda = \beta}$	$\boldsymbol{\gamma}$	$\boldsymbol{\eta}$
BBC	2	2^{-8}	2^{-4}
Reuters-1200	4	2^{-8}	2^{-8}
HW	4	2^{-8}	2^{-2}
Scene-15	4	2^{-6}	2^{-10}
Flower17	4	2^{-6}	2^{-6}
ORL	2^{-4}	2^{-8}	2^{-4}

All experiments are repeated 10 times and the convergence tolerance is 10^{-5}. For practicality, we set $\lambda = \beta$ throughout the experiments, which reduces the parameter-search space from two dimensions to one. As illustrated by the joint sensitivity analysis on BBC, the model remains stable over moderate combinations of λ and β, and the best performance is obtained at $\lambda = \beta = 2$. We therefore adopt the coupled setting $\lambda = \beta$, with dataset-specific values given in Table 7.

C ADDITIONAL CLUSTERING RESULTS

For completeness, Tables 8 and 9 report the Purity results on all six datasets.

Table 8: Purity comparison with unsupervised federated MVC methods under 20% label ratio on six datasets. The best and suboptimal results are highlighted in bold and underlined, respectively.

Dataset	FedCMv	FedFCMv	FedMVL	CeFMC	ASFMVC
BBC	$40.85_{\pm 10.1}$	$63.41_{\pm 4.5}$	$36.13_{\pm 2.8}$	$\underline{77.24_{\pm 0.1}}$	$\mathbf{84.26}_{\pm 0.5}$
Reuters	$33.96_{\pm 5.0}$	$30.29_{\pm 1.6}$	$22.51_{\pm 1.6}$	$\underline{47.34_{\pm 0.4}}$	$\mathbf{53.11}_{\pm 2.3}$
HW	$24.32_{\pm 8.6}$	$54.85_{\pm 1.4}$	$13.38_{\pm 0.3}$	$\underline{94.17_{\pm 0.2}}$	$\mathbf{95.59}_{\pm 0.3}$
Scene-15	$26.20_{\pm 3.8}$	$18.10_{\pm 0.3}$	$10.59_{\pm 0.3}$	$\underline{32.63_{\pm 5.2}}$	$\mathbf{39.97}_{\pm 4.9}$
Flower17	$7.69_{\pm 1.7}$	$13.68_{\pm 0.0}$	$11.04_{\pm 0.4}$	$\underline{35.06_{\pm 1.9}}$	$\mathbf{42.30}_{\pm 1.6}$
ORL	$8.55_{\pm 0.9}$	$27.50_{\pm 0.0}$	$17.43_{\pm 0.9}$	$\underline{75.72_{\pm 2.2}}$	$\mathbf{76.72}_{\pm 2.5}$

Table 9: Purity comparison with centralized semi-supervised MVC methods under 20% label ratio on six datasets. The best and suboptimal results are highlighted in bold and underlined, respectively.

Dataset	AMGL	MLAN	WAGE	SMGC	ASFMVC
BBC	$42.47_{\pm 2.6}$	$34.43_{\pm 0.5}$	$\underline{77.45_{\pm 6.9}}$	$33.58_{\pm 0.0}$	$\mathbf{84.26}_{\pm 0.5}$
Reuters	$28.74_{\pm 1.2}$	$24.17_{\pm 2.1}$	$\mathbf{59.30}_{\pm 4.5}$	$19.03_{\pm 0.5}$	$\underline{53.11_{\pm 2.3}}$
HW	$86.85_{\pm 1.5}$	$\mathbf{97.30}_{\pm 0.3}$	$91.69_{\pm 0.4}$	$81.16_{\pm 0.2}$	$\underline{95.59_{\pm 0.3}}$
Scene-15	$45.51_{\pm 1.1}$	$34.44_{\pm 2.5}$	$\mathbf{51.73}_{\pm 1.3}$	$\underline{49.30_{\pm 0.9}}$	$39.97_{\pm 4.9}$
Flower17	$27.37_{\pm 1.5}$	$20.86_{\pm 2.0}$	$\mathbf{63.42}_{\pm 2.2}$	$20.29_{\pm 0.7}$	$\underline{42.30_{\pm 1.6}}$
ORL	$\mathbf{82.92}_{\pm 1.4}$	$73.11_{\pm 0.7}$	$53.16_{\pm 3.4}$	$\underline{81.03_{\pm 1.8}}$	$76.72_{\pm 2.5}$

D Additional Theoretical Analysis

D.1 Detailed Convergence Analysis

We provide further analysis of the convergence behavior of ASFMVC. For the t-th outer iteration, the adaptive coefficients $\{\alpha_v^{(t)}\}_{v=1}^V$ are treated as fixed while updating $\{\mathbf{H}_v\}_{v=1}^V$, $\{\mathbf{Y}_v\}_{v=1}^V$, $\mathbf{C}$, and $\mathbf{Y}$. Let

$$\mathcal{L}_t\left(\{\mathbf{H}_v\}_{v=1}^V, \{\mathbf{Y}_v\}_{v=1}^V, \mathbf{Y}, \mathbf{C}\right)$$

denote the objective in Eq. (14) under the fixed coefficients $\{\alpha_v^{(t)}\}_{v=1}^V$.

According to the alternating optimization procedure, each block is updated while the remaining variables are fixed. The SVD-based representation step performs a non-decreasing surrogate update, the pseudo-label update maximizes the corresponding row-wise score, the centroid update gives the closed-form orthogonal Procrustes solution, and the consensus-label update performs coordinate ascent. Therefore, within the t-th outer iteration,

$$\begin{aligned}
&\mathcal{L}_t\left(\{\mathbf{H}_v^{(t)}\}_{v=1}^V, \{\mathbf{Y}_v^{(t)}\}_{v=1}^V, \mathbf{Y}^{(t)}, \mathbf{C}^{(t)}\right) \\
\le\ &\mathcal{L}_t\left(\{\mathbf{H}_v^{(t+1)}\}_{v=1}^V, \{\mathbf{Y}_v^{(t)}\}_{v=1}^V, \mathbf{Y}^{(t)}, \mathbf{C}^{(t)}\right) \\
\le\ &\mathcal{L}_t\left(\{\mathbf{H}_v^{(t+1)}\}_{v=1}^V, \{\mathbf{Y}_v^{(t+1)}\}_{v=1}^V, \mathbf{Y}^{(t)}, \mathbf{C}^{(t)}\right) \\
\le\ &\mathcal{L}_t\left(\{\mathbf{H}_v^{(t+1)}\}_{v=1}^V, \{\mathbf{Y}_v^{(t+1)}\}_{v=1}^V, \mathbf{Y}^{(t)}, \mathbf{C}^{(t+1)}\right) \\
\le\ &\mathcal{L}_t\left(\{\mathbf{H}_v^{(t+1)}\}_{v=1}^V, \{\mathbf{Y}_v^{(t+1)}\}_{v=1}^V, \mathbf{Y}^{(t+1)}, \mathbf{C}^{(t+1)}\right).
\end{aligned} \tag{24}$$

We next show that the objective under fixed adaptive coefficients is upper bounded. The objective in Eq. (14) consists of five terms, which are analyzed separately.

1) Local representation term. Since $\mathbf{H}_v^\top \mathbf{H}_v = \mathbf{I}_k$, we have $\|\mathbf{H}_v\|_2 = 1$. Therefore,

$$\begin{aligned}
\mathrm{Tr}\left(\mathbf{H}_v^\top \mathbf{X}_v \mathbf{X}_v^\top \mathbf{H}_v\right) &= \|\mathbf{X}_v^\top \mathbf{H}_v\|_F^2 \\
&\le \|\mathbf{X}_v\|_F^2.
\end{aligned} \tag{25}$$

2) Label-guided representation term. Let N_l denote the number of labeled samples. Since each labeled sample contributes one nonzero entry to the observed-label matrix, $\|\mathbf{Y}_o\|_F^2 = N_l$. Hence,

$$\begin{aligned}
\mathrm{Tr}\left(\mathbf{H}_v^\top \mathbf{Y}_o \mathbf{Y}_o^\top \mathbf{H}_v\right) &= \|\mathbf{Y}_o^\top \mathbf{H}_v\|_F^2 \\
&\le \|\mathbf{Y}_o\|_F^2 = N_l.
\end{aligned} \tag{26}$$

3) Local clustering approximation term. Let $c_v = \|\mathbf{C}_v\|_2$. Because $\mathbf{Y}_v$ is one-hot, $\|\mathbf{Y}_v\|_F = \sqrt{N}$, and $\|\mathbf{H}_v\|_F = \sqrt{k}$. By the Cauchy–Schwarz inequality,

$$\begin{aligned}
\mathrm{Tr}\left(\mathbf{H}_v^\top \mathbf{Y}_v \mathbf{C}_v\right) &\le \|\mathbf{H}_v\|_F \|\mathbf{Y}_v \mathbf{C}_v\|_F \\
&\le \sqrt{kN}\, c_v.
\end{aligned} \tag{27}$$

4) Adaptive federated consensus term. Since $\mathbf{C}\mathbf{C}^\top = \mathbf{I}_k$, $\|\mathbf{C}\|_2 = 1$, and thus each block $\bar{\mathbf{C}}_v$ satisfies $\|\bar{\mathbf{C}}_v\|_2 \leq 1$. Moreover, from Eq. (5),

$$\alpha_v = \frac{1}{2\sqrt{d_v + \epsilon}} \leq \frac{1}{2\sqrt{\epsilon}} \triangleq \alpha_{\max}, \tag{28}$$

where $d_v = \|\mathbf{Y}_v\mathbf{C}_v - \mathbf{Y}\bar{\mathbf{C}}_v\|_F^2 \geq 0$. Therefore,

$$\begin{aligned} &\sqrt{\alpha_v}\,\mathrm{Tr}\left[(\mathbf{Y}_v\mathbf{C}_v)^\top \mathbf{Y}\bar{\mathbf{C}}_v\right] \\ &\leq \sqrt{\alpha_{\max}}\|\mathbf{Y}_v\mathbf{C}_v\|_F\|\mathbf{Y}\bar{\mathbf{C}}_v\|_F \\ &\leq N\sqrt{\alpha_{\max}}\,c_v. \end{aligned} \tag{29}$$

5) Signed constraint term. Since $\mathbf{Y}$ is a one-hot indicator matrix,

$$\mathrm{Tr}(\mathbf{Y}^\top \mathbf{S}\mathbf{Y}) = \sum_{i,j} S_{ij}\mathbb{I}(y_i = y_j) \leq \sum_{i,j} |S_{ij}|. \tag{30}$$

Combining Eqs. (25)–(30), we have

$$\begin{aligned} \mathcal{L}_t \leq\ & \sum_{v=1}^{V} \|\mathbf{X}_v\|_F^2 + \eta V N_l + \lambda\sqrt{kN}\sum_{v=1}^{V} c_v \\ &+ \beta N\sqrt{\alpha_{\max}}\sum_{v=1}^{V} c_v + \gamma\sum_{i,j}|S_{ij}| \\ \triangleq\ & \mathcal{B} < \infty. \end{aligned} \tag{31}$$

Hence, for fixed adaptive coefficients, the alternating block updates generate a non-decreasing objective sequence that is upper bounded. Therefore, the fixed-weight block optimization converges to a coordinate-wise stable solution. After the client and server variables have been updated, the adaptive coefficients are refreshed according to Eq. (5) for the next outer iteration. The empirical convergence behavior of the complete adaptive procedure is reported in Section 4.

D.2 Detailed Communication Overhead

We further derive the communication overhead of ASFMVC. The analysis is performed for one client, since all clients follow the same communication protocol. We distinguish the initialization stage from the subsequent alternating clustering stage. Here, "float" denotes a floating-point value and "uint" denotes an integer cluster label.

Initialization. The v-th client first sends its initialized local representation $\mathbf{H}_v^{(0)} \in \mathbb{R}^{N\times k}$ to the server, requiring

$$Nk \tag{32}$$

floating-point values. The server performs k-means on the concatenated representations and returns the initial consensus labels $\mathbf{Y}^{(0)}$ together with the corresponding centroid block $\bar{\mathbf{C}}_v^{(0)}$. Since the consensus labels can be represented by N integer cluster indices and $\bar{\mathbf{C}}_v^{(0)} \in \mathbb{R}^{k\times k}$, this step requires

$$k^2 \text{ floats} \qquad \text{and} \qquad N \text{ uints}. \tag{33}$$

Therefore, the initialization communication cost per client is

$$Nk + k^2 \text{ floats}, \qquad N \text{ uints}. \tag{34}$$

Alternating clustering. Assume that ASFMVC converges after t outer iterations. In each iteration, the v-th client uploads its pseudo-label vector containing N integer assignments and one scalar local statistic, while the server returns the updated consensus labels and the corresponding global centroid block. Hence, each outer iteration requires

$$k^2 + 1 \text{ floats}, \qquad 2N \text{ uints}. \tag{35}$$

Over t iterations, the clustering stage therefore requires

$$t(k^2+1) \text{ floats}, \qquad 2tN \text{ uints}. \tag{36}$$

Combining Eqs. (34) and (36), the total communication overhead between one client and the server is

$$\boxed{Nk + (t+1)k^2 + t} \quad \text{floats}, \tag{37}$$

and

$$\boxed{(2t+1)N} \quad \text{uints}. \tag{38}$$

The proposed label-guided representation mechanism does not introduce additional iterative communication because the observed supervision is used locally for representation learning. Likewise, the signed constraint graph is constructed and retained on the server, and thus does not need to be transmitted during the alternating optimization. The adaptive coefficients are computed from the already available local and global clustering structures and therefore introduce no additional high-dimensional communication variables.