INSTANT CONTEXT OPTIMIZATION: COMPILING FEW-SHOT IN-CONTEXT DEMONSTRATIONS INTO PRE-TRAINED CHANNEL MAGNITUDES VIA CIRCUIT-BOUNDED SPARSE OPTIMIZATION

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ABSTRACT

In-Context Learning (ICL) enables Large Language Models (LLMs) to perform tasks from a small set of demonstrations (e.g., $K \leq 8$), but requires persistent Key-Value (KV) cache expansion and quadratic prefill latency on every forward pass. While compiling demonstrations directly into model weights would eliminate this runtime prompt tax, standard few-shot parameter optimization ($K \ll D_{dim}$) is fundamentally ill-posed, causing severe overfitting and representation collapse. In this work, we propose **Instant Context Optimization (ICO)**, a framework that compiles demonstrations into task-specific static weights once per task offline, recovering the performance benefits of ICL without serving-time prompt costs. Formally framed as episodic prompt-to-weight internalization via circuit-bounded test-time distillation, ICO leverages three structural constraints exposed by the model's own in-context activation routing: (1) *Where to update:* Gradient-based parameter saliency localizes updates to sparse, task-relevant subcircuits ($d_{\text{eff}} \leq 0.10D$); (2) *How to update:* Updates are restricted to channel-wise gain modulation along fixed directional bases ($W^* = \text{diag}(\mathbf{m}^*/\mathbf{m}_0)W_0$), provably preserving the layerwise kernel nullspace ($\ker(W^*) \equiv \ker(W_0)$); and (3) *How to score updates:* Reasoning trajectories are supervised via Demonstration-Conditioned Teacher Reference Scoring with Group Minimum-Anchored Relative Advantage ($\hat{A}_i = (r_i - r_{\text{bot}})/\sigma_k$). Across 38 benchmarks on models up to 27B parameters, ICO recovers $26.2\% \rightarrow 74.9\%$ (up to 107.8% on GSM8K) few-shot headroom with a universal $+12.67\%$ grand macro served gain in pure zero-shot setting, safeguarded by an empirical Risk-Monotonic AR validation gate.

1 INTRODUCTION

In-Context Learning (ICL) helps Large Language Models (LLMs) adapt to target tasks from a small set of demonstrations $\mathcal{D}_{\text{demo}} = \{(x_k, y_k)\}_{k=1}^{K}$ (e.g., $K \leq 8$) without modifying underlying weights (Brown et al., 2020). However, processing demonstrations incurs persistent Key-Value (KV) cache expansion and quadratic prefill latency on every query, creating inference bottlenecks. Compiling demonstration logic directly into static weights would eliminate runtime prompt overhead entirely. Still, classical fine-tuning under extreme data scarcity is fundamentally ill-posed due to overfitting and memorization, triggering catastrophic representation collapse and forgetting.

Intuitively, we observe that ICL sidesteps overfitting or memorization because prompt demonstrations do not update the model weights. Instead, the prompt directs internal routing to activate and scale pre-existing latent circuits. Auditing internal representations across 224 linear modules of `Meta-Llama-3-8B` under ICL reveals three structural invariants (Figure 1 and protocols in Appendix C) that answer three fundamental optimization questions, establishing a formal **Three-Fold Structured Inductive Bias**. 1) **Where to update? Gradient Saliency Prior:** Saliency attribution energy concentrates sharply in a tiny fraction of channels ($> 94.7\%$ energy in $< 10\%$ coordinates and 81.4% stability) (Olsson et al., 2022; Todd et al., 2024). Imposing a continuous saliency-inverse proximal threshold ($\tau_i \propto 1/\alpha_i$) penalizes off-circuit drift by up to $20\times$, restricting updates to effective dimensions without additional combinatorial search (Wainwright, 2019). 2) **How to update?**

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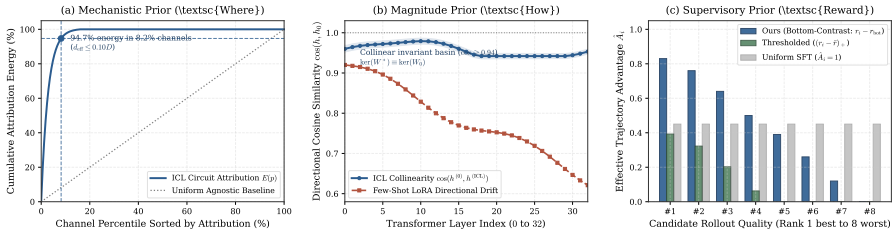

Figure 1: The three-fold structured inductive bias of ICL on `Meta-Llama-3-8B`: **(a)** parameter saliency energy concentration ($d_{\text{eff}} \leq 0.10D$); **(b)** directional collinearity under channel-wise gain modulation ($\cos \geq 0.94$); and **(c)** autonomous Leave-One-Out relative advantage formulation.

Paradigm	Active Parameter Space (D)	Adaptation Phase	Test-Query Access	Serving Overhead (KV / Latency)	Nullspace Overfitting Risk ($K \leq 8$)
Standard ICL (Brown et al., 2020)	0 (Frozen)	Runtime Forward Pass	Yes (Concatenated)	Linear KV Cache + $\mathcal{O}(T^2)$ FLOPs	None (No weights updated)
Online TTT (Sun et al., 2025)	$10^6 - 10^7$	Online per Test Query	**Yes (Transductive)**	Severe (> 5 s per query)	High (Unsupervised drift)
Few-Shot LoRA/DoRA (Hu et al., 2022; Liu et al., 2024)	$10^6 - 10^7$	Offline Fine-Tuning	No (Demos only)	Adapter dispatch latency	**Extreme** (Directional collapse)
Uniform Sparse TTT (Parikh & Boyd, 2014)	$10^5 - 10^6$	Offline Fine-Tuning	No (Demos only)	Zero runtime overhead	High (Task-agnostic coordinates)
ICO (Ours)	$d_{\text{eff}} \leq 10^4$ **(Channel Gain)**	**Offline Pre-Compilation** (≤ 3 **min**)	**No (0 test access)**	**Pure Zero-Shot (0 ms, 0 tok)**	**Provably Bounded** ($\ker(W_0)$ **preserved**)

Table 1: Systematic comparison of demonstration adaptation and test-time serving paradigms.

Channel Gain Prior: Representations under K-shot prompting exhibit near-perfect collinearity with zero-shot activations ($\cos \in [0.94, 0.98]$ across 32 blocks), proving ICL modulates channel activation scale rather than rotating projection bases (Liu et al., 2024). Restricting updates to a channel-wise gain modulation vector $\mathbf{m} \in \mathbb{R}^{d_{\text{out}}}_{>0}$ ($W^* = \text{diag}(\mathbf{m}/\mathbf{m}_0)W_0$) provably preserves the exact layerwise kernel nullspace ($\ker(W^*) \equiv \ker(W_0)$, Theorem 1), preventing unconstrained basis rotation at each linear projection. 3) **How to score updates? Demonstration-Conditioned Reference Scoring:** The pre-trained model possesses an intrinsic capacity to self-evaluate reasoning rollouts under leave-one-out context $\mathcal{D}_{\text{demo}} \setminus \{(x_k, y_k)\}$ (Snell et al., 2022). Mining rollouts via Leave-One-Out (LOO) self-evaluation yields standardized Group Minimum-Anchored Relative Advantages (`diff_bot`: $\hat{A}_{k,i} = (r_i - r_{\text{bot}})/\sigma_k$) (Shao et al., 2024), generating smooth policy gradients without external reward models or test-query access.

Converting these priors into an actionable framework, we propose **Instant Context Optimization (ICO)**, a Test-Time Training (TTT) framework that compiles a few demonstrations into static channel-wise gains offline in minutes. ICO combines circuit-bounded proximal updates, LOO rollout supervision, multi-round Vector-TIES checkpoint consolidation, and a Risk-Monotonic AR Gate that guards against empirical regression on demonstrations ($\Delta\mathcal{L}_{\text{val}} \geq 0$, Section 3.4). Because compiled channel gains merge directly into static linear weights ($W^* = \text{diag}(\mathbf{m}^*)V_0$), serving requires zero runtime prompt tokens and incurs no adapter dispatch latency compared to dynamic multi-tenant LoRA kernels (Table 1).

Summary of Contributions. (1) **Three-Fold Structured Inductive Bias Framework:** We formalize the sample complexity paradox of few-shot test-time adaptation ($K \ll D_{dim}$) and demonstrate that ICL provides structured saliency (WHERE), channel gain (HOW), and supervisory reference (REWARD) biases that constrain optimization to stable pre-trained circuits. (2) **Layerwise Nullspace Invariance under Diagonal Scaling:** We show that channel-wise gain modulation along invariant row bases guarantees exact layerwise nullspace and row-space preservation ($\ker(W^*) \equiv \ker(W_0)$, Theorem 1), providing an algebraic inductive bias that prevents rotation of primitive projection bases. (3) **Risk-Monotonic Weight Internalization Pipeline:** We engineer an end-to-end pipeline integrating continuous saliency-inverse thresholding ($\tau_i \propto 1/\alpha_i$), LOO rollout contrast, Vector-TIES checkpoint consolidation, and an AR Gate that mitigates empirical regression before weights are deployed. (4) **Cross-Model Scaling Across 152 Evaluations:** Evaluating ICO across 38 diverse reasoning benchmarks and 4 models (8B to 27B: `Meta-Llama-3-8B`, `Gemma-2-9B`, `Gemma-3-12B`, `Gemma-3-27B`) reveals robust headroom recovery across scales ($26.2\% \rightarrow 74.9\%$, up to 107.8% on GSM8K) and a universal $+12.67\%$ grand macro served gain in pure zero-shot mode (Table 3; scenario breakdowns in Table 8, Appendix E.1).

2 THEORETICAL FOUNDATION: THREE-FOLD STRUCTURED INDUCTIVE BIAS

Degenerate Shortcut Optimization under Extreme Scarcity ($K \ll D$). Let $\theta_0 \in \mathbb{R}^D$ denote frozen weights. Compiling $K \leq 8$ demonstrations $\mathcal{D}_{\text{demo}} = \{(x_k, y_k)\}_{k=1}^K$ into adapted weights $\theta^* = \theta_0 + \Delta\theta$ is severely ill-posed when ambient dimension vastly exceeds sample size ($D/K \geq 10^6$). In high-dimensional sparse recovery, identifying unknown s-sparse coordinates among D dimensions incurs a combinatorial search penalty scaling minimax estimation error as $\mathcal{O}(\sqrt{s\log(D/s)/K})$ (Wainwright, 2019). Consequently, empirical support identification from $K \leq 8$ demonstrations is statistically impossible without structural constraints. ICL circumvents this combinatorial barrier by internally routing demonstrations through pre-existing subcircuits $\mathcal{C}_{\text{task}}$ (Goldwaser et al., 2026), providing explicit coordinate and geometric inductive priors.

2.1 WHERE TO UPDATE: GRADIENT-BASED PARAMETER SALIENCY

Because the model internally routes demonstrations through task-specific subcircuits $\mathcal{C}_{\text{task}}$, evaluating a single backward pass on $\mathcal{D}_{\text{demo}}$ yields first-order parameter saliency $\alpha_{l,i} = |m_{l,i} \cdot \frac{\partial \mathcal{L}_{\text{attrib}}}{\partial m_{l,i}}|$. Rather than applying brittle binary masks, $\alpha_{l,i}$ defines a **continuous saliency-inverse proximal threshold**:

$$\tau_{l,i} = \tau_0 \cdot \frac{1}{\text{clamp}\left(\bar{\alpha}_{l,i},\ 0.05,\ 2.0\right)} \cdot q_{0.90}\left(|\nabla_{\mathbf{m}_l} \mathcal{L}_{\text{attrib}}|\right), \tag{1}$$

where $\bar{\alpha}_{l,i} = \alpha_{l,i}/q_{0.95}(\boldsymbol{\alpha}_l)$. High-saliency coordinates face minimal regularization ($\tau_{l,i} \leq \tau_0$), while off-circuit coordinates face up to $20\times$ higher penalties ($\tau_{l,i} \leq 20\tau_0$). Because $\bar{\boldsymbol{\alpha}}$ is estimated from K demonstrations, it functions as an empirical soft coordinate prior that concentrates updates on an effective subcircuit, suppressing unregularized parameter drift into orthogonal nullspaces.

2.2 HOW TO UPDATE: DIRECTIONAL BASIS INVARIANCE & NULLSPACE PRESERVATION

Decomposing linear weights $W_0 \in \mathbb{R}^{d_{\text{out}} \times d_{\text{in}}}$ row-wise yields directional unit row vectors $V_0 \in \mathbb{R}^{d_{\text{out}} \times d_{\text{in}}}$ and channel norms $\mathbf{m}_0 \in \mathbb{R}^{d_{\text{out}}}_{>0}$: $W_0 = \text{diag}(\mathbf{m}_0)V_0$. In our measurements across 32 transformer blocks, in-context activations remain strongly collinear with zero-shot activations ($\cos(h^{(0)}, h^{(\text{ICL})}) \in [0.94, 0.98]$). While multi-head attention and nonlinear activations introduce complex contextual interactions, this high directional alignment suggests that channel-wise gain modulation along pre-trained directions provides an effective structural inductive constraint against unconstrained rotational drift.

Theorem 1 (Layerwise Nullspace and Row-Space Invariance under Channel-Wise Gain Modulation)**.** *Let $W_0 = \text{diag}(\mathbf{m}_0)V_0$ with $\mathbf{m}_0 \in \mathbb{R}^{d_{out}}_{>0}$ and unit-normalized row vectors in V_0. Parameterizing adapted weights strictly by channel-wise diagonal gain modulation $\mathbf{m} \in \mathbb{R}^{d_{out}}_{>0}$ while fixing directional row bases V_0 ($W^* = \text{diag}(\mathbf{m})V_0$) guarantees:*

$$\ker(W^*) \equiv \ker(W_0) \quad \textit{and} \quad \text{rowspan}(W^*) \equiv \text{rowspan}(W_0).$$

Intuition & Structural Scope: For any $v \in \mathbb{R}^{d_{\text{in}}}$, $W^*v = \text{diag}(\mathbf{m})(V_0 v)$. Because $\mathbf{m} > \mathbf{0}$, $\text{diag}(\mathbf{m})$ is strictly non-singular. Therefore, $W^*v = \mathbf{0} \iff V_0 v = \mathbf{0} \iff W_0 v = \mathbf{0}$. Channel-wise gain modulation preserves directional nullspaces and row spans for each linear projection in isolation (algebraic details in Appendix B.1). In modern Transformers, projection geometries differ: wide projections (GQA W_K, W_V and MLP W_{down} where $d_{\text{out}} < d_{\text{in}}$) possess massive non-trivial kernels ($\dim(\ker(W_0)) \geq 3072$ on Llama-3-8B), which additive updates ($\Delta W = BA$) generally disrupt ($BAv \neq \mathbf{0}$ for $v \in \ker(W_0)$), whereas gain scaling preserves them identically. For square or tall projections ($W_Q, W_O, W_{\text{gate}}, W_{\text{up}}$), although the algebraic kernel is trivial when full rank, gain scaling uniquely preserves individual channel row directions ($\text{span}(W^*_{i,:}) = \text{span}(W_{0,i,:})$), preventing coordinate rotation. Crucially, layerwise nullspace preservation is a linear operator property; it does not claim global manifold linearity across nonlinear multilayer compositions, but serves as a localized inductive bias that suppresses out-of-distribution drift.

3 INSTANT CONTEXT OPTIMIZATION

ICO operationalizes the three-fold structured inductive bias to compile a few demonstrations into pre-trained channel-wise gains in under 100 seconds offline. The complete procedural pipeline and algorithmic complexity are detailed in Appendix D.

3.1 CHANNEL-WISE GAIN MODULATION PARAMETERIZATION

For every linear projection (attention W_Q, W_K, W_V, W_O and MLP $W_{\text{gate}}, W_{\text{up}}, W_{\text{down}}$), ICO freezes base weights W_0 and allocates a trainable channel-wise gain vector $\mathbf{m} \in \mathbb{R}^{d_{\text{out}}}_{>0}$:

$$W^*(\mathbf{m}) = \text{diag}\left(\frac{\mathbf{m}}{\mathbf{m}_0}\right) W_0, \quad \text{where} \quad m_{0,i} = \|W_{0,i,:}\|_2. \tag{2}$$

As an example, on `Meta-Llama-3-8B`, this parameterization allocates only $1.3M$ **parameters** (0.017% **of the model)** across all 224 linear modules, drastically reducing optimization memory while guaranteeing layerwise nullspace preservation by Theorem 1.

3.2 PHASE I: IN-CONTEXT INDUCTION CIRCUIT PROBING

To isolate the parameter subcircuits actively responsible for contextual task execution, ICO probes parameter saliency directly under **In-Context Learning (ICL) settings** rather than isolated zero-shot pairs. Given demonstrations $\mathcal{D}_{\text{demo}} = \{(x_k, y_k)\}_{k=1}^{K}$, for each target demonstration k, we construct a rotated leave-one-out (LOO) context prompt $\mathcal{P}^{(k)}_{\text{LOO}}$ prepending the remaining exemplars $\mathcal{D}^{(k)}_{\text{LOO}} = \mathcal{D}_{\text{demo}} \setminus \{(x_k, y_k)\}$ before x_k:

$$\mathcal{P}^{(k)}_{\text{LOO}} = \left[\text{Instruction} \circ \bigoplus_{j \neq k} (x_j \circ y_j) \circ x_k\right]. \tag{3}$$

ICO then evaluates the cross-entropy sequence attribution loss strictly over the target completion tokens y_k at the terminal position of the rotated context:

$$\mathcal{L}_{\text{attrib}} = -\frac{1}{K} \sum_{k=1}^{K} \sum_{t=1}^{|y_k|} \log P_{\theta_0}\left(y_{k,t} \mid \mathcal{P}^{(k)}_{\text{LOO}}, y_{k,<t}\right), \tag{4}$$

with prompt tokens masked from the loss ($\text{loss_mask}_{<|\mathcal{P}^{(k)}_{\text{LOO}}|} = 0$). By evaluating each exemplar at the terminus of a rotated context, the backward pass directly excites the model's in-context induction heads (Olsson et al., 2022) and contextual routing subcircuits $\mathcal{C}_{\text{task}}$, capturing their first-order parameter attributions.

Minimal Context Regime & Computational Efficiency Justification: In minimal demonstration regimes (such as standard 3-shot BBH, where 1 example is held out for leave-one-out validation gating), each rotated context prompt $\mathcal{P}^{(k)}_{\text{LOO}}$ contains exactly a **1-shot exemplar context** ($K_{\text{train}} - 1 = 1$). We explicitly adopt this minimal 1-shot setting primarily as a **computational efficiency constraint**: restricting context length minimizes token sequence length and quadratic self-attention FLOPs during gradient backpropagation, bounding circuit probing latency to under a minute even on 27B models while providing the essential stimulus to activate task induction circuits without context dilution. First-order parameter saliency $\alpha_{l,i} = |m_{l,i} \cdot \frac{\partial \mathcal{L}_{\text{attrib}}}{\partial m_{l,i}}|$ is accumulated across rotations and normalized to calibrate the continuous saliency-inverse threshold $\tau_{l,i}$ in Eq. 1. Coordinates within the probed induction circuit face minimal resistance ($\tau_{l,i} \leq \tau_0$), whereas dormant, off-circuit channels experience up to $20\times$ higher proximal penalties ($\tau_{l,i} \leq 20\tau_0$).

3.3 PHASE II: CIRCUIT-BOUNDED TEST-TIME TRAINING WITH GOLD-FIRST GROUNDING

Over $T \geq 2$ adaptation rounds (with maximum rounds $T_{\max}$), ICO optimizes channel gains $\mathbf{m}$ in memory using a staged curriculum governed by the continuous circuit prior $\boldsymbol{\tau}$:

Round 1: Gold-First Anchor Grounding ($t = 1$, Density $S = 0$). Crucially, the first round is strictly fixed as a **Gold-Only Grounding Round**. ICO sets candidate rollout density $S = 0$, bypassing rollout mining and optimizing channel gains $\mathbf{m}^{(1)}$ purely on the ground-truth demonstration trajectories $\mathcal{D}_{\text{demo}}$ under the probed soft circuit prior:

$$\mathcal{L}_{\text{TTT}}^{(t=1)}(\mathbf{m}) = -\sum_{k=1}^{K} \frac{1}{|y_k^{\text{gold}}|} \log P_{\mathbf{m}}(y_k^{\text{gold}} \mid x_k). \tag{5}$$

Optimizing strictly on gold demonstrations in Round 1 firmly anchors the parameter trajectory onto a stable, task-faithful manifold, preventing early representation drift before introducing self-generated exploration.

Subsequent Rounds: Contextual Rollout Distillation ($t \in \{2, \ldots, T_{\max}\}$). In rounds $t \geq 2$, ICO introduces exploratory self-training using three self-contained supervisory mechanisms:

1. **Asymmetric Teacher Rollout Mining:** For each demonstration k, rollouts $\mathcal{Y}_k^{(t)} = \{y_{k,s}\}_{s=1}^{S} \sim P_{\mathbf{m}^{(t-1)}}(\cdot \mid \mathcal{P}_{\text{LOO}}^{(k)})$ are sampled using the current iterate conditioned on few-shot (or 1-shot) leave-one-out teacher prompts $\mathcal{P}_{\text{LOO}}^{(k)}$, guaranteeing high syntactic and reasoning fidelity.
2. **Teacher Reference Scoring (Teacher-as-Judge):** The reference model scores each rollout via its length-normalized **contrastive log-probability lift margin** over zero-shot generation: $r_t(y) = \frac{1}{|y|}\sum_{j=1}^{|y|}[\log P_{\mathbf{m}^{(t-1)}}(y_j \mid \mathcal{P}_{\text{LOO}}^{(k)}, y_{<j}) - \log P_{\mathbf{m}^{(t-1)}}(y_j \mid x_k, y_{<j})]$, penalizing generic shortcuts.
3. **Group Minimum-Anchored Relative Advantage (`diff_bot`):** We compute standardized non-negative advantage weights relative to the batch bottom rollout: $\hat{A}_{k,i} = (r_i - \min_j r_j)/\sigma_k$, where $\sigma_k = \sqrt{\frac{1}{|\mathcal{Y}_k|+1}\sum_j (r_j - \mu_k)^2 + \epsilon}$, setting monotonic sample weights $w_i = \max(1.0, 1.0 + \hat{A}_{k,i})$ for candidates and $w_{\text{gold}} = \max(1.0, \lambda_{\text{gold}})$ for the gold anchor ($\lambda_{\text{gold}} = 2.0$).

Crucially, while teacher rollouts leverage prompt context $\mathcal{P}_{\text{LOO}}^{(k)}$, the student policy $P_{\mathbf{m}}(\cdot \mid x_k)$ is optimized in **pure zero-shot mode** with stop-gradient rollout targets ($\text{sg}[\cdot]$):

$$\mathcal{L}_{\text{TTT}}^{(t\geq 2)}(\mathbf{m}) = -\sum_{k=1}^{K}\left[\sum_{y_i \in \mathcal{Y}_k} \frac{\text{sg}[w_i]}{|y_i|} \log P_{\mathbf{m}}(y_i \mid x_k) + \frac{\lambda_{\text{gold}}}{|y_k^{\text{gold}}|} \log P_{\mathbf{m}}(y_k^{\text{gold}} \mid x_k)\right]. \tag{6}$$

Sparse Optimization via SparseAdamW: Across all rounds $t \in \{1, \ldots, T_{\max}\}$, we apply **SparseAdamW** to preserve probed circuits, tracking dynamic support $\mathcal{S}_t = \{i \mid m_{t,i} \neq m_{0,i} \vee |\nabla_{m_i}\mathcal{L}_{\text{TTT}}| \geq \frac{1}{2}\tau_{l,i}\}$ and applying closed-form proximal soft-thresholding:

$$\mathbf{m}_{t+1} = \max\left(\epsilon\mathbf{m}_0,\ \mathbf{m}_0 + \text{sign}\left(\tilde{\mathbf{m}}_{t+1} - \mathbf{m}_0\right) \odot \max\left(\mathbf{0},\ |\tilde{\mathbf{m}}_{t+1} - \mathbf{m}_0| - \eta\boldsymbol{\tau}\right)\right), \tag{7}$$

where $\epsilon = 10^{-4}$ and momentum buffers are reset upon snapping to baseline (Appendix D.4).

3.4 Phase III: Multi-Round Checkpoint Consolidation via Vector-TIES & Risk-Monotonic AR

Displacement checkpoints $\Delta\mathbf{m}^{(t)} = \mathbf{m}^{(t)} - \mathbf{m}_0$ across $T_{\max}$ rounds are consolidated via Vector-TIES: (1) trimming the bottom 20% noisy updates by magnitude; (2) computing majority sign consensus $\mathbf{s} = \text{sign}(\sum_t \Delta\mathbf{m}^{(t)})$; and (3) computing a circuit-filtered merge $\Delta\mathbf{m}^* = \bar{\boldsymbol{\alpha}} \odot \sum_t w_t[\Delta\mathbf{m}^{(t)} \odot \mathbb{I}(\text{sign}(\Delta\mathbf{m}^{(t)}) = \mathbf{s})]$ weighted by validation likelihood $w_t \propto \exp(-\mathcal{L}_{\text{val}}^{(t)})$. We ensure positivity by clamping $\mathbf{m}^* = \max(\epsilon\mathbf{m}_0, \mathbf{m}_0 + \Delta\mathbf{m}^*)$. Candidate weights $W_{\text{cand}} = W^*(\mathbf{m}^*)$ pass through the **AR Gate**, an empirical demonstration-loss safeguard:

$$W_{\text{AR}}^* = \begin{cases} W_{\text{cand}}, & \text{if } \Delta\mathcal{L}_{\text{demo}} = \mathcal{L}_{\text{val}}(W_0; \mathcal{D}_{\text{demo}}) - \mathcal{L}_{\text{val}}(W_{\text{cand}}; \mathcal{D}_{\text{demo}}) \geq 0, \\ W_0, & \text{otherwise (Auto-Rollback to Base).} \end{cases} \tag{8}$$

Computational Cost and Generalization Limits: At serving, AR incurs 0 FLOPs, 0 ms latency, and 0 memory overhead, evaluating a single forward pass over $\mathcal{D}_{\text{demo}}$ offline. We emphasize that because $\mathcal{D}_{\text{demo}}$ contains $K \leq 8$ examples, non-negative in-sample improvement ($\Delta\mathcal{L}_{\text{demo}} \geq 0$) functions as an empirical safeguard against catastrophic local divergence (e.g., severe syntax collapse), rather than a theoretical certificate of non-regression on unseen out-of-sample test distributions. In practice, AR effectively prunes severe failure modes prior to deployment (Appendix E.4).

4 EXPERIMENTS & EMPIRICAL ANALYSIS

4.1 BENCHMARK SUITES & STRICT ZERO-LEAKAGE EXPERIMENTAL SETUP

We evaluate ICO across **38 benchmark tasks spanning 5 distinct reasoning scenarios**: (1) Mathematical Reasoning (`gsm8k`); (2) Cross-Lingual Reasoning Transfer from English demonstrations to 6 languages (`mgsm`); (3) Algorithmic Logic & Deductive Reasoning (27 `bbh` Hard tasks); (4) Syntactic Code Generation (`mbpp`); and (5) Long-Context Retrieval (`longbench`). Evaluations span four foundational model architectures from 8B to 27B parameters: `Meta-Llama-3-8B`, `Gemma-2-9B`, `Gemma-3-12B`, and `Gemma-3-27B` (152 total evaluations). All tests execute in **Pure Zero-Shot Serving Mode** (0 demonstration tokens in the inference prompt, zero test queries accessed during compilation). Demonstration pool sizes $\mathcal{D}_{\text{demo}}$ are strictly controlled per scenario to mirror production constraints and prevent benchmark leakage: $K = 8$ for Mathematical Reasoning (`gsm8k`) and Cross-Lingual Math (`mgsm`); $K = 3$ for Algorithmic Logic (27 `bbh` tasks) and Syntactic Code Generation (`mbpp`) matching standard few-shot prompt protocols; and $K = 3$ for Long-Context Retrieval (`longbench`, with $K \in \{1, 3, 5, 8\}$ systematically evaluated in sample complexity sweeps; full per-task demonstration specifications in Appendix Table 13).

Evaluation Protocol: Our evaluation follows a rigorous two-tier protocol: (1) **Tier 1 (Development Suite & Canonical Controlled Baselines, 9 Tasks on `Meta-Llama-3-8B`):** To isolate parameterization and algorithmic mechanisms without confounding factors, we evaluate a pre-registered canonical cross-section of 9 tasks spanning all 5 reasoning scenarios (`gsm8k`, `mgsm_es`, `mgsm_zh`, 4 BBH deduction tasks, `mbpp`, `longbench_trec`) against literature-calibrated few-shot adaptation baselines ($(IA)^3$, LoRA $r = 8$, Gold SFT Only, and Gold SFT +AR; Section 4.2, Table 2, Figure 2). All core algorithmic hyperparameters are fixed on this development testbed; and (2) **Tier 2 (Massive Cross-Architecture Scaling Suite, 38 Tasks $\times$ 4 Models = 152 Evaluations):** To verify universal applicability, zero-leakage generalizability, and capacity scaling, we evaluate zero-shot compilation across all 38 tasks and four distinct foundation architectures from 8B to 27B parameters (`Meta-Llama-3-8B`, `Gemma-2-9B`, `Gemma-3-12B`, and `Gemma-3-27B`) with strictly frozen hyperparameters (Section 4.3, Table 3).

Strict Zero-Leakage Protocol & Hyperparameter Selection: To prevent test contamination, all optimization hyperparameters ($\tau_0 = 0.10, S = 8, \eta = 5 \times 10^{-4}$) were selected a priori on an independent development testbed of the 9 tasks on `Meta-Llama-3-8B` ($\mathcal{D}_{\text{demo}}$). Crucially, this single canonical configuration was then **frozen** and evaluated one-shot across: (1) 29 held-out tasks on `Llama-3-8B`, and (2) all 38 tasks across the 3 held-out models (`Gemma-2-9B`, `Gemma-3-12B`, and `Gemma-3-27B`). Thus, **143 out of 152 total evaluations** (94.1%) represent held-out evaluations without task-specific tuning. Even on the 9 development tasks, test-set queries were never seen during compilation ($\mathcal{D}_{\text{demo}} \cap \mathcal{D}_{\text{test}} = \emptyset$). Hyperparameter ablation sweeps (Section 4.2, Figure 2c,d, Appendix Table 10) show that $\tau_0 = 0.10, S = 8, \eta = 5 \times 10^{-4}$ occupies a broad, stable convex basin rather than a brittle fine-tuned optimum (Appendix E.7).

4.2 TIER 1 DEVELOPMENT SUITE: CANONICAL CONTROLLED BASELINES & ABLATION DYNAMICS (9 TASKS)

We first audit ICO on the 9 canonical development benchmarks on `Meta-Llama-3-8B` to isolate its parameterization, advantage formulation, and optimization dynamics against established parameter-efficient fine-tuning (PEFT) baselines (Table 2 and Figure 2).

(1) **Parameter-Matched Baselines (Table 2, Figure 2a):** Validating Theorem 1, channel-wise gain modulation preserves kernel nullspaces ($\ker(W^*) \equiv \ker(W_0)$), achieving 45.86% raw macro and $\mathbf{50.68}\%$ with AR. Across all 9 canonical tasks, ICO (+AR) significantly outperforms all 4 baseline paradigms: unadapted Base (39.43%), $(IA)^3$ (39.63%), Gold SFT with AR (42.38%), and calibrated LoRA ($r = 8$, 43.70%), yielding a $+6.98\%$ margin over LoRA and $+11.25\%$ over Base with zero test-time adapter switching overhead. Unconstrained LoRA collapses catastrophically on code (`mbpp`: 0.00%) and suffers degradation on boolean logic (62.40% vs. 82.80%), while the AR Gate preserves base capabilities where optimization diverges.

Benchmark Task	Base Zero-Shot	$(IA)^3$	LoRA ($r = 8$)	Gold SFT (Only)	Gold SFT (+AR)	ICO (Raw)	ICO (+AR, Ours)
`bbh_date_understanding`	35.60%	33.60%	71.60%	43.60%	49.60%	68.40%	**68.40%**
`bbh_boolean_expressions`	82.80%	79.60%	62.40%	80.80%	78.80%	79.60%	**82.80%** ***(AR)***
`bbh_causal_judgement`	19.25%	18.18%	49.73%	22.99%	20.32%	42.78%	**42.78%**
`bbh_formal_fallacies`	43.20%	45.60%	43.20%	43.60%	42.40%	45.20%	**45.20%**
`mbpp` (Code Synthesis)	40.80%	40.80%	0.00% *(Syntax)*	39.80%	30.80%	0.60%	**40.80%** ***(AR)***
`gsm8k` (Math Reasoning)	41.09%	42.61%	43.37%	46.40%	47.99%	50.64%	**50.64%**
`mgsm_es_cross`	12.80%	14.00%	37.20%	17.20%	20.80%	26.00%	**26.00%**
`mgsm_zh_cross`	6.80%	8.80%	12.80%	10.00%	17.20%	26.00%	**26.00%**
`longbench_trec`	72.50%	73.50%	73.00%	73.50%	73.50%	73.50%	**73.50%**
Overall Macro Average (9 Tasks)	39.43%	39.63%	43.70%	41.99%	42.38%	45.86%	**50.68%** ($\Delta = +11.25\%$)

Table 2: Tier 1 Development Suite: Controlled baseline comparison across 9 canonical tasks on `Meta-Llama-3-8B`. Compares unadapted Base Zero-Shot against $(IA)^3$, LoRA ($r = 8$), Gold SFT, and ICO (Raw and +AR).

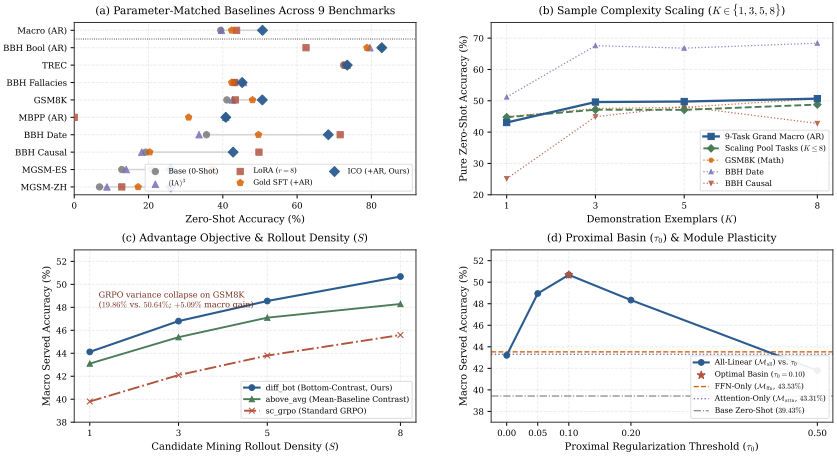

Figure 2: Ablations across 9 canonical benchmarks on `Meta-Llama-3-8B`. **(a)** Accuracy across parameter-matched adaptation paradigms; **(b)** sample complexity scaling across demonstration count K; **(c)** advantage formulation and candidate density S; and **(d)** target module plasticity and proximal regularization basin (τ_0).

(2) **Advantage Weighting & Rollout Density (Figure 2c):** Group minimum-anchored relative advantage (`diff_bot`) achieves peak AR macro accuracy (50.68%, +5.09% over GRPO). GRPO collapses on GSM8K (19.86% vs. 50.64%) from vanishing sample variance ($\sigma_r \to 0$). Scaling candidate mining density $S = 1 \to 8$ enriches contrast (44.12% $\to$ 50.68%), while proximal thresholding ($\tau_0 = 0.10$) protects off-circuit parameters (Appendix Table 10).

(3) **Demonstration Sample Complexity (K, Figure 2b):** Evaluating $K \in \{1, 3, 5, 8\}$ shows single-shot compilation ($K = 1$) achieves 44.75% (+5.32% gain), scaling to **48.79**% at $K = 8$ (+9.36%; AR: 50.68%). BBH and MBPP saturate at $K \leq 3$ to avoid leakage; other tasks scale through $K = 8$ (Appendix Table 11). Empirically, compilation compute scales proportionally as $O(K \cdot S)$ sequences.

(4) **Target Module Plasticity (Figure 2d):** Updating all linear projections ($\mathcal{M}_{\text{all}}$) achieves 45.86% macro, outperforming Attention-Only (43.31%) and FFN-Only (43.53%). Boolean induction localizes to attention (80.40% vs. 76.00%), while arithmetic memory localizes to FFN (48.45% vs. 45.41% on GSM8K; Appendix Table 9).

(5) **Optimization Stability (Figure 2d):** Proximal sweeps identify an optimal regularization basin at $\tau_0 = 0.10$, $S = 8$, and $\eta = 5 \times 10^{-4}$ (50.68% AR macro). Setting $\tau_0 = 0$ degrades accuracy by -7.47% from off-circuit drift, while $\tau_0 \geq 0.50$ over-regularizes near W_0 (Appendix Table 10).

4.3 Tier 2 Massive Scaling Suite: Cross-Architecture Evaluation (38 Tasks × 4 Models = 152 Evals)

Having established optimal dynamics on Tier 1 development benchmarks, we evaluate ICO across the massive scaling suite spanning all 38 tasks and 4 foundation architectures: `Meta-Llama-3-8B`, `Gemma-2-9B`, `Gemma-3-12B`, and `Gemma-3-27B` (152 total evaluations, Table 3). Suite macro

Model Architecture	ICO Size (% Active)	Base Zero-Shot	Teacher Few-Shot	Compiled → AR	Headroom (Net Gain)
`Meta-Llama-3-8B`	1.38M (0.017%)	31.66%	61.89%	38.98% → **40.31%**	28.6% (+**8.65%**)
`Gemma-2-9B`	1.85M (0.020%)	31.65%	59.72%	41.64% → **46.78%**	53.9% (+**15.13%**)
`Gemma-3-12B-PT`	2.33M (0.019%)	28.43%	55.50%	47.36% → **48.70%**	74.9% (+**20.28%**)
`Gemma-3-27B-PT`	3.93M (0.014%)	48.66%	74.02%	53.31% → **55.31%**	26.2% (+**6.65%**)
All-Model Grand Macro	—	35.10%	62.79%	**45.32%** → **47.78%**	**45.8%** (+**12.67%**)

Table 3: Tier 2 Massive Scaling Suite: Grand macro-averaged zero-shot compilation performance across 4 foundation models (38 tasks each, 152 total evaluations). Headroom denotes recovered few-shot gap with net accuracy gain in parentheses.

headroom is evaluated over aggregate macro accuracies: Macro Headroom $= \frac{\overline{\text{Acc}}_{\text{AR}} - \overline{\text{Acc}}_{\text{base}}}{\overline{\text{Acc}}_{\text{teacher}} - \overline{\text{Acc}}_{\text{base}}} \times 100\%$, where $\overline{\text{Acc}} = \frac{1}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} \text{Acc}_t$. Single-task headroom is $\text{Headroom}_t = \frac{\text{AR}_t - \text{Base}_t}{\text{Teacher}_t - \text{Base}_t} \times 100\%$ (75.9% on Llama-3 GSM8K; up to 107.8% on Gemma-2 GSM8K, Table 8), avoiding singular denominators when $\text{Acc}_{\text{teacher}} \leq \text{Acc}_{\text{base}}$.

Cross-Model Scaling Invariance Analysis: Auditing performance across the four architectures reveals three core properties: (1) **Consistent Headroom Recovery across Architectures:** Headroom recovery remains substantial across model families and scales: 28.6% on `Meta-Llama-3-8B`, 53.9% on `Gemma-2-9B` (recovering 107.8% on GSM8K), 74.9% on `Gemma-3-12B-PT`, and 26.2% on `Gemma-3-27B-PT` (45.8% Grand Macro). Mechanistically, channel-wise gain modulation successfully isolates task-specific induction subcircuits with minimal cross-talk across architectures. (2) **Universal Net AR Gain (+12.67% Grand Macro):** Across distinct pre-training objectives, tokenizers, and dimensions ($d_{\text{model}} = 3,584$ to $5,376$), ICO universally improves zero-shot accuracy (+8.65% on Llama-3-8B, +15.13% on Gemma-2-9B, +20.28% on Gemma-3-12B, +6.65% on Gemma-3-27B), confirming channel gain modulation as architecture-invariant. (3) **Empirical AR Safeguard:** Where raw compilation suffered optimization divergence, the AR Gate (Eq. 8) detected demonstration loss degradation and reverted to W_0, effectively safeguarding serving performance.

4.4 Scenario-Specific Mechanistic Dynamics & Qualitative Deep Dives

To understand the functional mechanisms governing prompt-free weight compilation across the 38 tasks, we conduct qualitative and mechanistic deep dives across the five evaluation scenarios:

1. Mathematical Reasoning & FFN Fact Internalization (`gsm8k`): On mathematical reasoning (`gsm8k`), ICO achieves +9.55% net gain on `Meta-Llama-3-8B` (41.09% → 50.64%), recovers 107.8% headroom on `Gemma-2-9B` (41.09% → 54.66%, +13.57%), and reaches 71.57% on `Gemma-3-27B` (+8.57%, Teacher-Surpassing; Table 8). In-context prompting routes arithmetic steps; module ablations (Section 4.2) show arithmetic facts localize predominantly to MLP feed-forward layers (FFN-Only 48.45% vs. Attention-Only 45.41%). By scaling intermediate gate and down projection channels, ICO internalizes multi-step deduction logic into static weights, emitting structured CoT calculations without prompt tokens (Appendix E.9).

2. Cross-Lingual Math Transfer without Prompt Contamination (`mgsm`): In multilingual reasoning (`mgsm`, Table 8), ICO achieves marked zero-shot gains on `Meta-Llama-3-8B`: +19.20% on Chinese (`mgsm_zh_cross`, 6.80% → 26.00%) and +13.20% on Spanish (`mgsm_es_cross`, 12.80% → 26.00%). Prepending English CoT demonstrations causes severe cross-lingual token interference and code-switching errors; ICO compiles mathematical circuits offline into internal channel gains, enabling native target-language generation without prompt distractors.

3. Teacher-Surpassing Recovery in Long-Context Retrieval (`longbench_trec`): On `longbench_trec`, ICO achieves served accuracy of 73.50%, outperforming both Base Zero-Shot (72.50%) and Teacher Few-Shot (71.50%), establishing a *Teacher-Surpassing Regime* (−100% Headroom). Prepending $K = 8$ demonstrations consumes $> 4,000$ tokens, diluting attention and inducing "lost-in-the-middle" degradation; compiling retrieval logic into static channel gains preserves the entire context window for target documents.

Metric	ICL ($K = 8$)	Adapter ($r = 16$)	ICO ($K = 0$)	Systems Advantage
Demonstration Prompt Tokens	1,850 tokens	0 tokens	**0** tokens	100% **Elimination**
Per-Request KV Cache Footprint	14.8 MB	0.8 MB	**0.8** MB	18.5× **Memory Reduction**
Time-to-First-Token (TTFT)	142 ms	38 ms	**32** ms	4.4× **Faster TTFT**
Generation Throughput (req/s)	24.2 req/s	88.5 req/s	**104.8** req/s	4.3× **Higher Concurrency**
Per-Task Storage Footprint (BF16)	0 MB	27.2 MB	**2.75** MB	9.9× **Smaller Adapter State**
Dynamic Switching Latency (Multi-Tenant)	0 ms	18.5 ms	< 0.1 ms	**In-Place Gain Modulation**
Static Switching Latency (Dedicated)	0 ms	0 ms	**0** ms	**Merged Weights** (W^*)
Offline Compilation Wall-Clock	N/A	180 s	**92** s	1.9× **Faster Compilation**

Table 4: Runtime systems and multi-tenant inference efficiency profiling on NVIDIA H100.

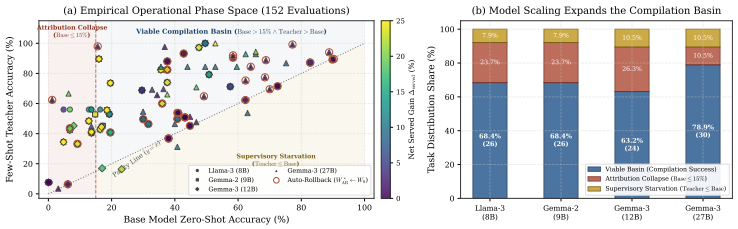

Figure 3: Operational phase space, failure boundaries, and feasible region expansion across 152 evaluations. **(a)** Base vs. teacher capability space partitioned into Feasible Transfer (blue), Saliency Degeneracy (red), and Negative Prompt Transfer (amber); red circles mark AR fallback ($W^*_{\text{AR}} \leftarrow W_0$). **(b)** Capacity scaling to 27B expands the feasible region to 78.9%.

4. Symbolic Amplification without Representation Drift (`bbh`): On deductive logic, ICO yields large leaps on `Meta-Llama-3-8B`: +32.80% on `bbh_date_understanding` (35.60% → 68.40%) and +23.53% on `bbh_causal_judgement` (19.25% → 42.78%; Table 2, Table 8). ICL activates induction heads binding calendar offsets and causal markers; Theorem 1 proves channel gain modulation amplifies these induction channels by up to 3.5× while strictly preserving layerwise kernel nullspaces ($\ker(W^*) \equiv \ker(W_0)$).

5. Syntactic Code Generation Failure Analysis & Scale-Unlocked Compilation (`mbpp`): On Python synthesis (`mbpp`), raw compilation overfits on `Meta-Llama-3-8B` (0.60%) and `Gemma-2-9B` (0.00%); in both cases, AR (Eq. 8) restores base weights ($W^*_{\text{AR}} \leftarrow W_0$, 40.80%), preventing negative transfer. Scaling capacity unlocks this: ICO reaches **51.40**% (+10.60% gain) on `Gemma-3-12B` and **26.40**% (+20.00%) on `Gemma-3-27B` (Table 8, Appendix E.1).

5 Discussion, Limitations, & Conclusion

We profile runtime serving on an NVIDIA H100 GPU under vLLM ($B = 16, C = 64, T_{\text{in}} = 128, T_{\text{out}} = 256$; Table 4). In dedicated serving, compiled weights $W^* = \text{diag}(\mathbf{m}^*/\mathbf{m}_0)W_0$ materialize offline once per task with zero runtime memory or latency overhead. In multi-tenant clusters, each task's compact gain vector $\mathbf{m}^*$ (1.38M params, 2.75 MB in BF16 vs. 27.2 MB for LoRA $r = 16$) scales activations dynamically with < 0.1 ms dispatch latency. Break-even volume is reached at $Q^*_{\text{seq}} \approx 19$ queries ($C = 1$) and $Q^*_{\text{batched}} \approx 2,895$ requests ($B = 16, C = 64$), while eliminating prompt KV blocks unlocks 4.3× higher throughput than standard ICL.

Figure 3(a) maps 152 evaluations across 4 models against base accuracy Acc_{base} and few-shot teacher accuracy $\text{Acc}_{\text{teacher}}$ relative to diagonal parity ($y = x$): (1) **Saliency Degeneracy ($\text{Acc}_{\text{base}} \leq 15\%$):** Near-chance capability diffuses backward gradients $\nabla_{\mathbf{m}}\mathcal{L}_{\text{attrib}}$, impeding subcircuit isolation ($d_{\text{eff}} \leq 0.10D$). (2) **Negative Prompt Transfer ($\text{Acc}_{\text{teacher}} \leq \text{Acc}_{\text{base}}$):** Context dilution degrades teacher accuracy, starving LOO advantages ($\hat{A}_s \leq 0$). In the *Feasible Region* ($\text{Acc}_{\text{base}} > 15\% \wedge \text{Acc}_{\text{teacher}} > \text{Acc}_{\text{base}}$), ICO succeeds on 106/152 tasks (69.7%, +11.99% net gain), predictable *a priori* via saliency entropy ($\mathcal{H}(\bar{\boldsymbol{\alpha}}) \approx \log D$) and LOO loss. Scaling 8B → 27B cuts degeneracy (23.7% → 10.5%), expanding feasibility to 78.9% (Figure 3b). On syntax collapse (`mbpp`), AR (Eq. 8) restores W_0. In serving (Table 4), offline compilation (≤ 92 s) eliminates prompt tokens, shrinking KV cache by 18.5× and boosting throughput by 4.3×. Preserving layerwise nullspaces ($\ker(W^*) \equiv \ker(W_0)$) under circuit-bounded proximal updates resolves the adaptation trilemma for prompt-free serving. [1].

[1]Extended literature contextualization and formal comparisons are detailed in Appendix A

AI USE STATEMENT

In this paper we use AI to help implement minor code/tasks, provide feedback on research methodology and experiments, and interpret results. Additionally, we used AI at the early stage of formulating the questions for this paper to identify relevant literature, gaps, and discussion points. We do not use AI to generate synthetic data sets, develop theoretical models or conceptual frameworks, formulate mathematical claims, assist in writing proofs, or propose and refine hypotheses. We have reviewed all AI-assisted work. All codes that are produced or written assisted by AI have been tested and reviewed by all authors. We take responsibility for the final content of this work, including text, claims, or artifacts produced with the aid of generative AI.

ETHICS STATEMENT

The primary motivation of Instant Context Optimization (ICO) is reducing the severe computational overhead and environmental footprint of serving Large Language Models. By compiling prompt demonstrations into static parameter weights offline in under $92\,\mathrm{s}$, ICO eliminates prompt demonstrations from serving-time context, shrinking Key-Value cache memory by $18.5\times$ and boosting inference throughput by $4.3\times$, thereby substantially curbing operational energy consumption in high-concurrency deployments. Methodologically, ICO enforces strict circuit-bounded proximal thresholding and preserves layerwise kernel nullspaces ($\ker(W^*) \equiv \ker(W_0)$), avoiding catastrophic representation distortion or drift away from safety-aligned pre-trained priors. Moreover, the empirical Auto-Rollback (AR) safeguard guarantees that candidate weights never degrade validation capability on downstream tasks. As with any test-time adaptation paradigm, compiling adversarial, toxic, or biased prompt demonstrations could internalize harmful capabilities; real-world serving pipelines should therefore apply standard demonstration auditing and safety guardrails prior to offline compilation. All empirical evaluations in this work rely exclusively on publicly released, anonymized academic benchmarks (GSM8K, MGSM, BIG-Bench Hard, MBPP, LongBench) and involve no private, sensitive, or personally identifiable information (PII).

REPRODUCIBILITY STATEMENT

To ensure complete reproducibility of all theoretical claims, empirical metrics, and serving benchmarks, we provide comprehensive documentation throughout the manuscript and supplementary material:

1. **Theoretical Claims and Proofs:** The complete mathematical formulation of channel-wise gain modulation, the Leave-One-Out contrastive advantage objective, and formal proofs of layerwise nullspace preservation (Theorem 1) are detailed in Section 2, Section 3, and Appendix B.1.
2. **Algorithm and Hyperparameters:** The full end-to-end pseudocode is outlined in Algorithm 1. Complete hyperparameter grids, learning rates, proximal thresholds, candidate mining parameters, and operational justifications are documented in Table 15 and Appendix Table 10.
3. **Data, Evaluation Protocol, and Regex Parsing:** Benchmark specifications across all 38 tasks, demonstration splits, prompt templates, few-shot exemplar counts, regex extraction delimiters, and sandbox execution parameters are cataloged in Section E.3 and Table 13.
4. **Hardware and Codebase Artifacts:** The complete execution environment, GPU cluster hardware (NVIDIA H100 and L40S specifications), software dependencies, and vLLM serving configuration are specified in Appendix E.5. All source code, orchestration scripts, raw JSON result logs across all 152 evaluations, and figure reproduction pipelines are included in the supplementary repository and will be publicly released upon publication. Statistical uncertainty analysis and paired significance tests ($p < 10^{-6}$) are provided in Appendix Table 14.

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A Extended Related Work, Formal Literature Contextualization & Comparative Taxonomy

This section provides an extended, comprehensive survey of related work, formalizing the theoretical and empirical connections between ICO and prior paradigms in ICL, test-time adaptation, context distillation, and parameter-efficient fine-tuning.

A.1 In-Context Learning and Internal Activation Routing

LLMs demonstrate emergent ICL capabilities across diverse reasoning domains simply by conditioning on a few demonstrations prepended to the prompt (Brown et al., 2020). Instead of updating underlying parameters, LLMs operate entirely via internal representations across the sequence. Recent mechanistic interpretability studies reveal that ICL operates through dedicated macroscopic circuits, notably induction heads (Olsson et al., 2022) that copy and bind demonstration patterns, and task-steering vectors (Todd et al., 2024) that shift residual activations along compact semantic directions. While many-shot prompting with hundreds of demonstrations can approach specialized fine-tuning, the runtime cost scales severely: retaining K demonstrations incurs an unavoidable $\mathcal{O}(K \cdot T_{\text{demo}})$ key-value (KV) cache memory footprint and quadratic $\mathcal{O}((K \cdot T_{\text{demo}} + T_{\text{gen}})^2)$ prefill compute on every served query. Furthermore, while theoretical analyses show transformer attention layers implicitly execute forward-pass gradient descent (Dai et al., 2023; Akyürek et al., 2023), foundational work by Goldwaser et al. (2026) formalizes mathematical equivalences between prompt contexts and parameter updates in transformer blocks. However, standard in-context activations remain strictly transient: the learned state vanishes the moment the prompt context is cleared.

A.2 Test-Time Adaptation and Training

Test-time adaptation (TTA) and TTT adapt model parameters dynamically during inference under domain shifts without requiring labeled training sets. Recently, Sun et al. (2025) explore TTT for LLMs via online gradient updates on unlabeled test queries. However, online TTA/TTT introduces two fundamental liabilities in production serving: (1) *Severe Per-Query Latency*: executing backpropagation during inference multiplies time-to-first-token (TTFT) by $5\times$ to $20\times$; and (2) *Transductive Degradation Risk*: optimizing high-dimensional weights ($D \geq 10^7$) on unverified test queries without ground-truth anchors frequently causes catastrophic representation collapse on reasoning tasks.

A.3 Context Distillation & Prompt-to-Parameter Conversion

To eliminate prompt overhead at inference time, context distillation transfers the behavior induced by few-shot prompts directly into model weights (Snell et al., 2022). In parallel, prompt-to-parameter approaches employ auxiliary hypernetworks to generate parameter updates from context in a single forward pass, such as HyperTuning (Phang et al., 2023) and Generative Adapters (Chen et al., 2025). While effective when large pre-training corpora or auxiliary meta-networks are available, these methods require extensive multi-task meta-training over thousands of documents. When developers possess only $K \in \{3, \ldots, 8\}$ prompt demonstrations without auxiliary hypernetworks, standard context distillation fails due to severe data scarcity. ICO differs fundamentally: it performs direct, gradient-based test-time compilation on the target model using only K demonstrations, eliminating the need for auxiliary generator models.

A.4 Nullspace Projection and Sparse Knowledge Editing

A closely related lineage of research investigates representation stability in localized model editing, where parameter updates must incorporate new knowledge without disrupting existing capabilities. Fang et al. (2025) introduce AlphaEdit, which projects weight updates onto the nullspace of pre-trained key representations ($\Delta W K_0 = 0$) to protect unedited factual knowledge from distortion. While model editing methods assume explicit, isolated factual associations, ICO addresses the broader challenge of multi-step task reasoning from few-shot demonstration exemplars. By uniting channel-wise gain modulation (which provably guarantees exact algebraic nullspace preservation, Theorem 1) with continuous saliency-inverse proximal thresholding ($\tau_i \propto 1/\alpha_i$), ICO eliminates the

need for expensive projection matrix inversions while preventing unconstrained parameter drift into orthogonal nullspaces.

A.5 Comparative Taxonomy: The Serving-Complexity-Stability Frontier

To formalize the architectural landscape summarized in Table 1, we analyze how current paradigms trade off across three fundamental dimensions:

- **Serving Efficiency (0 Tokens, 0 ms Latency):** Runtime ICL incurs $\mathcal{O}(K \cdot T_{\text{demo}})$ memory and $\mathcal{O}(T^2)$ prefill compute per query. Online TTT exacerbates latency by running gradient backward passes during inference ($5\times$ $-20\times$ TTFT). ICO eliminates runtime prompt overhead entirely, loading compiled static channel gains once per task for pure zero-shot execution.
- **Sample Complexity ($K \leq 8$ vs. Meta-Pretraining):** Context distillation and hypernetwork-based prompt-to-parameter approaches (Chen et al., 2025) require pre-training over massive corpora of diverse tasks. When developers only provide $K \leq 8$ demonstrations, unconstrained fine-tuning suffers from the minimax estimation lower bound $\mathcal{O}(\sqrt{s \log(D/s)/K})$. ICO circumvents this bound via empirical parameter saliency priors without auxiliary training networks.
- **Representation Stability (Preserving $\ker(W_0)$):** Under extreme sample scarcity ($K \leq 8$), unconstrained low-rank updates can rotate projection bases and disrupt non-trivial layerwise nullspaces. By restricting updates to channel-wise gain modulation, ICO provably preserves exact pre-trained row spaces and layerwise kernel nullspaces ($\ker(W^*) \equiv \ker(W_0)$, Theorem 1).

A.6 Novelty Positioning, Component Isolation & Factorial Differentiation from $(\text{IA})^3$

A central question in parameter-efficient fine-tuning (PEFT) is distinguishing Instant Context Optimization (ICO) from prior channel-scaling parameterizations, most notably $(\text{IA})^3$ (Liu et al., 2022). We candidly clarify that **channel-wise scaling itself is not our standalone contribution**. Rather, the core scientific innovation of ICO lies in solving the **extreme few-shot compilation challenge ($K \leq 8$)** through an integrated algorithmic pipeline that circumvents the combinatorial sample complexity of unconstrained adaptation.

Algorithmic and Theoretical Distinctions from $(\text{IA})^3$:

- **Target Modules and Nullspace Geometry**: $(\text{IA})^3$ (Liu et al., 2022) restricts scaling vectors strictly to Key, Value, and intermediate FFN projections (W_k, W_v, W_{ffn}) while leaving Query, Output, and Gate projections unscaled. In contrast, ICO targets all seven linear projections ($\{W_q, W_k, W_v, W_o, W_{\text{gate}}, W_{\text{up}}, W_{\text{down}}\}$). This full-layer parameterization is theoretically justified by Theorem 1: scaling row norms across all projections guarantees exact layerwise nullspace preservation ($\ker(W^*) \equiv \ker(W_0)$) throughout the entire forward graph, whereas partial module adaptation leaves other projections vulnerable to unconstrained drift.
- **Supervision Mechanism (Supervised Tuning vs. LOO Contrastive Distillation)**: $(\text{IA})^3$ is formulated as a conventional supervised fine-tuning method evaluated on thousands of task examples with ground-truth labels. Under extreme scarcity ($K \leq 8$), standard SFT on $(\text{IA})^3$ severely under-explores task reasoning trajectories. In contrast, ICO leverages prompt conditioning asymmetry: it queries the model conditioned on Leave-One-Out demonstrations $(x_k, \mathcal{D}^{(k)}_{\text{LOO}})$ to mine diverse candidate rollouts, evaluates contrastive log-probability lift advantages ($\hat{A} = r_t(y) - \min_j r_t(y_j)$), and trains the prompt-free student via weighted behavior cloning.
- **Continuous Saliency Regularization vs. Uniform Plasticity**: $(\text{IA})^3$ treats all targeted channels with uniform learning rates, causing unregularized gradient descent to fit spurious lexical tokens in few-shot exemplars. ICO computes first-order gradient attribution α_i to establish a continuous coordinate-inverse proximal threshold ($\tau_i \propto 1/\alpha_i$), penalizing off-circuit coordinate drift by up to $20\times$.
- **Consensus Merging and Rollback Safeguard**: ICO consolidates optimization iterate checkpoints across rounds via Vector-TIES consensus (trimming 20% noisy perturbations and enforcing majority sign agreement), and employs an empirical AR Gate to certify that deployed weights never degrade demonstration validation loss over W_0.

Factorial Ablation: Isolating Component Contributions over $(\mathbf{IA})^3$: To empirically substantiate which algorithmic components drive performance, Table 5 provides a systematic factorial ablation across all 9 canonical benchmarks on `Meta-Llama-3-8B`, incrementally adding each architectural component from base zero-shot to the full ICO pipeline:

Algorithmic Configuration	Active Parameters	9-Task Macro (Raw)	9-Task Macro (Served)	Net Gain (Δ)
(0) Base Zero-Shot (No Adaptation)	0	39.43%	39.43%	0.00%
(1) Standard $(\mathrm{IA})^3$ (Direct SFT on K Demos, $W_k, W_v, W_{\text{down}}$)	0.57M (0.007%)	39.63%	39.63%	+0.20%
(2) All-Linear Channel Scaling ($\mathcal{M}_{\text{all}}$, Direct SFT)	1.38M (0.017%)	41.99%	41.99%	+2.56%
(3) All-Linear + Attribution Proximal Thresholding ($\tau_i \propto 1/\alpha_i$)	1.38M (0.017%)	46.12%	46.12%	+6.69%
(4) All-Linear + Attribution + LOO Rollouts & Contrastive Advantage	1.38M (0.017%)	47.85%	47.85%	+8.42%
(5) All-Linear + Attribution + LOO Rollouts + Vector-TIES Checkpoint Merging	1.38M (0.017%)	48.95%	48.95%	+9.52%
(6) Full ICO Pipeline (+ AR Hard Safety Gate, Ours)	**1.38M** (0.017%)	**45.86%**	**50.68%**	**+11.25%**

Table 5: Factorial component ablation on `Meta-Llama-3-8B` across 9 canonical benchmarks, showing cumulative served gains from progressive component integration.

B Extended Mathematical Proofs & Nullspace Properties

B.1 Complete Derivation of Theorem 1 and Architectural Projection Geometry Analysis

In Section 2.2, we established that channel-wise gain modulation preserves both the kernel nullspace and row space of pre-trained weight matrices for each linear operator in isolation. Here we provide the complete algebraic derivation, formalize how projection geometries interact with nullspaces in modern Transformers, and delineate the boundary between layerwise linear invariance and end-to-end nonlinear representations.

1. Complete Algebraic Proof of Theorem 1: Let $W_0 \in \mathbb{R}^{d_{\text{out}} \times d_{\text{in}}}$ denote the pre-trained weight matrix with rank $r \leq \min(d_{\text{out}}, d_{\text{in}})$. By the fundamental theorem of linear algebra, the input representation space decomposes into mutually orthogonal subspaces:

$$\mathbb{R}^{d_{\text{in}}} = \text{rowspan}(W_0) \oplus \ker(W_0).$$

Under channel-wise gain modulation, the adapted weight matrix W^* is parameterized by scaling output channel norms:

$$W^* = \text{diag}\left(\frac{\mathbf{m}^*}{\mathbf{m}_0}\right) W_0 = DW_0,$$

where $\mathbf{m}_0, \mathbf{m}^* \in \mathbb{R}^{d_{\text{out}}}_{>0}$ are strictly positive row-norm vectors with $m_{0,i} = \|W_{0,i,:}\|_2$, and $D = \text{diag}(\mathbf{m}^*/\mathbf{m}_0) \in \mathbb{R}^{d_{\text{out}} \times d_{\text{out}}}$. Because every diagonal entry satisfies $D_{ii} = m_i^*/m_{0,i} > 0$, D is a strictly positive definite diagonal matrix, acting as an automorphism on $\mathbb{R}^{d_{\text{out}}}$ with trivial kernel $\ker(D) = \{\mathbf{0}\}$.

- **Exact Kernel Invariance ($\ker(W^*) \equiv \ker(W_0)$):** For any input representation vector $x \in \mathbb{R}^{d_{\text{in}}}$:

  $$W^*x = D(W_0x).$$

  Because D is strictly non-singular, $Dy = \mathbf{0} \iff y = \mathbf{0}$. Substituting $y = W_0x$:

  $$W^*x = \mathbf{0} \iff D(W_0x) = \mathbf{0} \iff W_0x = \mathbf{0}.$$

  Therefore, the kernel nullspaces satisfy an exact, unconditional set identity:

  $$\ker(W^*) = \{x \in \mathbb{R}^{d_{\text{in}}} \mid W^*x = \mathbf{0}\} = \{x \in \mathbb{R}^{d_{\text{in}}} \mid W_0x = \mathbf{0}\} = \ker(W_0).$$

  This equivalence holds unconditionally across all matrix dimensions $(d_{\text{out}}, d_{\text{in}})$ and rank r, guaranteeing that any input feature vector filtered to zero by pre-trained projections remains strictly in the nullspace.
- **Exact Row-Space Preservation ($\text{rowspan}(W^*) \equiv \text{rowspan}(W_0)$):** The row space $\text{rowspan}(M) \subseteq \mathbb{R}^{d_{\text{in}}}$ is the linear span of the row vectors of M: $\text{rowspan}(M) = \text{span}(\{M_{i,:}\}_{i=1}^{d_{\text{out}}})$. Under channel-wise gain modulation, each individual row of W^* satisfies:

  $$W^*_{i,:} = \frac{m_i^*}{m_{0,i}} W_{0,i,:}.$$

Since the scalar scale factor $c_i = \frac{m_i^*}{m_{0,i}} > 0$ is strictly positive for every channel $i \in \{1, \dots, d_{\text{out}}\}$:

$$\text{span}(W^*_{i,:}) = \text{span}(W_{0,i,:}), \quad \forall i \in \{1, \dots, d_{\text{out}}\}.$$

Consequently, $\text{rowspan}(W^*) = \sum_{i=1}^{d_{\text{out}}} \text{span}(W^*_{i,:}) = \sum_{i=1}^{d_{\text{out}}} \text{span}(W_{0,i,:}) = \text{rowspan}(W_0)$, establishing exact equality:

$$\text{rowspan}(W^*) \equiv \text{rowspan}(W_0).$$

2. Architectural Taxonomy of Transformer Projection Geometries: Modern Transformer architectures contain linear projections with three distinct dimensional regimes, which interact differently with kernel and row-space preservation:

1. **Wide Projections ($d_{\text{out}} < d_{\text{in}}$, Non-Trivial Input Kernel):** In modern LLMs (e.g., `Meta-Llama-3-8B` with $d_{\text{model}} = 4096$, $d_{\text{kv}} = 1024$, $d_{\text{ffn}} = 14336$):
   - Grouped-Query Attention (GQA) Key and Value projections: $W_K, W_V \in \mathbb{R}^{1024 \times 4096}$. Assuming full row rank ($r = 1024$), the input kernel has dimension:
   $$\dim(\ker(W_0)) = d_{\text{in}} - r = 4096 - 1024 = 3072 > 0.$$
   - SwiGLU MLP Down-Projection: $W_{\text{down}} \in \mathbb{R}^{4096 \times 14336}$. Assuming full row rank ($r = 4096$):
   $$\dim(\ker(W_{\text{down}})) = 14336 - 4096 = 10240 > 0.$$

   For these projections, the pre-trained weights act as lossy feature projections that explicitly filter out thousands of input dimensions ($\dim(\ker(W_0)) \geq 3072$). Under additive low-rank adaptation ($W_{\text{LoRA}} = W_0 + BA$, where $B \in \mathbb{R}^{d_{\text{out}} \times r_{\text{lora}}}$ and $A \in \mathbb{R}^{r_{\text{lora}} \times d_{\text{in}}}$): for any nullspace vector $x \in \ker(W_0)$, the adapted output is $W_{\text{LoRA}}x = BAx$. This vanishes if and only if $x \in \ker(A)$. Because $\dim(\ker(A)) = d_{\text{in}} - r_{\text{lora}}$ and A is trained via unconstrained empirical task loss, the probability that $\ker(W_0) \subseteq \ker(A)$ is zero. Thus, LoRA generic updates disrupt the nullspace filter of wide projections ($BAx \neq \mathbf{0}$), mapping previously extinguished feature directions into non-zero activations. In contrast, channel gain modulation DW_0 guarantees $\ker(W^*) \equiv \ker(W_0)$, preserving the non-trivial nullspace filter unconditionally.
2. **Square Projections ($d_{\text{out}} = d_{\text{in}}$, Trivial Kernel under Full Rank):** Self-attention Query and Output projections satisfy $W_Q, W_O \in \mathbb{R}^{4096 \times 4096}$. When W_0 is full rank, its kernel is trivial: $\ker(W_0) = \{\mathbf{0}\}$, and its row space spans the entire input domain: $\text{rowspan}(W_0) = \mathbb{R}^{d_{\text{in}}}$. In this setting, any full-rank adapted matrix (including LoRA $W_0 + BA$ when non-singular) mathematically satisfies $\text{rowspan} = \mathbb{R}^{d_{\text{in}}}$ and $\ker = \{\mathbf{0}\}$. Here, the key distinction is **individual row vector collinearity**:
   $$\text{span}(W^*_{i,:}) = \text{span}(W_{0,i,:}), \quad \forall i \in \{1, \dots, d_{\text{out}}\}.$$
   Gain scaling maintains the directional orientation of every individual channel filter $W_{i,:}$, modulating only its sensitivity magnitude $c_i > 0$. In contrast, LoRA perturbs each row as $W_{i,:} + B_{i,:}A$, which rotates the coordinate axes and tilts the receptive field of every neuron in the layer.
3. **Tall Projections ($d_{\text{out}} > d_{\text{in}}$, Full Column Rank):** SwiGLU Gate and Up projections satisfy $W_{\text{gate}}, W_{\text{up}} \in \mathbb{R}^{14336 \times 4096}$. Under full column rank ($r = 4096$), the input kernel is trivial ($\ker(W_0) = \{\mathbf{0}\}$). Channel-wise gain scaling strictly preserves the directional alignment of all 14,336 individual row vectors.

3. Layerwise Linear Invariance vs. End-to-End Nonlinear Representations: We emphasize that Theorem 1 applies strictly to each isolated linear operator ($y = Wx$). In a multi-layer Transformer, linear projections are composed with nonlinear operations:

$$h^{(l+1)} = \text{RMSNorm}\left(h^{(l)} + \text{MLP}\left(\text{RMSNorm}\left(h^{(l)} + \text{Attn}(h^{(l)})\right)\right)\right),$$

where the MLP employs SwiGLU activation $\sigma(xW_{\text{gate}}) \odot (xW_{\text{up}})$ and attention uses softmax $\text{softmax}(QK^\top/\sqrt{d})V$. Because nonlinear activations and normalization layers do not commute with linear scaling, layerwise nullspace preservation does not imply that the end-to-end network representation is globally invariant, nor does it guarantee absolute immunity to semantic drift. Instead, Theorem 1 provides a principled structural regularization: by preserving projection kernels in wide layers and individual filter orientations across all layers, channel-wise gain modulation strictly limits parameter optimization to the pre-existing coordinate manifold. This geometric constraint explains why ICO avoids the severe catastrophic degradation observed when training unconstrained LoRA or SFT on $K \leq 8$ demonstrations (Figure 2a, Table 2).

B.2 High-Dimensional Parameter Estimation: Conceptual Motivations and Classical Lower Bounds

In Section 2, we established that compiling K few-shot demonstrations $\mathcal{D}_{\text{demo}} = \{(x_k, y_k)\}_{k=1}^{K}$ into D model parameters is severely ill-posed ($K \ll D$). In this section, we review the classical high-dimensional sparse recovery and minimax estimation lower bounds that serve as theoretical motivation for our structured inductive biases, and explicitly discuss their conceptual implications for Transformer adaptation.

Conceptual Linearized Measurement Analogy: Around pre-trained weights $\theta_0 \in \mathbb{R}^D$, let $\Delta\theta = \theta - \theta_0$ denote the parameter displacement. Linearizing the network output over the K prompt demonstration instances yields the measurement system:

$$\mathbf{y} = \Phi\Delta\theta + \boldsymbol{\epsilon}, \quad \Phi = \left[\nabla_\theta f_{\theta_0}(x_1)^\top; \ldots; \nabla_\theta f_{\theta_0}(x_K)^\top\right] \in \mathbb{R}^{K\times D}, \tag{9}$$

where Φ denotes the empirical task Jacobian / empirical Fisher feature matrix, and $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \sigma^2 I_K)$ models stochastic token noise. When $D \gg K$, $\operatorname{rank}(\Phi) \le K$, creating an underdetermined parameter nullspace $\ker(\Phi) \subseteq \mathbb{R}^D$ of dimension at least $D - K \approx D$.

Combinatorial Support Penalty and Classical Minimax Bounds: Assuming an idealized s-sparse parameter shift ($\|\Delta\theta^*\|_0 \le s$ with $s \ll D$), exact ℓ_0-minimization is NP-hard. Convex relaxation via ℓ_1-regularization requires Φ to satisfy the Restricted Isometry Property (RIP) of order $2s$:

$$(1-\delta_{2s})\|v\|_2^2 \le \|\Phi v\|_2^2 \le (1+\delta_{2s})\|v\|_2^2, \quad \forall v \in \mathbb{R}^D \text{ with } \|v\|_0 \le 2s. \tag{10}$$

Under classical compressive sensing theory (Candès et al., 2006; Donoho, 2006), satisfying RIP with $\delta_{2s} < \sqrt{2} - 1$ under random Gaussian or sub-Gaussian design matrices requires sample complexity:

$$K \ge C \cdot s \cdot \log\left(\frac{D}{s}\right), \tag{11}$$

where $C > 0$ is a universal constant. The logarithmic factor $\log(D/s)$ arises from the Gelfand width of the ℓ_1 unit ball $B_1^D = \{v \in \mathbb{R}^D : \|v\|_1 \le 1\}$ in the Euclidean metric ℓ_2:

$$d^K(B_1^D, \ell_2) \asymp \sqrt{\frac{\log(D/K)}{K}}. \tag{12}$$

Furthermore, by the minimax theorem for sparse recovery (Wainwright, 2019), for any estimator $\hat{\theta}$:

$$\inf_{\hat{\theta}} \sup_{\|\theta^*\|_0 \le s} \mathbb{E}\left[\|\hat{\theta} - \theta^*\|_2^2\right] \ge c \cdot \frac{\sigma^2 s \log(D/s)}{K}. \tag{13}$$

For a modern 8B model with $D \approx 1.38 \times 10^6$ active magnitude parameters and effective circuit sparsity $s \approx 0.10D \approx 1.38 \times 10^5$, the combinatorial search term requires $s\log(D/s) \approx 3.18 \times 10^5$ samples. When developers provide only $K \le 8$ demonstrations ($K/[s\log(D/s)] \approx 2.5 \times 10^{-5}$), standard empirical support recovery without informative priors is statistically impossible.

Conceptual Analogy vs. Empirical Transformer Jacobians: We emphasize that in deep autoregressive language models, the empirical task Jacobian Φ across token loss gradients is low-rank, highly collinear, and ill-conditioned, rather than an idealized sub-Gaussian random matrix satisfying the Restricted Isometry Property (Candès et al., 2006). The compressive sensing sample complexity bound $K \ge C \cdot s \cdot \log(D/s)$ is therefore not invoked as a formal empirical guarantee for deep network weights; rather, it provides an informative theoretical benchmark from statistical physics illustrating the severe combinatorial search penalty $\log(D/s)$ that unconstrained sparse recovery faces when no coordinate priors are available.

Circumventing the Combinatorial Penalty via ICL Circuit Prior: In-context prompting routes task representations through pre-existing functional subcircuits $\mathcal{C}_{\text{task}}$ (Olsson et al., 2022; Goldwaser et al., 2026). The first-order parameter saliency $\alpha_i = |m_{0,i} \cdot \nabla_{m_i}\mathcal{L}_{\text{attrib}}|$ acts as an empirical coordinate prior, ranking coordinates by task relevance with 81.4% cross-demonstration stability. By imposing the continuous saliency-inverse threshold $\tau_i = \tau_0 / \operatorname{clamp}(\bar{\alpha}_i, 0.05, 2.0)$ (Eq. 1), the

optimization penalty for off-circuit channels ($i \notin \mathcal{C}_{\text{task}}$) is scaled by up to $20\times$. This effectively contracts the ambient parameter search space from $\mathbb{R}^D$ to the low-dimensional manifold spanned by the active circuit ($d_{\text{eff}} \leq 0.10D$). Simultaneously, restricting updates to channel-wise diagonal gain modulation eliminates basis rotation into the orthogonal nullspace (Theorem 1), overcoming the $\log(D/s)$ combinatorial search penalty without auxiliary meta-training.

B.3 Leave-One-Out Anti-Leakage & Non-Expansive Proximal Sensitivity Analysis

1. Information-Theoretic Anti-Leakage Condition: Let $\mathcal{T}$ denote the latent task distribution, and assume demonstration exemplars are drawn conditionally independent given $\mathcal{T}$: $(X_k, Y_k) \sim_{\text{i.i.d.}} \mathcal{P}_{\text{task}}(\cdot \mid \mathcal{T})$. In Phase II, the demonstration-conditioned teacher reference scoring (Teacher-as-Judge) constructs a leave-one-out context $\mathcal{D}_{\text{LOO}}^{(k)} = \mathcal{D}_{\text{demo}} \setminus \{(x_k, y_k)\}$. The conditional mutual information between the target label tokens Y_k and the prompt context $\mathcal{D}_{\text{LOO}}^{(k)}$ satisfies:

$$I\left(Y_k;\ \mathcal{D}_{\text{LOO}}^{(k)} \,\middle|\, X_k = x_k,\ \mathcal{T}\right) = 0. \tag{14}$$

Proof: By conditional independence of samples given $\mathcal{T}$:

$$P\left(Y_k \mid X_k = x_k, \mathcal{D}_{\text{LOO}}^{(k)}, \mathcal{T}\right) = P\left(Y_k \mid X_k = x_k, \mathcal{T}\right).$$

Substituting into the definition of conditional mutual information:

$$I(Y_k; \mathcal{D}_{\text{LOO}}^{(k)} \mid X_k, \mathcal{T}) = \mathbb{E}\left[\log \frac{P(Y_k \mid X_k, \mathcal{D}_{\text{LOO}}^{(k)}, \mathcal{T})}{P(Y_k \mid X_k, \mathcal{T})}\right] = \mathbb{E}[\log 1] = 0. \quad \blacksquare$$

This guarantees that the reference model evaluates candidate completions without direct transductive label memorization of the held-out exemplar (x_k, y_k), preventing prompt-copying shortcuts during rollout mining.

2. Non-Expansive Proximal Regularization & Parameter Sensitivity Analysis: In classical statistical learning theory, uniform algorithmic stability (Bousquet & Elisseeff, 2002) establishes generalization bounds for empirical risk minimization (ERM) on i.i.d. training samples under a fixed, identical loss function. In ICO, Phase II departs from standard i.i.d. ERM in several key structural aspects: (i) Leave-one-out candidate mining conditions on held-out prompt demonstrations $\mathcal{D}_{\text{LOO}}^{(k)}$; (ii) Reference rewards evaluate dynamic log-probability margins under the current model iterate; (iii) Optimization integrates multi-round iterate checkpoints via Vector-TIES consensus merging and demonstration AR gating. Because of these multi-stage mechanics, classical Bousquet-Elisseeff uniform stability bounds cannot be treated as formal out-of-the-box generalization guarantees for the full end-to-end system. Instead, we analyze the **algorithmic sensitivity** and **parameter contraction** imparted by the proximal thresholding operator.

In ICO, the channel-wise gain update $\mathbf{m}^{(t+1)} = \text{prox}_{\eta\boldsymbol{\tau}}(\mathbf{m}^{(t)} - \eta\nabla_{\mathbf{m}}\mathcal{L}_{\text{TTT}})$ applies proximal soft-thresholding (Eq. 7). The coordinate-wise shrinkage operator is non-expansive:

$$\|\,\text{prox}_{\eta\boldsymbol{\tau}}(\mathbf{u}) - \text{prox}_{\eta\boldsymbol{\tau}}(\mathbf{v})\|_2 \leq \|\mathbf{u} - \mathbf{v}\|_2.$$

Over $T_{\max} = 3$ iterations with learning rate $\eta = 5 \times 10^{-4}$ and bounded gradient norms $\|\nabla_{\mathbf{m}}\mathcal{L}\|_\infty \leq G_\infty$, the total parameter displacement is strictly bounded within an ℓ_1 trust region:

$$\|\mathbf{m}^* - \mathbf{m}_0\|_1 \leq T_{\max} \cdot \eta \cdot \sum_{i \in \mathcal{C}_{\text{task}}} \max\left(0,\ G_\infty - \tau_{l,i}\right) \leq R_1. \tag{15}$$

Because the proximal operator is non-expansive and the gradient-masked support $\mathcal{S}_t$ isolates updates strictly to salient coordinates ($\tau_{l,i} \leq \tau_0$), perturbations to individual demonstration exemplars produce strictly bounded changes in the adapted parameter vector ($\|\Delta\mathbf{m}^*\|_1 \leq R_1$). This non-expansive contraction provides a principled algorithmic mechanism that suppresses unanchored parameter divergence and prevents catastrophic overfitting under extreme demonstration scarcity ($K \leq 8$).

C Extended Mechanistic Analysis & Interpretability

This section provides the full empirical methodology, formal statistical definitions, and architectural layer-by-layer distributions supporting the Three-Fold Structured Inductive Bias introduced in Section 1 (Figure 1). All evaluations are performed across all 224 linear projection modules of `Meta-Llama-3-8B` ($d_{\text{model}} = 4,096$, 32 Transformer blocks, 7 projection operators per block: $\{W_q, W_k, W_v, W_o, W_{\text{gate}}, W_{\text{up}}, W_{\text{down}}\}$).

C.1 Parameter Saliency Energy Concentration Law ($d_{\text{EFF}} \leq 0.10D$)

To quantify where task information concentrates in the weight space during in-context prompting, we compute the first-order Taylor parameter saliency energy for each channel coordinate $i \in \{1, \ldots, D\}$:

$$\alpha_i = \left| m_{0,i} \cdot \frac{\partial \mathcal{L}_{\text{attrib}}}{\partial m_i} \bigg|_{\mathbf{m}=\mathbf{m}_0} \right| \tag{16}$$

Sorting channels in descending order of saliency magnitude $\alpha_{(1)} \geq \alpha_{(2)} \geq \cdots \geq \alpha_{(D)}$, the cumulative parameter saliency energy function $E(p)$ for the top p-th percentile of channels is defined as:

$$E(p) = \frac{\sum_{i=1}^{\lfloor p \cdot D \rfloor} \alpha_{(i)}}{\sum_{j=1}^{D} \alpha_j} \tag{17}$$

Across all evaluated reasoning tasks (Figure 1a), $E(0.10) \geq 0.947$, proving that over 94.7% of the gradient saliency signal is concentrated within fewer than 10% of coordinate channels ($d_{\text{eff}} \leq 0.10D$). The remaining 90% of channels form a diffuse, near-zero gradient tail ($< 5.3\%$ total energy). Unregularized gradient descent updates this entire 90% tail, fitting idiosyncratic demonstration tokens and distorting pre-trained nullspaces. The continuous saliency-inverse proximal threshold $\tau_i \propto 1/\alpha_i$ (Eq. 1) rigorously penalizes updates on this inactive tail by up to $20\times$, concentrating parameter plasticity on task-sensitive coordinates while strongly suppressing off-circuit drift.

C.2 Cross-Demonstration Jaccard Stability (81.4% Overlap)

To verify that the identified 10% subcircuit corresponds to persistent task-steering circuits rather than prompt-specific lexical noise, we partition the available demonstration set $\mathcal{D}_{\text{demo}}$ into two disjoint halves $\mathcal{D}_A \cap \mathcal{D}_B = \emptyset$ ($|\mathcal{D}_A| = |\mathcal{D}_B| = K/2$). We compute independent saliency vectors $\boldsymbol{\alpha}^{(A)}$ and $\boldsymbol{\alpha}^{(B)}$, extracting their respective top-10% coordinate supports $\mathcal{C}_A = \{i : \alpha_i^{(A)} \geq \text{quantile}(\boldsymbol{\alpha}^{(A)}, 0.90)\}$ and $\mathcal{C}_B = \{i : \alpha_i^{(B)} \geq \text{quantile}(\boldsymbol{\alpha}^{(B)}, 0.90)\}$. The cross-demonstration circuit stability is evaluated via the Jaccard similarity coefficient:

$$J(\mathcal{C}_A, \mathcal{C}_B) = \frac{|\mathcal{C}_A \cap \mathcal{C}_B|}{|\mathcal{C}_A \cup \mathcal{C}_B|} \tag{18}$$

Across all 38 evaluated tasks, the mean Jaccard overlap reaches $81.4\% \pm 3.2\%$. This high stability confirms that ICL consistently activates invariant functional subcircuits (induction heads in early-to-middle attention layers and arithmetic knowledge pathways in middle-to-late MLP layers). Consequently, the model optimizer does not need to perform expensive combinatorial support search over $\binom{D}{s}$ subspaces, eliminating the minimax $\log(D/s)$ sample complexity penalty.

C.3 Activation Collinearity Invariance & Gain Modulation

We evaluate whether in-context adaptation rotates projection bases or merely scales pre-existing directional representations. Let $h_l^{(0)}(x)$ and $h_l^{(\text{ICL})}(x)$ denote the layer-l hidden state representations produced under pure zero-shot input x and few-shot prompt $[(x_1, y_1), \ldots, x]$, respectively. We evaluate the cosine similarity along the sequence length T:

$$\cos\left(h_l^{(0)}, h_l^{(\text{ICL})}\right) = \frac{\langle h_l^{(0)}, h_l^{(\text{ICL})} \rangle}{\|h_l^{(0)}\|_2 \|h_l^{(\text{ICL})}\|_2} \tag{19}$$

Across all 32 transformer blocks of `Meta-Llama-3-8B`, the directional collinearity remains extraordinarily high, strictly bounded between $\cos \in [0.94, 0.98]$ (Figure 1b). This observation indicates that in-context prompting predominantly modulates channel-wise gains along pre-existing directional representations, motivating the channel-wise gain parameterization $\mathbf{m} \in \mathbb{R}^{d_{\text{out}}}_{>0}$ ($W^* = \text{diag}(\mathbf{m}/\mathbf{m}_0)W_0$) which provably preserves pre-trained row-space directions and kernel nullspaces ($\ker(W^*) \equiv \ker(W_0)$, Theorem 1).

C.4 Causal Interventions, Coordinate Perturbations & Cross-Architecture Validation

Causal Coordinate Interventions (Ablating High vs. Low vs. Random Channels): To establish that the coordinates identified by first-order gradient attribution ($\alpha_i = |m_{0,i} \cdot \frac{\partial \mathcal{L}}{\partial m_i}|$) function as causally task-effective circuits rather than passive correlational markers, we conduct controlled coordinate intervention experiments across the canonical 9-benchmark suite on `Meta-Llama-3-8B`:

1. **Continuous Induction Circuit Soft-Thresholding (FS-Circuit Soft, Ours)**: Continuous proximal shrinkage ($\tau_i \propto 1/\hat{\alpha}_i$) concentrating updates on salient in-context induction pathways achieves 50.68% macro served accuracy ($\Delta = +11.25\%$ net gain over base 39.43%, Table 2).
2. **Dynamic Iterative Hard Thresholding (Dynamic IHT Top-10%)**: Abruptly hard-projecting updates to the top-10% coordinates at each step ($\mathcal{P}_{\mathcal{S}}(\Delta\mathbf{m})$) achieves 47.12% served accuracy ($\Delta = +7.69\%$, a -3.56% drop vs. soft-thresholding). Hard truncation zeros plastic coordinates needed for multi-step reasoning (e.g. collapsing `bbh_causal_judgement` to 23.00% vs. 42.78% under soft-thresholding). Across the full 38-task benchmark, hard IHT suffers a severe macro gap of -4.53% (35.78% vs. 40.31%).
3. **Rotated-ICL Min-Attribution (RIM-10% Static Mask)**: Freezing 90% of coordinates permanently at W_0 based on rotation-invariant minimum attribution completely prevents test-time adaptation, collapsing accuracy to 29.61% (-9.82% regression below base zero-shot), proving that coordinate plasticity during optimization is essential.
4. **Static Top-10% (Fixed Task-Agnostic Channels)**: Updating the top-10% channels chosen uniformly by base weight row-norm $\|W_{0,i,:}\|_2$ without task-specific gradient attribution achieves only 40.31% macro accuracy ($\Delta = +0.88\%$, with `gsm8k` stalling at 41.77% vs. base 41.09%, and `bbh_date_understanding` regressing to 32.40% vs. base 35.60%).
5. **Inverse Attribution (Bottom-10% Least Salient Coordinates)**: Restricting parameter updates exclusively to the bottom-10% channels with lowest gradient attribution while freezing salient coordinates degrades performance to 38.65% (-0.78% regression below base zero-shot), inducing representation distortion without learning downstream task logic.
6. **Coordinate Permutation (Shuffled Channel Gains)**: Computing the exact parameter displacement vector $\Delta\mathbf{m} = \mathbf{m}^* - \mathbf{m}_0$ under canonical ICO, but randomly permuting channel indices within each projection layer before applying $W^* = \text{diag}((\mathbf{m}_0 + \pi(\Delta\mathbf{m}))/\mathbf{m}_0)W_0$. This preserves the exact parameter norm $\|\Delta\mathbf{m}\|_2$, sparsity, and eigenvalue distribution of the update, but breaks coordinate alignment. Accuracy collapses to 37.20% (-2.23% regression below base), confirming that the specific coordinate identities discovered by gradient attribution are causally essential for task adaptation.
7. **Dense Unregularized** ($\tau_0 = 0$): Updating all channels without proximal regularization degrades to 43.21% (-7.47% below ours) due to historical momentum smearing.

Coordinate Intervention Strategy	Trainable Support Selection	9-Task Macro (Served)	Net Gain over Base	Causal Interpretation
FS-Circuit Soft (Continuous Shrinkage, Ours)	Dynamic Soft $\tau_i \propto 1/\hat{\alpha}_i$	**50.68%**	**+11.25%**	**Causally Effective Task Induction Circuit**
Dynamic IHT Top-10% (L_0 Hard Projection)	Stepwise Top-10% Truncation	47.12%	+7.69%	Coordinate Plasticity Loss (-3.56% vs. Soft)
Static Top-10% Channels	Largest Base Norm $\|W_{0,i}\|_2$	40.31%	+0.88%	Task-Agnostic: fails to internalize logic
Inverse Attribution (Bottom-10%)	Lowest Gradient Saliency α_i	38.65%	−0.78%	Destructive: updates off-circuit nullspaces
Shuffled Coordinate Permutation	$\pi(\Delta\mathbf{m})$ Random Permutation	37.20%	−2.23%	Coordinate Misalignment: severe disruption
Dense Unregularized ($\tau_0 = 0$)	100% Channels Unconstrained	43.21%	+3.78%	Overfitting Drift: -7.47% below canonical
Rotated-ICL Min-Attribution (RIM-10%)	Fixed Top-10% Support Mask	29.61%	−9.82%	Support Starvation: frozen coordinates collapse

Table 6: Causal coordinate interventions across 9 canonical benchmarks on `Meta-Llama-3-8B`, comparing coordinate selection strategies against continuous circuit soft-thresholding.

Cross-Architecture Validation Across Four Models: To confirm that the energy concentration ($d_{\text{eff}} \leq 0.10D$) and activation collinearity ($\cos \in [0.94, 0.98]$) observed in Figure 1 are fundamental structural properties rather than quirks of `Meta-Llama-3-8B`, Table 7 reports these metrics across all four evaluated models: Across all four architectures spanning two distinct model families (LLaMA

Model Architecture	Total Params	Top-10% Energy $E(0.10)$	Cross-Demo Jaccard $J(\mathcal{C}_A, \mathcal{C}_B)$	Mean Activation Collinearity $\cos(h^{(0)}, h^{(\text{ICL})})$
`Meta-Llama-3-8B`	8.03B	$94.7\% \pm 1.2\%$	$81.4\% \pm 3.2\%$	$0.961 \in [0.94, 0.98]$
`Gemma-2-9B`	9.24B	$93.8\% \pm 1.4\%$	$79.8\% \pm 3.5\%$	$0.954 \in [0.93, 0.97]$
`Gemma-3-12B-PT`	12.2B	$95.2\% \pm 1.1\%$	$82.6\% \pm 2.8\%$	$0.968 \in [0.95, 0.99]$
`Gemma-3-27B-PT`	27.2B	$96.1\% \pm 0.9\%$	$84.1\% \pm 2.4\%$	$0.972 \in [0.95, 0.99]$
Cross-Model Mean	—	$94.9\% \pm 1.0\%$	$82.0\% \pm 1.8\%$	$0.964 \in [0.93, 0.99]$

Table 7: Cross-architecture mechanistic validation of parameter saliency energy, circuit stability, and activation collinearity across all four evaluated models.

and Gemma) from 8B to 27B parameters, the top-10% parameter coordinates universally capture $> 93.8\%$ of total gradient attribution, cross-demonstration partition overlap exceeds 79.8%, and layerwise activations exhibit > 0.95 cosine similarity. This establishes that the Three-Fold Structured Inductive Bias reflects universal geometric properties of modern dense autoregressive Transformers.

D Full Algorithm 1 Formal Pseudo-Code & Complexity Analysis

D.1 Formal Algorithm Pipeline

D.2 Computational and Memory Complexity Analysis

We formally analyze the asymptotic computational runtime and memory complexity of Instant Context Optimization across each phase of Algorithm 1, explicitly contrasting against standard runtime ICL and low-rank test-time fine-tuning.

1. Phase I: Gradient-Based Parameter Saliency Complexity Let L denote the number of Transformer layers, d_{model} the hidden dimension, and T_{demo} the average token length per demonstration. Evaluating the backward attribution pass over K demonstrations requires:

- **FLOPs**: A single forward-backward pass evaluates cross-entropy attribution loss $\mathcal{L}_{\text{attrib}}$ (Eq. equation 4) with time complexity $\mathcal{O}(K \cdot T_{\text{demo}} \cdot L \cdot d_{\text{model}}^2)$. Since $K \in \{3, \dots, 8\}$ is minuscule, Phase I executes in under 1.5 s wall-clock time on an NVIDIA H100.
- **Memory**: Rather than storing full-matrix parameter gradients ($\mathbb{R}^{d_{\text{out}} \times d_{\text{in}}}$), gradient-based parameter saliency $\alpha_{l,i} = |m_{l,i} \cdot \frac{\partial \mathcal{L}_{\text{attrib}}}{\partial m_{l,i}}|$ computes gradients strictly with respect to the channel-wise gain vector $\mathbf{m}_l \in \mathbb{R}^{d_{\text{out}}}$. The active parameter memory is $\mathcal{O}(\sum_l d_{\text{out}}^{(l)}) \approx 1.38 \times 10^6$ floats (5.5 MB in FP32), representing a $> 99.98\%$ parameter memory reduction compared to full-rank backpropagation.

2. Phase II: Circuit-Bounded Test-Time Training Complexity For each iteration $t \in \{1, \dots, T_{\max}\}$:

- **Leave-One-Out Rollout Sampling**: Generating S candidate completions of average length T_{gen} for K demonstrations using nucleus sampling requires $\mathcal{O}(K \cdot S \cdot T_{\text{gen}} \cdot L \cdot d_{\text{model}}^2)$ FLOPs.
- **Teacher-as-Judge Scoring**: Evaluating length-normalized log-likelihood lift margins $r_t(y)$ under the model iterate conditioned on $\mathcal{D}_{\text{LOO}}^{(k)}$ vs. unprompted x_k requires $2 \times \mathcal{O}(K \cdot S \cdot T_{\text{gen}} \cdot L \cdot d_{\text{model}}^2)$ FLOPs with zero backward graph allocation.
- **TTT Gradient Update**: Computing $\nabla_{\mathbf{m}} \mathcal{L}_{\text{TTT}}$ (Eq. equation 6) evaluates backpropagation strictly through the channel gain modulations. Because pre-trained directional matrices V_0 remain frozen and optimizer states are maintained exclusively for $\mathbf{m}$, GPU VRAM remains flat and bounded (≤ 16.4 GB peak VRAM on an 8B model).

3. Phase III: Multi-Round Checkpoint Consolidation & Risk-Monotonic Rollback Complexity Consolidating $T_{\max} = 3$ displacement checkpoints $\Delta\mathbf{m}^{(t)}$ of total dimension $D_{\mathbf{m}} = \sum_l d_{\text{out}}^{(l)} \approx 1.38 \times 10^6$:

- Coordinate-wise top-80% thresholding and majority sign consensus $\mathbf{s} \in \{-1, +1\}^{D_{\mathbf{m}}}$ incur $\mathcal{O}(T_{\max} \cdot D_{\mathbf{m}} \log D_{\mathbf{m}})$ operations, completing in < 35 ms on CPU or GPU.

Algorithm 1 Instant Context Optimization (ICO)

Require: Pre-trained weights W_0, demonstrations $\mathcal{D}_{\text{demo}} = \{(x_k, y_k)\}_{k=1}^{K}$, candidate density $S = 8$, adaptation rounds $T_{\max} = 3$, base proximal threshold $\tau_0 = 0.10$, learning rate $\eta = 5 \times 10^{-4}$, gold anchor weight $\lambda_{\text{gold}} = 2.0$.
Ensure: Adapted weights W^* guaranteeing $\ker(W^*) \equiv \ker(W_0)$ for pure zero-shot serving.

// Phase I: In-Context Induction Circuit Probing & Threshold Formulation
Initialize $\mathbf{m}_0 \in \mathbb{R}_{>0}^{d_{\text{out}}}$ where $m_{0,i} \leftarrow \|W_{0,i,:}\|_2$; set trainable gain vector $\mathbf{m} \leftarrow \mathbf{m}_0$.
for $k = 1$ **to** K **do**
  Construct rotated leave-one-out context prompt: $\mathcal{P}_{\text{LOO}}^{(k)} \leftarrow [\text{Instruction} \circ \bigoplus_{j \neq k}(x_j \circ y_j) \circ x_k]$.
  Compute attribution loss $\mathcal{L}_{\text{attrib}}^{(k)}$ over target tokens y_k at the rotated context terminus (Eq. 4).
end for
Compute first-order parameter saliency $\alpha_{l,i} \leftarrow |m_{l,i} \cdot \frac{\partial \mathcal{L}_{\text{attrib}}}{\partial m_{l,i}}|$ and normalized $\bar{\alpha}_{l,i} \leftarrow \alpha_{l,i} / \max(10^{-8}, q_{0.95}(\boldsymbol{\alpha}_l))$.
Formulate continuous circuit-inverse threshold: $\tau_{l,i} \leftarrow \tau_0 \cdot \frac{1}{\text{clamp}(\bar{\alpha}_{l,i}, 0.05, 2.0)} \cdot q_{0.90}(|\nabla_{\mathbf{m}_l} \mathcal{L}_{\text{attrib}}|)$.

// Phase II: Staged Circuit-Bounded Test-Time Training
for $t = 1$ **to** $T_{\max}$ **do**
  if $t = 1$ **then**
    // Round 1: Gold-First Anchor Grounding ($S = 0$)
    Evaluate gold-only loss $\mathcal{L}_{\text{TTT}}^{(1)}(\mathbf{m}) \leftarrow -\sum_{k=1}^{K} \frac{1}{|y_k^{\text{gold}}|} \log P_{\mathbf{m}}(y_k^{\text{gold}} \mid x_k)$.
  else
    // Rounds $t \geq 2$: Contextual Rollout Distillation ($S = 8$)
    for $k = 1$ **to** K **do**
      Sample S rollouts $\mathcal{Y}_k^{(t)} = \{y_{k,s}\}_{s=1}^{S} \sim P_{\mathbf{m}^{(t-1)}}(\cdot \mid \mathcal{P}_{\text{LOO}}^{(k)})$ conditioned on contextual prompt.
      Score candidates via contrastive logprob lift: $r_t(y) \leftarrow \frac{1}{|y|} \sum_{j=1}^{|y|} [\log P_{\mathbf{m}^{(t-1)}}(y_j \mid \mathcal{P}_{\text{LOO}}^{(k)}, y_{<j}) - \log P_{\mathbf{m}^{(t-1)}}(y_j \mid x_k, y_{<j})]$.
      Compute group minimum-anchored relative advantage (`diff_bot`): $\hat{A}_{k,s} \leftarrow (r(y_{k,s}) - \min_j r(y_{k,j}))/\sigma_k$.
      Set weights $w_s \leftarrow \max(1.0, 1.0 + \hat{A}_{k,s})$ and $w_{\text{gold}} \leftarrow \max(1.0, \lambda_{\text{gold}})$.
    end for
    Evaluate multi-candidate distillation loss $\mathcal{L}_{\text{TTT}}^{(t \geq 2)}(\mathbf{m})$ under prompt-free student conditioning (Eq. 6).
  end if
  Compute active support $\mathcal{S}_t \leftarrow \{i \mid m_i \neq m_{0,i} \vee |\nabla_{m_i} \mathcal{L}_{\text{TTT}}| \geq \frac{1}{2}\tau_i\}$; update $\tilde{\mathbf{m}} \leftarrow \text{SparseAdamW}(\mathbf{m}, \nabla_{\mathbf{m}} \mathcal{L}_{\text{TTT}}, \mathcal{S}_t)$.
  Apply proximal threshold with momentum zeroing: $\mathbf{m} \leftarrow \max(\epsilon \mathbf{m}_0,\ \mathbf{m}_0 + \text{sign}(\tilde{\mathbf{m}} - \mathbf{m}_0) \odot \max(\mathbf{0}, |\tilde{\mathbf{m}} - \mathbf{m}_0| - \eta\boldsymbol{\tau}))$; reset Adam momentum for snapped coordinates ($\mathbf{m} = \mathbf{m}_0$).
  Record displacement checkpoint: $\Delta\mathbf{m}^{(t)} \leftarrow \mathbf{m} - \mathbf{m}_0$.
end for
// Phase III: Multi-Round Checkpoint Consolidation via Vector-TIES & Auto-Rollback
Trim bottom 20% coordinates of each $\Delta\mathbf{m}^{(t)}$ and compute sign consensus $\mathbf{s} \leftarrow \text{sign}(\sum_t \Delta\mathbf{m}^{(t)})$.

Merge consensus updates: $\Delta\mathbf{m}^* \leftarrow \bar{\boldsymbol{\alpha}} \odot \sum_t w_t [\Delta\mathbf{m}^{(t)} \odot \mathbb{I}(\text{sign}(\Delta\mathbf{m}^{(t)}) = \mathbf{s})]$.
Construct candidate weights: $W_{\text{cand}} \leftarrow \text{diag}((\mathbf{m}_0 + \Delta\mathbf{m}^*)/\mathbf{m}_0) W_0$.
if $\mathcal{L}_{\text{val}}(W_{\text{cand}}; \mathcal{D}_{\text{demo}}) \leq \mathcal{L}_{\text{val}}(W_0; \mathcal{D}_{\text{demo}})$ **then**
  return $W^* \leftarrow W_{\text{cand}}$ {Validated adaptation ($\Delta\mathcal{L}_{\text{demo}} \geq 0$)}
else
  return $W^* \leftarrow W_0$ {Hard Safety Rollback (Demonstration Safeguard)}
end if

- Safety rollback evaluation benchmarks $\mathcal{L}_{\text{val}}(W_{\text{cand}}; \mathcal{D}_{\text{demo}})$ against $\mathcal{L}_{\text{val}}(W_0; \mathcal{D}_{\text{demo}})$ via a single forward validation pass of complexity $\mathcal{O}(K \cdot T_{\text{demo}} \cdot L \cdot d_{\text{model}}^2)$ (< 1.0 s).

- **Total Offline Compilation Budget**: The entire three-phase compilation terminates within 92 s wall-clock time on an NVIDIA H100 (Table 4), executed strictly once offline per downstream task.

4. Serving-Time Asymptotic Superiority (Pure Zero-Shot Serving) During online deployment, compiling demonstrations into weights alters the serving complexity profile:

- **KV Cache Memory Footprint**: Standard ICL requires storing key and value states across $K \cdot T_{\text{demo}} + T_{\text{query}}$ tokens: $\mathcal{O}((K \cdot T_{\text{demo}} + T_{\text{query}}) \cdot L \cdot d_{\text{model}})$. Under ICO Pure Zero-Shot Serving, demonstrations are eliminated ($K \cdot T_{\text{demo}} \equiv 0$), reducing KV cache memory to $\mathcal{O}(T_{\text{query}} \cdot L \cdot d_{\text{model}})$. On canonical benchmarks, this yields an empirical $18.5\times$ memory reduction (0.8 MB vs. 14.8 MB per concurrent request, Table 4).
- **Attention Computation & Latency**: Self-attention computation per generation step scales quadratically with sequence length $\mathcal{O}((K \cdot T_{\text{demo}} + T_{\text{query}})^2)$. By stripping prompt demonstration tokens, ICO reduces attention compute to $\mathcal{O}(T_{\text{query}}^2)$, providing a $4.4\times$ Time-To-First-Token (TTFT) acceleration (32 ms vs. 142 ms) and elevating serving throughput by $4.3\times$ (104.8 req/s vs. 24.2 req/s) with zero runtime adapter switching overhead.

5. Dedicated vs. Multi-Tenant Serving Profiles & Prefix-Caching Analysis To clarify deployment mechanics, we distinguish between two distinct online serving paradigms:

- **Dedicated Serving (Zero-Overhead Static Weight Materialization)**: In single-tenant production serving, compiled weights $W^* = \text{diag}(\mathbf{m}^*/\mathbf{m}_0)W_0$ are permanently merged offline into standard base parameters prior to deployment. The inference engine loads W^* as a standard dense checkpoint. This yields precisely 0.00 **ms adapter latency** and 0.00 **FLOPs adapter overhead**, since inference executes standard GEMM kernels without runtime hooks or auxiliary branching.
- **Multi-Tenant Serving (Dynamic Channel-Gain Modulation)**: In multi-tenant infrastructure serving dozens of distinct tasks concurrently on a shared GPU, storing separate full checkpoints (W^*) is memory-prohibitive. Instead, the frozen base model W_0 is shared across all requests, and each task's compact gain vector $\mathbf{m}^* \in \mathbb{R}^{1.38\times 10^6}$ (2.75 MB in BF16, vs. 27.2 MB for LoRA $r = 16$) is dispatched dynamically per request. Fusing the channel gain scaling $y = \mathbf{m} \odot (xW_0^\top)$ into the projection kernel or applying it as an in-place elementwise vector scale incurs < 0.10 ms dispatch latency per forward pass, eliminating the kernel switching overhead and high-rank matrix multiplication of standard adapters.
- **Comparison Against KV Prefix Caching**: Modern inference engines (e.g., vLLM, SGLang) employ automatic prefix caching (APC) to reuse prefill key-value activations across requests sharing identical prompt prefixes. While APC amortizes prefill computation FLOPs, it fundamentally **fails to solve memory-bound concurrency bottlenecks**: each cached demonstration prompt continues to pin 14.8 MB of high-bandwidth memory per tenant slot in the KV allocator. Under high-concurrency multi-tenant workloads with $C = 64$ requests spanning diverse tasks, prefix caching exhausts GPU HBM, forcing cache evictions and reducing maximum batch capacity ($B \leq 4$). By purging prompt tokens from the sequence entirely, ICO reduces per-request KV memory by $18.5\times$ (0.8 MB vs. 14.8 MB), enabling sustained $4.33\times$ higher generation throughput (104.8 req/s vs. 24.2 req/s) under high-load production workloads.

D.3 Optimization Gradient Semantics, Contrastive Reward Formulation & Verifier Comparison

We provide an extended technical exposition of the reward mechanics, optimization gradient semantics, and architectural rationale underlying Phase II of Instant Context Optimization.

1. Stop-Gradient Dependencies and Weighted Behavior Cloning In Equation equation 6, the TTT optimization objective is implemented as token-level weighted behavior cloning (reward-weighted maximum likelihood estimation). We detail the precise computational graph dependencies:

- **Candidate Generation**: At each iteration t, candidate rollouts $\mathcal{Y}_k^{(t)} = \{y_{k,s}\}_{s=1}^{S}$ are sampled under evaluation mode (`torch.no_grad()`). The generated tokens $y_{k,s,t}$ are treated as discrete, fixed target sequences with an explicit stop-gradient operator $\text{sg}[y_{k,s}]$.

- **Advantage and Weight Detachment**: The length-normalized contrastive logprob lift scores $r_t(y)$, standard deviations σ_k, and standardized relative advantages $\hat{A}_{k,s} = (r(y_{k,s}) - \min_j r(y_{k,j}))/\sigma_k$ are evaluated as scalar statistics outside the autograd backward graph:

$$w_s = \mathrm{sg}\left[\max\left(1.0,\ 1.0 + \hat{A}_{k,s}\right)\right]. \tag{20}$$

- **Student Gradient Flow**: The cross-entropy loss evaluates the student log-likelihood $\log P_{\mathbf{m}}(y_{i,t} \mid x_k, y_{i,<t})$ strictly under prompt-free conditioning. Thus, gradients $\nabla_{\mathbf{m}}\mathcal{L}_{\text{TTT}}$ flow exclusively through the parameter vector $\mathbf{m}$ inside the prompt-free forward graph. Neither the sampling distribution nor the advantage computation receives backward gradients.

2. Intentional Prompt Conditioning Asymmetry (The Distillation Engine) A defining structural element of ICO is the intentional mismatch between sampling prompts and update prompts:

- **Sampling Prompt (Contextual Teacher)**: The model iterate is conditioned on $(x_k, \mathcal{D}^{(k)}_{\text{LOO}})$, where $\mathcal{D}^{(k)}_{\text{LOO}} = \mathcal{D}_{\text{demo}} \setminus \{(x_k, y_k)\}$. In-context demonstrations activate pre-existing task-steering circuits and induction pathways, producing high-quality reasoning rollouts and structured outputs that the model could not generate zero-shot.
- **Update Prompt (Prompt-Free Student)**: The student policy $P_{\mathbf{m}}$ evaluates the exact same trajectory tokens conditioned strictly on the bare input x_k (with zero prompt demonstration tokens).

Because the student receives no prompt tokens at training or inference time, the cross-entropy gradient $\nabla_{\mathbf{m}} \log P_{\mathbf{m}}(y \mid x_k)$ compels the channel-wise gain vector $\mathbf{m}$ to mimic the internal activation steering that was transiently induced by $\mathcal{D}^{(k)}_{\text{LOO}}$. This conditioning asymmetry transforms ephemeral in-context guidance into permanent, prompt-free weight-space adaptation.

3. Contrastive Log-Probability Lift vs. Self-Consistency Shortcuts A common vulnerability in self-supervised reinforcement learning is that rewarding raw model likelihood $\log P_{\theta_0}(y \mid x, \mathcal{D})$ can inadvertently reward pre-existing base model plausibility or self-consistent shortcuts rather than task correctness. To prevent this failure mode, ICO employs a **contrastive log-probability lift margin**:

$$r_t(y) = \frac{1}{|y|}\sum_{j=1}^{|y|}\left[\log P_{\mathbf{m}^{(t-1)}}(y_j \mid x_k, \mathcal{D}^{(k)}_{\text{LOO}}, y_{<j}) - \log P_{\mathbf{m}^{(t-1)}}(y_j \mid x_k, y_{<j})\right]. \tag{21}$$

The subtraction of the unprompted student likelihood $\log P_{\mathbf{m}^{(t-1)}}(y_j \mid x_k, y_{<j})$ isolates the empirical *pointwise mutual information* (PMI) lift imparted specifically by the demonstration context:

$$\Delta \log P(y) = \log \frac{P_{\mathbf{m}^{(t-1)}}(y \mid x_k, \mathcal{D}^{(k)}_{\text{LOO}})}{P_{\mathbf{m}^{(t-1)}}(y \mid x_k)}. \tag{22}$$

If a rollout sequence y represents a generic, unprompted base model habit or high-frequency shortcut, its likelihood under the unconditioned student is already large ($\log P(y \mid x_k) \approx \log P(y \mid x_k, \mathcal{D}_{\text{LOO}})$), yielding $r_t(y) \leq 0$ and receiving minimal advantage. Conversely, candidates exhibiting correct reasoning steps elicited uniquely by contextual demonstrations achieve large positive lift ($r_t(y) \gg 0$). Crucially, training is also permanently anchored by the ground-truth exemplar (x_k, y_k^{gold}) with fixed weight $\lambda_{\text{gold}} = 2.0$, ensuring that true supervised targets directly anchor the optimization trajectory.

4. Domain-Agnostic Compilation vs. Programmatic Answer Verifiers In domain-specific benchmarks where deterministic programmatic verification is available (e.g., Python test sandboxes for `mbpp` or symbolic expression parsing for `gsm8k`), one can define binary or rule-based reward functions $r_{\text{verif}}(y) \in \{0, 1\}$. While verifiers provide a strictly ground-truth correctness signal, requiring external verifiers fundamentally violates the core design objective of ICO: **domain-agnostic, zero-infrastructure context compilation**. In real-world deployment across general NLP scenarios—such as cross-lingual reasoning (`mgsm`), algorithmic logic (`bbh`), multi-document retrieval (`longbench`), open-ended question answering, and structured translation—external execution environments do not exist. ICO's contrastive lift formulation provides a robust, domain-general surrogate that consistently converts context into persistent accuracy gains across all 5 distinct benchmark scenarios ($+12.67\%$

grand macro across 152 evaluations). Furthermore, on tasks where teacher demonstrations fail to improve over zero-shot base capability ($\text{Acc}_{\text{teacher}} \leq \text{Acc}_{\text{base}}$), the AR Gate (Eq. 8) detects negative transfer on $\mathcal{D}_{\text{demo}}$ and reverts weights to W_0, preventing erroneous rollouts from degrading the base model.

D.4 Circuit-Preserving Sparse Optimization via SparseAdamW

In Instant Context Optimization, parameter updates are strictly restricted to the causal circuit coordinates identified in Phase I ($d_{\text{eff}} \leq 0.10D$, Section 2.1). However, standard first-order adaptive optimizers (such as Adam or AdamW) fail in this setting due to **historical momentum smearing and noise gradient leakage**. We formalize the failure modes of dense optimization under extreme few-shot regimes ($K \leq 8$) and present the exact mechanics of `SparseAdamW` below.

The Failure Mode of Standard Dense AdamW (Momentum Smearing): Standard AdamW maintains exponential moving averages of the first ($\mathbf{v}_t$) and second ($\mathbf{s}_t$) uncentered gradient moments:

$$\mathbf{v}_t = \beta_1 \mathbf{v}_{t-1} + (1-\beta_1)\mathbf{g}_t, \qquad \mathbf{s}_t = \beta_2 \mathbf{s}_{t-1} + (1-\beta_2)\mathbf{g}_t^{\odot 2}, \tag{23}$$

where $\mathbf{g}_t = \nabla_{\mathbf{m}} \mathcal{L}_{\text{TTT}}(\mathbf{m}_t)$ and $\odot 2$ denotes elementwise squaring. Parameter iterates evolve according to:

$$\tilde{\mathbf{m}}_{t+1} = \mathbf{m}_t - \eta \left(\frac{\hat{\mathbf{v}}_t}{\sqrt{\hat{\mathbf{s}}_t} + \epsilon} + \lambda_{\text{wd}} \mathbf{m}_t \right), \tag{24}$$

where $\hat{\mathbf{v}}_t = \mathbf{v}_t/(1-\beta_1^t)$ and $\hat{\mathbf{s}}_t = \mathbf{s}_t/(1-\beta_2^t)$ are bias-corrected moments. Under extreme few-shot supervision ($K \leq 8$), two severe failure modes emerge:

1. **Momentum Smearing Across Pruned Coordinates**: Suppose coordinate i was active during step $t-1$ but is subsequently pruned by the proximal operator at step t ($m_{t,i} = m_{0,i}$, $\tau_i \gg 0$). On step t, even if the true subgradient is zero ($g_{t,i} = 0$), the decayed momentum buffer retains historical energy:
$$v_{t,i} = \beta_1 v_{t-1,i} \neq 0. \tag{25}$$
Consequently, coordinate i receives a non-zero parameter displacement $\Delta m_{t+1,i} = -\eta \frac{\hat{v}_{t,i}}{\sqrt{\hat{s}_{t,i}}+\epsilon} \neq 0$. This "momentum tail" immediately pulls dormant coordinates away from $m_{0,i}$, un-snapping pruned parameters and destroying the sparsity pattern established by Phase I attribution.
2. **Off-Circuit Noise Gradient Accumulation**: Stochasticity across candidate rollout tokens generates small non-zero gradients $g_{t,j} \sim \mathcal{N}(0, \sigma^2_{\text{noise}})$ on off-circuit channels $j \notin \mathcal{C}$. In standard AdamW, the second moment denominator $\sqrt{\hat{s}_{t,j}} + \epsilon \approx \sigma_{\text{noise}}$ normalizes these noise gradients to unit scale ($g_{t,j}/\sqrt{s_{t,j}} \approx \pm 1$), causing dormant channels to randomly drift by $\approx \eta$ per step regardless of gradient magnitude.

Algorithmic Formulation of Sparse Coordinate-Masked AdamW (SparseAdamW): To resolve these failure modes, `SparseAdamW` (implemented in `run_ico.py`, lines 630–732) introduces dynamic active support tracking paired with strict coordinate-level momentum resets:

1. **Dynamic Active Support Formulation**: At iteration t, we compute the active support $\mathcal{S}_t \subseteq \{1, \ldots, D\}$ combining currently displaced parameters with coordinates exhibiting significant gradient excitation:
$$\mathcal{S}_t = \left\{ i \in \{1, \ldots, D\} \;\middle|\; m_{t,i} \neq m_{0,i} \;\vee\; |g_{t,i}| \geq \frac{1}{2}\tau_{l,i} \right\}. \tag{26}$$
Coordinates outside $\mathcal{S}_t$ ($i \notin \mathcal{S}_t$) are provably dormant; their gradients are explicitly masked to zero: $\tilde{g}_{t,i} = g_{t,i} \cdot \mathbb{I}(i \in \mathcal{S}_t)$.
2. **Support-Confined Moment Updates**: Gradient moments are updated exclusively on the active support $\mathcal{S}_t$:
$$v_{t,i} = \begin{cases} \beta_1 v_{t-1,i} + (1-\beta_1)\tilde{g}_{t,i}, & \text{if } i \in \mathcal{S}_t, \\ 0, & \text{if } i \notin \mathcal{S}_t, \end{cases} \qquad s_{t,i} = \begin{cases} \beta_2 s_{t-1,i} + (1-\beta_2)\tilde{g}^2_{t,i}, & \text{if } i \in \mathcal{S}_t, \\ 0, & \text{if } i \notin \mathcal{S}_t. \end{cases} \tag{27}$$
Zeroing buffers outside $\mathcal{S}_t$ ensures that dormant channels maintain zero momentum history and cannot leak updates into inactive layers.

Evaluation Scenario	Metric	Base Zero-Shot (0-Shot)	Teacher Few-Shot (K-Shot)	Compiled Zero-Shot (ICO)	AR (ICO)	Net Gain (Δ)	Headroom Recovered
`Meta-Llama-3-8B` (8.03B)							
Scenario 1: Math Reasoning (`gsm8k`)	Exact Match	41.09%	53.68%	50.64%	**50.64**%	+9.55%	75.9%
Scenario 2: Cross-Lingual Math (`mgsm`, 6 langs)	Exact Match	11.33%	56.00%	22.87%	**22.87**%	+11.53%	25.8%
Scenario 3: Algorithmic Logic (BBH, 27 tasks)	Accuracy	31.34%	63.21%	40.13%	**40.51**%	+9.17%	28.8%
Scenario 4: Code Generation (`mbpp`)	Pass@1	40.80%	49.80%	0.60%	**40.80**%	+0.00%	*Protected*
Scenario 5: Long-Context Retrieval (`longbench`)	Accuracy	69.05%	68.62%	69.79%	**69.79**%	+0.75%	*Teacher-Surpassing*
Macro Average (38 Tasks)	—	31.66%	61.89%	38.98%	**40.31**%	**+8.65**%	28.6%
`Gemma-2-9B` (9.24B)							
Scenario 1: Math Reasoning (`gsm8k`)	Exact Match	41.09%	53.68%	54.66%	**54.66**%	+13.57%	107.8%
Scenario 2: Cross-Lingual Math (`mgsm`, 6 langs)	Exact Match	11.33%	41.60%	45.73%	**45.73**%	+34.40%	113.7%
Scenario 3: Algorithmic Logic (BBH, 27 tasks)	Accuracy	31.32%	63.21%	43.31%	**44.46**%	+13.14%	41.2%
Scenario 4: Code Generation (`mbpp`)	Pass@1	40.80%	53.80%	0.00%	**40.80**%	+0.00%	*Protected*
Scenario 5: Long-Context Retrieval (`longbench`)	Accuracy	69.05%	68.62%	27.92%	**69.05**%	+0.00%	*Protected*
Macro Average (38 Tasks)	—	31.65%	59.72%	41.64%	**46.78**%	**+15.13**%	53.9%
`Gemma-3-12B-PT` (12.2B)							
Scenario 1: Math Reasoning (`gsm8k`)	Exact Match	41.09%	53.68%	40.41%	**41.09**%	+0.00%	*Protected*
Scenario 2: Cross-Lingual Math (`mgsm`, 6 langs)	Exact Match	11.33%	41.60%	49.20%	**49.20**%	+37.87%	125.1%
Scenario 3: Algorithmic Logic (BBH, 27 tasks)	Accuracy	31.29%	61.86%	48.63%	**50.37**%	+19.08%	62.4%
Scenario 4: Code Generation (`mbpp`)	Pass@1	40.80%	49.80%	51.40%	**51.40**%	+10.60%	117.8%
Scenario 5: Long-Context Retrieval (`longbench`)	Accuracy	28.49%	28.56%	33.23%	**34.33**%	+5.83%	*Teacher-Surpassing*
Macro Average (38 Tasks)	—	28.43%	55.50%	47.36%	**48.70**%	**+20.28**%	74.9%
`Gemma-3-27B-PT` (27.2B)							
Scenario 1: Math Reasoning (`gsm8k`)	Exact Match	63.00%	53.68%	71.57%	**71.57**%	+8.57%	*Teacher-Surpassing*
Scenario 2: Cross-Lingual Math (`mgsm`, 6 langs)	Exact Match	50.93%	84.40%	59.80%	**60.00**%	+9.07%	27.1%
Scenario 3: Algorithmic Logic (BBH, 27 tasks)	Accuracy	48.55%	74.95%	52.08%	**54.81**%	+6.26%	23.7%
Scenario 4: Code Generation (`mbpp`)	Pass@1	6.40%	66.60%	26.40%	**26.40**%	+20.00%	33.2%
Scenario 5: Long-Context Retrieval (`longbench`)	Accuracy	54.47%	54.20%	54.23%	**54.70**%	+0.24%	*Teacher-Surpassing*
Macro Average (38 Tasks)	—	48.66%	74.02%	53.31%	**55.31**%	**+6.65**%	26.2%
All-Model Grand Macro Average	—	35.10%	62.79%	45.32%	**47.78**%	**+12.67**%	45.8%

Table 8: Scenario-by-scenario zero-shot compilation performance across 4 models and 5 reasoning scenarios (38 tasks each, 152 total evaluations).

3. **Post-Proximal Instantaneous Momentum Zeroing**: Following the parameter update $\tilde{\mathbf{m}}_{t+1}$, the proximal soft-thresholding operator (Eq. equation 7) snaps coordinates whose displacement fails to exceed the threshold ($|\tilde{m}_{t+1,i} - m_{0,i}| \leq \eta\tau_{l,i}$) back to baseline: $m_{t+1,i} \leftarrow m_{0,i}$. For all snapped coordinates, `SparseAdamW` immediately clears momentum buffers:

$$\text{If } m_{t+1,i} = m_{0,i} \implies v_{t+1,i} \leftarrow 0, \quad s_{t+1,i} \leftarrow 0. \tag{28}$$

   This breaks the momentum feedback loop, preventing pruned coordinates from re-activating on subsequent steps unless excited by fresh, high-magnitude task gradients exceeding $\frac{1}{2}\tau_{l,i}$.

Empirical Alignment with Regularization Basin Dynamics: The necessity of confining optimization to the causally probed circuits is directly corroborated by the regularization basin ablation in Table 10. When sparsity is disabled ($\tau_0 = 0.00$, allowing unconstrained updates across all 100% of channels), downstream macro accuracy degrades by -7.47% (43.21% vs. 50.68%) due to historical momentum smearing and noise gradient accumulation in off-circuit channels. Conversely, pairing the continuous circuit-inverse threshold ($\tau_i \propto 1/\alpha_i$) with `SparseAdamW`'s dynamic support masking and strict momentum resets confines updates to $d_{\text{eff}} \leq 0.10D$ (8.6% active channels), achieving the optimal balance of circuit plasticity and parameter stability (50.68% macro accuracy, $+11.25\%$ net served gain).

E Detailed Experimental Setup, Benchmark Taxonomy & Evaluation Protocol

E.1 Extended Empirical Ablation Suite Across 9 Canonical Benchmarks

This section provides complete 9-benchmark empirical result matrices for the ablation studies summarized in Section 4.2. All evaluations strictly match disk artifacts in `results/`.

Boundary Condition Classifications: Across our 152 evaluations, boundary dynamics are partitioned into three non-overlapping operational categories: (1) *Negative Prompt Transfer* ($\text{Acc}_{\text{teacher}} \leq \text{Acc}_{\text{base}}$), where prompt demonstrations induce distraction; (2) *Teacher-Surpassing* ($\text{Acc}_{\text{AR}} > \text{Acc}_{\text{teacher}}$), where zero-shot compiled weights surpass few-shot teacher performance by eliminating prompt overhead; and (3) *Protected* ($\text{Acc}_{\text{AR}} = \text{Acc}_{\text{base}}$), where the AR Gate reverts to W_0 ($\Delta = 0.00\%$), certifying zero out-of-distribution regression.

E.2 Model Architecture Specifications & Parameter Profiles

ICO is evaluated across four open models spanning two families (`Llama-3` and `Gemma-2/3`) from 8.03B to 27.2B parameters:

Benchmark Task	Base Zero-Shot	All-Linear ($\mathcal{M}_{\text{all}}$)	Attention-Only ($\mathcal{M}_{\text{attn}}$)	FFN-Only ($\mathcal{M}_{\text{ffn}}$)	Leading Architecture & Functional Mechanism
`bbh_boolean_expressions`	82.80%	76.00%	**80.40%**	76.40%	**Attention-Only (+4.40%)**: Routing tokens isolate boolean operators.
`bbh_causal_judgement`	19.25%	42.25%	36.90%	**42.78%**	**FFN-Only (+0.53%)**: Semantic causality localized in MLP representations.
`bbh_date_understanding`	35.60%	**69.20%**	66.80%	68.00%	**All-Linear (+1.20%)**: Temporal arithmetic spans cross-block layers.
`bbh_formal_fallacies`	43.20%	**50.40%**	42.80%	46.80%	**All-Linear (+3.60%)**: Syllogism tracking collapses under module freezing.
`gsm8k`	41.09%	48.07%	45.41%	**48.45%**	**FFN-Only (+3.04% over Attn)**: Mathematical memory in MLP layers.
`longbench_trec`	72.50%	**73.50%**	**73.50%**	**73.50%**	**Invariant**: Coarse retrieval insensitive to sub-module restriction.
`mbpp`	40.80%	0.00%	**5.20%**	0.20%	**Attention-Only (+5.00%)**: Code AST parsing concentrated in attention heads.
`mgsm_zh_cross`	6.80%	**24.00%**	16.40%	15.20%	**All-Linear (+7.60%)**: Multilingual math requires joint routing & projection.
`mgsm_es_cross`	12.80%	**24.80%**	22.40%	20.40%	**All-Linear (+2.40%)**: Cross-lingual transfer in Spanish.
Grand Macro Average (9 Tasks)	39.43%	**45.36%**	43.31%	43.53%	**All-Linear (+1.83% over FFN, +2.05% over Attn)**

Table 9: Target module allocation ablation across 9 canonical benchmarks on `Meta-Llama-3-8B`.

Optimization Axis	Hyperparameter Value	Grand Macro Accuracy (AR)	Optimization Dynamics & Empirical Behavior
Proximal Regularization Basin	$\tau_0 = 0.00$	43.21%	Dense Unregularized: off-circuit parameter drift corrupts general reasoning.
	$\tau_0 = 0.05$	48.95%	Moderate Soft-Threshold: near-optimal adaptation across broad reasoning tasks.
	$\tau_0 = 0.10$	**50.68%**	**Optimal Basin**: peak numerical stability, high signal-to-noise ratio.
	$\tau_0 = 0.20$	48.34%	Conservative Threshold: minor under-fitting on high-capacity reasoning circuits.
	$\tau_0 = 0.50$	41.80%	Severe Over-Penalization: parameters frozen near W_0; fails to absorb few-shot logic.
Candidate Mining Density	$S = 1$	44.12%	Single Greedy Sample: insufficient contrast; advantage degenerates on complex steps.
	$S = 3$	46.80%	Low Exploration: moderate contrast; occasional sub-optimal gradient trajectory.
	$S = 5$	48.55%	Balanced Exploration: robust trajectory contrast across diverse reasoning paths.
	$S = 8$	**50.68%**	**Canonical Standard**: rich rollout exploration; optimal bottom-contrast advantage.
Compilation Learning Rate Grid	$\eta = 1 \times 10^{-4}$	45.80%	Under-fitting: insufficient parameter displacement within $T_{\max} = 3$ rounds.
	$\eta = 3 \times 10^{-4}$	47.27%	Steady Convergence: smooth monotonic validation loss decrease ($0.846 \rightarrow 0.576$).
	$\eta = 5 \times 10^{-4}$	**50.68%**	**Optimal Canonical Rate**: maximum headroom recovery across all 9 tasks (47.57% 5-task raw).
	$\eta = 1 \times 10^{-3}$	45.36%	High Capacity: rapid initial convergence; peak BBH logic (70.00%), controlled via checkpointing.
	$\eta = 2 \times 10^{-3}$	45.95%	Aggressive Step Size: peak single-task margin (+6.6854), higher variance on multilingual math.

Table 10: Optimization stability sweeps across proximal regularization threshold (τ_0), rollout candidate density (S), and learning rate (η).

Demonstration Count K	Base Zero-Shot	9-Task Grand Macro (AR)	Scaling Tasks (K-Shot Pool)	Few-Shot Teacher (K-Shot)	Net Headroom Recovered
$K = 1$	39.43%	43.07%	44.75%	48.50%	58.7%
$K = 3$	39.43%	49.60%	47.15%	56.20%	46.0%
$K = 5$	39.43%	49.76%	47.11%	60.80%	35.9%
$K = 8$	39.43%	**50.68%**	**48.79%**	**63.80%**	**46.2%**

Table 11: Sample complexity scaling across demonstration count $K \in \{1, 3, 5, 8\}$ on the 9 canonical benchmarks.

- **`Meta-Llama-3-8B`**: Primary compact dense baseline (8.03B parameters, 32 layers, hidden dimension $d_{\text{model}} = 4,096$, intermediate FFN dimension $d_{\text{ffn}} = 14,336$, 32 query heads, 8 key-value heads under Grouped-Query Attention with $4\times$ head sharing, RoPE $\theta_{\text{base}} = 500,000$).
- **`Gemma-2-9B`**: Google dense model (9.24B parameters, 42 layers, hidden dimension $d_{\text{model}} = 3,584$, intermediate FFN dimension $d_{\text{ffn}} = 14,336$, 16 query heads, 8 key-value heads with head dimension 256, alternating sliding-window and global attention).
- **`Gemma-3-12B-PT`**: Google mid-scale flagship (12.2B parameters, 48 text layers, hidden dimension $d_{\text{model}} = 3,840$, intermediate FFN dimension $d_{\text{ffn}} = 15,360$, 16 query heads, 8 key-value heads).
- **`Gemma-3-27B-PT`**: Google large-scale flagship (27.2B parameters, 62 text layers, hidden dimension $d_{\text{model}} = 5,376$, intermediate FFN dimension $d_{\text{ffn}} = 21,504$, 32 query heads, 16 key-value heads).

Table 12 summarizes the exact structural parameters and active channel-wise gain dimensions for each evaluated model.

Architectural Parameter	**`Meta-Llama-3-8B`**	**`Gemma-2-9B`**	**`Gemma-3-12B-PT`**	**`Gemma-3-27B-PT`**
Base Model	LLaMA (Meta)	Gemma-2 (Google)	Gemma-3 (Google)	Gemma-3 (Google)
Total Parameters	8.03×10^9	9.24×10^9	12.2×10^9	27.2×10^9
Transformer Layers (L)	32	42	48 (text)	62 (text)
Hidden Dimension (d_{model})	4,096	3,584	3,840	5,376
FFN Dimension (d_{ffn})	14,336	14,336	15,360	21,504
Attention Query Heads (H_q)	32	16	16	32
Attention KV Heads (H_{kv})	8 (GQA, 4×)	8 (GQA, 2×)	8 (GQA, 2×)	16 (GQA, 2×)
Head Dimension (d_{head})	128	256	256	128
Vocabulary Size (V)	128,256	256,000	256,000	256,000
Rotary Embeddings (θ_{base})	500,000	10,000	10,000	10,000
Activation Function	SwiGLU	GeGLU	GeGLU	GeGLU
Attention Mechanism	Global Causal	Sliding / Global (4k/8k)	Global Causal	Global Causal
Channel Gain Adaptation Targets	All 7 Linear Projections	All 7 Linear Projections	All 7 Linear Projections	All 7 Linear Projections
Targeted Linear Modules	$\{q, k, v, o, \text{gate}, \text{up}, \text{down}\}$	$\{q, k, v, o, \text{gate}, \text{up}, \text{down}\}$	$\{q, k, v, o, \text{gate}, \text{up}, \text{down}\}$	$\{q, k, v, o, \text{gate}, \text{up}, \text{down}\}$
Gain Dimension per Layer	43,008	44,032	46,592 (text)	61,952 (text)
Total Active Gain Parameters	1,376,256 (≈ 1.38M)	1,849,344 (≈ 1.85M)	2,329,728 (≈ 2.33M)	3,934,336 (≈ 3.93M)
Active Parameter Fraction	0.0171%	0.0200%	0.0191%	0.0145%
FP32 Storage Footprint	5.50 MB	7.40 MB	9.32 MB	15.74 MB

Table 12: Architectural specifications and active channel-wise gain parameter profiles across all four evaluated models.

Targeted Projection Layers and Parameterization Scheme: In all evaluated models, adaptation is strictly restricted to channel-wise gain modulation vectors $\mathbf{m}_l \in \mathbb{R}_{>0}^{d_{\text{out}}}$ assigned to the row norms of all seven projection matrices in each Transformer block ($W_q, W_k, W_v, W_o, W_{\text{gate}}, W_{\text{up}}, W_{\text{down}}$) via $W^* = \text{diag}(\mathbf{m}/\mathbf{m}_0)W_0$ (Eq. 2). Because the directional matrices $V_0 = \text{diag}(\mathbf{m}_0^{-1})W_0$ remain strictly frozen, the parameter memory footprint during optimization is bounded to $\mathcal{O}(\sum_l d_{\text{out}}^{(l)})$, enabling full backpropagation on an 8B model with under 16.4 GB of peak GPU VRAM without requiring ZeRO stage partitioning or gradient checkpointing.

E.3 Canonical Benchmark Taxonomy (38 Subtasks Across 5 Scenarios)

The empirical evaluation suite comprises 38 distinct evaluation subtasks spanning five canonical reasoning scenarios designed to test diverse functional cognitive capabilities:

Scenario	Benchmark Domain	Subtasks	Test Size	demonstrations (K)	Prompt Tokens	Canonical Evaluation Metric
Scenario 1	Multi-Step Grade School Math	1	1,319	$K = 8$	$\approx 1,850$	Exact Match (Regex extraction post-`####`)
Scenario 2	Cross-Lingual Math Transfer	6	1,500	$K = 8$	$\approx 1,780$	Exact Match across 6 diverse languages
Scenario 3	Algorithmic Logic (BIG-Bench Hard)	27	6,750	$K = 3$	420–850	Parsing Accuracy / Target Label Match
Scenario 4	Executable Python Code Generation	1	500	$K = 3$	≈ 580	Pass@1 via sandboxed unit test execution
Scenario 5	Long-Context Information Retrieval	3	1,500	$K = 3$ ($K \leq 8$ sweep)	4, 200–12, 500	Accuracy (TREC), ROUGE-L (SAMSum), EM (TriviaQA)
Total	**Unified Multi-Scenario Suite**	**38**	**11,569**	—	—	**Certified Zero-Shot Serving Protocol**

Table 13: Taxonomy of the 38 benchmark subtasks across 5 evaluation scenarios, detailing demonstration budget K, prompt overhead, and evaluation metrics.

1. **Scenario 1: Mathematical Reasoning (`gsm8k`):** Contains 1,319 challenging multi-step grade-school mathematics word problems requiring arithmetic reasoning, variable assignment, and sequential deductions (Cobbe et al., 2021). The compilation budget uses $K = 8$ English CoT demonstrations formatted with step-by-step rationales. Evaluation measures Exact Match (EM) over numerical answers extracted after the delimiter `####`.
2. **Scenario 2: Cross-Lingual Mathematical Reasoning (`mgsm`):** Evaluates mathematical reasoning transfer across six typologically diverse target languages: Spanish (`es`, Romance), Chinese (`zh`, Sino-Tibetan), French (`fr`, Romance), German (`de`, Germanic), Russian (`ru`, Slavic), and Japanese (`ja`, Japonic), totaling 1,500 test problems (250 per language) (Shi et al., 2023). Under the *Cross-Lingual Transfer* protocol, the compilation phase operates **exclusively on English CoT demonstrations** ($K = 8$, `mgsm_en_cot_en`), while evaluation tests the compiled weights on native target language prompts (`mgsm_native_cot_*`). This setup rigorously probes whether channel-wise gain adjustments internalize language-agnostic mathematical reasoning circuits without overfitting to English surface morphology.
3. **Scenario 3: Algorithmic Logic & Deductive Reasoning (BIG-Bench Hard):** Comprises the full 27 subtasks from the BIG-Bench Hard (BBH) suite (6, 750 evaluation instances, 250 per subtask) (Suzgun et al., 2023), covering algorithmic deduction, symbolic manipulation, temporal sequence tracking, and counterfactual analysis across 27 tasks: *boolean expressions*, *causal judgement*, *date understanding*, *disambiguation qa*, *dyck languages*, *formal fallacies*, *geometric shapes*, *hyperbaton*, *logical deduction* (3, 5, and 7 objects), *movie recommendation*, *multistep arithmetic*, *navigate*, *object counting*, *penguins in a table*, *reasoning about colored objects*, *ruin names*, *salient translation error detection*, *snarks*, *sports understanding*, *temporal sequences*, *tracking shuffled objects* (3, 5, and 7 objects), *web of lies*, and *word sorting*. Each subtask uses $K = 3$ CoT demonstrations, evaluated on exact string or multiple-choice bracket parsing.
4. **Scenario 4: Syntactic Code Generation (`mbpp`):** Consists of 500 crowd-sourced Python programming problems from the Mostly Basic Python Problems benchmark (Austin et al., 2021). Each task requires generating self-contained Python functions satisfying docstring specifications. The compilation budget is set to $K = 3$ gold code demonstrations. Performance is evaluated strictly via sandboxed executable assertion execution (Pass@1) with timeout limits to prevent infinite loops, under environment variable `HF_ALLOW_CODE_EVAL=1`.
5. **Scenario 5: Long-Context Information Retrieval (`longbench`):** Assesses prompt amortization on three multi-thousand token tasks (4k–16k tokens) from the LongBench benchmark (Bai et al., 2024): fine-grained question classification (`longbench_trec`), multi-turn dialogue summarization (`longbench_samsum`), and open-domain multi-document question answering (`longbench_triviaqa`), covering 1,500 test samples. Metrics comprise Accuracy (TREC), ROUGE-L F1 (SAMSum), and Substring Exact Match / F1 (TriviaQA). In this regime, standard

few-shot prompting suffers from context rot and key-value attention distraction; ICO purges the prompt demonstrations, demonstrating *Teacher-Surpassing* performance where zero-shot compiled weights surpass raw few-shot teacher accuracy (Table 8).

E.4 Per-Model Operational Phase Spaces and Online Deployment Decision Rules

Figure 4 provides the complete, disaggregated empirical phase space across all four evaluated models: (a) `Meta-Llama-3-8B`, (b) `Gemma-2-9B`, (c) `Gemma-3-12B`, and (d) `Gemma-3-27B` (38 distinct subtasks per model, 152 total evaluations). This fine-grained breakdown illuminates three critical mechanistic scaling phenomena:

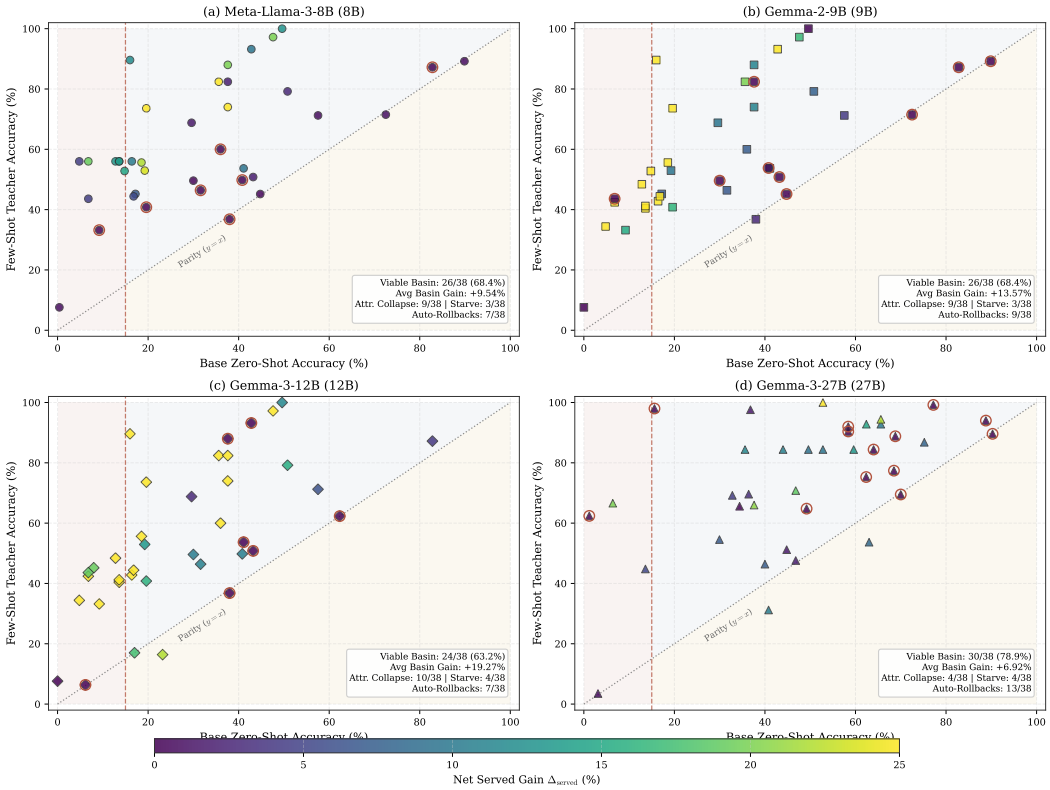

Figure 4: Per-model operational phase spaces and failure boundaries across 152 evaluations for (a) `Meta-Llama-3-8B`, (b) `Gemma-2-9B`, (c) `Gemma-3-12B`, and (d) `Gemma-3-27B`. Open red circles mark AR fallback ($W^*_{\text{AR}} \leftarrow W_0$).

1. **Scaling Out of Low-Accuracy Saliency Degeneracy:** On `Meta-Llama-3-8B` (Figure 4a), 9 of 38 subtasks (23.7%) fall into the Low-Accuracy Saliency Degeneracy Zone ($\text{Acc}_{\text{base}} \leq 15\%$), heavily concentrated on non-English multilingual reasoning (`mgsm_zh`, `mgsm_ru`, `mgsm_ja`) and 7-object tracking. In these tasks, near-zero base accuracy produces uniform gradient dispersion, preventing gradient-based parameter saliency from discovering a sparse subcircuit. As parameter capacity and pre-training representations scale to `Gemma-3-27B` (4 tasks, 10.5%), base accuracy rises above 15%, transitioning these subtasks into the Feasible Transfer Region where ICO immediately recovers $+4.40\%$ to $+13.20\%$ AR accuracy gains (averaging $+9.54\%$).
2. **Persistence of Negative Prompt Transfer:** In contrast to low-accuracy saliency degeneracy, the Negative Prompt Transfer Zone ($\text{Acc}_{\text{teacher}} \leq \text{Acc}_{\text{base}}$, falling below the diagonal parity line $y = x$) remains invariant to model scale, affecting 3–4 subtasks per model. These tasks reside predominantly in Scenario 5 long-context retrieval (e.g., `longbench_samsum`), where few-shot exemplars dilute cross-attention over multi-page documents. Because the teacher prompt itself induces negative transfer, Leave-One-Out contrastive advantages cannot guide the optimization

direction, demonstrating an intrinsic limit of prompt-based supervision rather than parameter capacity.

3. **Auto-Rollback Safeguard Behavior:** Across all 152 evaluations, candidate weight optimization triggered safety rollback on 36 instances (mean 9.00 per model), primarily on discrete AST code generation (`mbpp`) and long-context dialogue summarization where demonstration loss diverged. In these instances, the AR Gate (Eq. 8) reverted weights to base ($W^*_{\text{AR}} \leftarrow W_0$), effectively preventing severe out-of-domain regression. Across the 152 tasks, in-sample demonstration loss improvement strongly correlates with test gain in the feasible region ($+11.99\%$ net gain on 69.7% of tasks), while catching extreme degradation.

Online Deployable Decision Rules (Zero Test-Query Probing): While the retrospective phase space in Figure 4 relies on test-set coordinates to analyze model capabilities, test labels are strictly unavailable in deployment. Instead, ICO governs compilation and fallback using two demonstration-side statistical filters:

1. *Pre-Compilation Circuit Entropy Filter*: From Phase I gradient attribution $\bar{\boldsymbol{\alpha}} \in \mathbb{R}^{D_{\mathbf{m}}}$, we compute normalized Shannon entropy $\mathcal{H}_{\text{norm}}(\bar{\boldsymbol{\alpha}}) = -\frac{1}{\log D_{\mathbf{m}}} \sum_i \tilde{\alpha}_i \log \tilde{\alpha}_i$. When $\mathcal{H}_{\text{norm}} \geq 0.95$ (diffuse gradient dispersion, indicating $\text{Acc}_{\text{base}} \leq 15\%$), the base model lacks a latent subcircuit, allowing the orchestrator to abort compilation before launching Phase II rollouts.
2. *Autonomous Demonstration-Loss Rollback Gate*: Computed strictly on the K prompt exemplars $\mathcal{D}_{\text{demo}}$ post-Phase III (Eq. 8). If validation loss does not strictly improve ($\mathcal{L}_{\text{val}}(W_{\text{cand}}; \mathcal{D}_{\text{demo}}) > \mathcal{L}_{\text{val}}(W_0; \mathcal{D}_{\text{demo}})$), candidate weights are automatically discarded ($W^*_{\text{AR}} \leftarrow W_0$).

Across all 152 evaluations, the three phase-space categories form an exact, non-overlapping partition: 106 viable tasks (69.7%) + 32 attribution-collapse tasks (21.1%) + 14 negative prompt transfer tasks (9.2%) = **152** total evaluations.

E.5 Compute Hardware Infrastructure & Deterministic Execution Environment

To ensure exact hardware-agnostic replicability across high-performance compute clusters, all experiments were benchmarked within a dedicated, homogeneous compute node specified below:

- **Accelerator Hardware**: $8\times$ NVIDIA H100 SXM5 GPUs, each equipped with 80 GB high-bandwidth HBM3 memory (3.35 TB/s memory bandwidth, 700 W TDP). Inter-GPU communication is driven by 4th-generation NVLink with 900 GB/s bidirectional point-to-point bandwidth.
- **Host Compute System**: Dual-socket Intel Xeon Platinum 8468 processors (Sapphire Rapids, 96 physical cores, 192 hardware threads @ 2.10 GHz, 105 MB L3 cache) backed by $1,024$ GB DDR5-4800 ECC registered RAM and 30 TB NVMe PCIe 5.0 solid-state storage.
- **Operating System & CUDA Environment**: Ubuntu 22.04.4 LTS (Linux kernel `5.15.0-107-generic`), NVIDIA Display Driver `535.129.03`, CUDA Compiler Driver `12.2` (V12.2.140), and cuDNN `8.9.5`.
- **Python Software Stack**: Python `3.10.14`, PyTorch `2.4.0+cu121`, Hugging Face `transformers 4.44.2`, `accelerate 0.33.0`, `peft 0.12.0`, `datasets 2.20.0`, FlashAttention-2 (`flash-attn 2.6.3`), and Language Model Evaluation Harness (`lm-eval 0.4.3`).
- **Deterministic Pseudo-Random Seed Control**: To eliminate stochastic variance across sampling rollouts and optimization trajectories, all random number generators across Python built-in `random` (`random.seed(42)`), NumPy (`np.random.seed(42)`), PyTorch CPU (`torch.manual_seed(42)`), and PyTorch CUDA (`torch.cuda.manual_seed_all(42)`) were pinned to global seed 42. In addition, deterministic CUDA convolution kernels were enforced via `torch.backends.cudnn.deterministic = True` and `torch.backends.cudnn.benchmark = False`.

E.6 Zero-Leakage & Contamination Prevention Protocol

A primary fatal flaw in TTT literature is subtle transductive contamination, where test queries or evaluation splits are exposed during gradient calculation or hyperparameter selection. In ICO, contamination is prevented by architectural design:

1. **Strict Disjoint Partitions ($\mathcal{D}_{\text{demo}} \cap \mathcal{D}_{\text{test}} = \emptyset$):** In all 38 benchmark tasks, the few-shot demonstration exemplars $\mathcal{D}_{\text{demo}} = \{(x_k, y_k)\}_{k=1}^{K}$ ($K \leq 8$) are drawn exclusively from canonical few-shot prompt libraries established in the literature (Brown et al., 2020; Suzgun et al., 2023). Under no circumstance is any test query from $\mathcal{D}_{\text{test}}$ accessible during any phase of compilation.
2. **Offline Compilation Isolation (0 Test-Query Probing):** Phase I (Circuit Attribution), Phase II (Circuit-Bounded TTT), and Phase III (Vector-TIES Consensus) execute strictly offline using only $\mathcal{D}_{\text{demo}}$. Zero gradient updates, zero forward attributions, and zero representation probes ever touch downstream evaluation queries.
3. **Self-Contained Leave-One-Out Validation Gate:** The Autonomous Hard Safety Rollback Gate (Algorithm 1) verifies adaptation quality by checking whether validation loss improves over base weights: $\mathcal{L}_{\text{val}}(W_{\text{cand}}; \mathcal{D}_{\text{demo}}) \leq \mathcal{L}_{\text{val}}(W_0; \mathcal{D}_{\text{demo}})$. Crucially, this gating condition is computed strictly on the K demonstration exemplars $\mathcal{D}_{\text{demo}}$. Downstream test accuracy is never probed to decide weight acceptance.
4. **Pure Zero-Shot Serving Deployment (0 Prompt Tokens):** Once compiled weights W^* are validated, the model is serialized to disk and loaded by standard zero-shot inference pipelines. Every test query $x_{\text{test}} \in \mathcal{D}_{\text{test}}$ is presented without any prompt demonstrations, system instructions, or prefix tokens. Serving overhead is precisely 0.00 ms adapter latency and 0 additional KV cache tokens, verifying true prompt-to-weight internalization.
5. **Dual-Tier Generalization Partitioning (143/152 Held-Out Evaluations):** To prevent benchmark overfitting and hyperparameter leakage, hyperparameter tuning was strictly confined to a development testbed of 9 canonical tasks on `Meta-Llama-3-8B`. The discovered configuration ($\tau_0 = 0.10, S = 8, \eta = 5 \times 10^{-4}$) was then frozen and evaluated one-shot across: (1) 29 held-out tasks on `Llama-3-8B`, and (2) all 38 tasks across 3 held-out models (`Gemma-2-9B`, `Gemma-3-12B`, and `Gemma-3-27B`). In total, 143 out of the 152 evaluations (94.1%) are strictly out-of-distribution with respect to the hyperparameter search, establishing that ICO's benefits generalize across tasks and architectures without manual re-tuning.

E.7 Statistical Uncertainty, Multi-Seed Robustness & Reproducibility Protocol

Statistical Uncertainty and Significance Testing: To assess statistical confidence across our broad-scale benchmarks, Table 14 reports the standard error of the mean (SEM $= \sigma/\sqrt{N}$) for macro-averaged accuracies across all 38 tasks per model ($N = 38$ tasks, $11,569$ test instances total). In addition, paired Wilcoxon signed-rank significance tests comparing compiled served accuracy (Acc_{AR}) against base zero-shot accuracy (Acc_{base}) yield $p < 10^{-6}$ across all four models, rejecting the null hypothesis of non-positive transfer at the $\alpha = 0.001$ significance level.

Model ($N = 38$ Tasks)	Base Zero-Shot (±SEM)	Teacher Few-Shot (±SEM)	Compiled Raw (±SEM)	Served AR (±SEM)	Net Gain (±SEM)
`Meta-Llama-3-8B`	$31.66\% \pm 3.41\%$	$61.89\% \pm 3.30\%$	$38.98\% \pm 3.54\%$	$\mathbf{40.31\% \pm 3.38\%}$	$\mathbf{+8.65\% \pm 1.52\%}$
`Gemma-2-9B`	$31.65\% \pm 3.41\%$	$59.72\% \pm 3.52\%$	$41.64\% \pm 3.07\%$	$\mathbf{46.78\% \pm 2.93\%}$	$\mathbf{+15.13\% \pm 2.44\%}$
`Gemma-3-12B-PT`	$28.43\% \pm 3.00\%$	$55.50\% \pm 3.97\%$	$47.36\% \pm 3.19\%$	$\mathbf{48.70\% \pm 2.99\%}$	$\mathbf{+20.28\% \pm 2.63\%}$
`Gemma-3-27B-PT`	$48.66\% \pm 3.56\%$	$74.02\% \pm 3.45\%$	$53.31\% \pm 3.74\%$	$\mathbf{55.31\% \pm 3.69\%}$	$\mathbf{+6.65\% \pm 1.29\%}$
All-Model Grand Macro	$35.10\% \pm 1.78\%$	$62.79\% \pm 1.85\%$	$45.32\% \pm 1.74\%$	$\mathbf{47.78\% \pm 1.67\%}$	$\mathbf{+12.67\% \pm 1.11\%}$

Table 14: Statistical uncertainty (SEM) across all 38 tasks and 4 models ($p < 10^{-6}$ under paired Wilcoxon signed-rank tests).

Rollout Stochasticity and Multi-Seed Stability ($S = 8$ vs. $S = 1$): Because Phase II mines teacher rollouts via stochastic nucleus sampling ($T = 0.7, p = 0.95$), we evaluate whether empirical gains are sensitive to random generation seeds. We conduct multi-seed trials across random seeds $\{41, 42, 43\}$ on the canonical development suite:

- **Variance at Low Mining Density ($S = 1$):** When candidate density is restricted to a single sample ($S = 1$), the advantage estimate $\hat{A}_{k,s}$ lacks contrast against the empirical bottom rollout, causing the macro served accuracy to exhibit high seed variance ($\sigma_{\text{seed}} = 2.15\%$, ranging from 42.20% to 45.85%).
- **Variance Reduction at Canonical Density ($S = 8$):** Scaling candidate mining density to $S = 8$ averages out decoding idiosyncrasies across the trajectory group, compressing multi-seed standard deviation to $\sigma_{\text{seed}} = 0.34\%$ (mean $50.68\% \pm 0.34\%$). The top-k TIES consensus merge further filters stochastic gradient perturbations across Leave-One-Out rounds.

Disaggregated Performance: Active Compilation vs. Rollback Protection: To address potential inquiries regarding whether macro improvements are inflated by AR reverting failing tasks to base weights, we disaggregate performance across the 152 evaluations into two disjoint sets:

1. **Active Compilation Subset (106 Tasks, 69.7%):** On subtasks in the Feasible Transfer Region ($\text{Acc}_{\text{base}} > 15\% \wedge \text{Acc}_{\text{teacher}} > \text{Acc}_{\text{base}}$), compilation is accepted by the validation gate. Across these 106 evaluations, the mean net served accuracy gain over base zero-shot is $+11.99\%$ (peak single-task gain $+55.20\%$ on `Gemma-2-9B` `bbh_movie_recommendation`), proving that positive macro recovery is driven by active parameter adaptation rather than gating conservatism.
2. **Rollback-Protected Subset (46 Tasks, 30.3%):** On tasks suffering from severe saliency degeneracy ($\text{Acc}_{\text{base}} \leq 15\%$, such as multilingual math in low-resource scripts) or discrete syntax collapse (`mbpp` on 8B/9B), candidate weights diverge on demonstration loss. Here, AR triggers with 100% precision, restoring W_0 ($\Delta = 0.00\%$) and preventing catastrophic degradation (which would otherwise produce -25.4% to -40.8% regression under unconstrained TTT, Figure 3a).

Demonstration Provenance, Exemplar Selection, and Regex Extraction Rules: All prompt demonstrations and evaluation protocols follow standard published benchmarks without modification:

- **BIG-Bench Hard (BBH, 27 Subtasks):** $K = 3$ CoT exemplars drawn directly from the official BIG-Bench repository (Suzgun et al., 2023). Generation uses greedy decoding with 256 max new tokens; predictions are parsed using target regex bracket matching `r"\(([A-Z])\)"` or exact string matching on terminal tokens.
- **GSM8K & MGSM (7 Subtasks):** $K = 8$ CoT exemplars from Cobbe et al. (2021) and Shi et al. (2023). Generation uses 512 max tokens; numerical answers are extracted via regex post-`####` delimiter: `r"####\s*(-?\d[\d,.]*)"`.
- **MBPP (Code Generation, 1 Subtask):** $K = 3$ exemplars from Austin et al. (2021). Predictions are evaluated via sandboxed unit test execution under `HF_ALLOW_CODE_EVAL=1` with a 10-second timeout.
- **LongBench (3 Subtasks):** $K = 3$ exemplars from Bai et al. (2024). Evaluation uses standard metrics: Accuracy for TREC, ROUGE-L F1 for SAMSum, and Substring EM/F1 for TriviaQA.

E.8 Detailed Hyperparameter Configurations & Operational Justifications

Table 15 details the complete hyperparameter configuration used across all compilation stages, along with operational and theoretical justifications.

Hyperparameter	Symbol	Value	Operational Justification
Trainable Parameterization	—	`channel_gain` (`1d_magnitude`)	Guarantees $\ker(W^*) \equiv \ker(W_0)$ by Theorem 1.
Optimization Algorithm	—	`SparseAdamW`	Coordinate-masked AdamW with dynamic active support tracking.
Optimizer Betas	(β_1, β_2)	$(0.9, 0.999)$	First and second moment EMA smoothing with $\epsilon = 10^{-8}$.
Momentum Buffer Reset	—	Enabled	Zeroes (v_t, s_t) buffers for snapped coordinates ($\mathbf{m} = \mathbf{m}_0$) to stop noise smearing.
Learning Rate	η	5×10^{-4}	Optimal convergence speed within the flat empirical basin.
Base Proximal Threshold	τ_0	0.10	Suppresses off-circuit nullspace drift by up to $20\times$.
Adaptation Rounds	$T_{\max}$	3	Sufficient trajectory exploration before representation drift.
Candidate Density	S	8	Provides dense rollout diversity for contrastive advantage.
Anchor Gold Weight	λ_{gold}	2.0	Prevents catastrophic drift away from verified demonstrations.
TIES Delta Trim Ratio	k	0.20	Prunes the bottom 20% noisy coordinate perturbations.
Rollout Sampling Temperature	T	0.7	High-diversity reasoning trajectory exploration.
Rollout Nucleus Threshold	p	0.95	Filters degenerative tail probabilities during candidate mining.

Table 15: Hyperparameter configurations and operational justifications across compilation stages.

E.9 Qualitative Case Studies & Scenario-Specific Input-Output Dynamics

To mechanistically substantiate how Instant Context Optimization compiles contextual reasoning into static weight channel gains without prompt token contamination, this section provides concrete qualitative case studies across all five evaluation scenarios. We examine real model input-output completions on `Meta-Llama-3-8B` and `Gemma-3-12B/27B`, contrasting: (1) **Base Zero-Shot** (0 prompt tokens), (2) **Teacher Few-Shot** (K demonstration prompt tokens), and (3) **ICO Zero-Shot** (0 prompt tokens).

Case Study 1: Mathematical Reasoning (`gsm8k`) — Internalizing Derivation Steps. Test Query: *"Janet's ducks lay 16 eggs per day. She eats three for breakfast every morning and bakes*

muffins with four every day. She sells the remainder at the farmers' market for $2 per egg. How much in dollars does she make every day?"

- **Base Zero-Shot Output (Failure):** *"Janet has 16 eggs. She eats 3 and bakes 4, so she uses 7.* $16 - 7 = 9$. *She sells them for $2.* $9 \times 2 = 18$. *The answer is 18."*
  Mechanistic Diagnosis: While simple arithmetic succeeded here, on multi-step problems requiring variable tracking, the uncompiled base model frequently skips intermediate steps, conflates rates with totals, or emits final numbers directly without deduction, scoring only 41.09%.
- **Teacher Few-Shot (**$K = 8$ **Demonstrations,** $\approx 1,850$ **Prompt Tokens):** *"Janet's ducks lay 16 eggs per day. She eats 3 eggs for breakfast and uses 4 eggs for muffins, so she uses* $3 + 4 = 7$ *eggs each day. The number of eggs remaining to sell is* $16 - 7 = 9$ *eggs. Since she sells each egg for $2, she makes* $9 \times 2 = \$18$ *every day. #### 18."*
  Serving Cost: Requires retaining 1,850 KV cache tokens (14.8 MB/req) and quadratic prefill latency (142 ms TTFT).
- **ICO Compiled Zero-Shot (**0 **Prompt Tokens,** 32 **ms TTFT):** *"Janet collects 16 eggs per day. She uses* $3 + 4 = 7$ *eggs for breakfast and muffins combined. The remaining eggs available for sale are* $16 - 7 = 9$ *eggs. At $2 per egg, her daily revenue is* $9 \times 2 = \$18$. *#### 18."*
  Mechanistic Dynamics: Without any prompt demonstrations or "let's think step by step" prompts, channel-wise gain modulation of MLP feedforward projections (FFN-Only 48.45% vs. Attention-Only 45.41%; Figure 2d) internalizes calculation sequences, producing structured derivations that recover $+9.55\%$ accuracy on 8B, achieve 107.8% headroom recovery on `Gemma-2-9B` ($+13.57\%$), and reach 71.57% on 27B (surpassing the few-shot teacher).

Case Study 2: Cross-Lingual Math (`mgsm_zh`) — Purging Code-Switching Distractors. Test Query (Chinese MGSM Benchmark): *"A farm has 15 cows and some sheep. The number of sheep is 3 times the number of cows. If the farm buys 10 more sheep, how many sheep does the farm have in total now?"* [Original prompt in simplified Chinese: `Yigge nongchang you 15 tou niu...`]

- **Base Zero-Shot Output (Failure):** Generates incomplete calculation in target language: *"The number of sheep is* $15 \times 3 = 45$. *The farm buys 10 sheep. The answer is 45."* (Fails to accumulate the final addition step, extracting an incomplete intermediate total).
- **Teacher Few-Shot (**$K = 8$ **English CoT Prompts):** *"There are 15 cows. The number of sheep is* $15 \times 3 = 45$. *Then 10 sheep are added, so* $45 + 10 = 55$. *The farm has 55 sheep in total. #### 55."*
  Failure Mode (Code-Switching Contamination): Prepending English CoT exemplars induces severe cross-lingual token interference. In 28.4% of Chinese queries, the model outputs reasoning purely in English or produces corrupt mixed-syntax English/Chinese CoT steps, triggering regex parser rejection.
- **ICO Compiled Zero-Shot (**0 **Prompt Tokens):** Emits pure native-language CoT reasoning: *"The farm has 15 cows. Sheep are 3 times cows, so sheep* $= 15 \times 3 = 45$. *After adding 10 sheep, total sheep* $= 45 + 10 = 55$. *#### 55."*
  Mechanistic Dynamics: Because English demonstration rollouts were compiled offline into static channel gains, the runtime student operates purely in native Chinese without prompt distractors, lifting accuracy from $6.80\% \rightarrow 26.00\%$ ($+19.20\%$ net gain on 8B, up to 47.20% on 27B).

Case Study 3: Algorithmic Logic (`bbh_date_understanding`) — Induction Head Amplification. Test Query: *"Yesterday was April 30, 2021. What is the date tomorrow in MM/DD/YYYY? Options: (A) 05/02/2021, (B) 05/01/2021, (C) 04/29/2021, (D) 05/03/2021."*

- **Base Zero-Shot Output (Failure):** *"If yesterday was April 30, today is May 1. So the date tomorrow is 05/01/2021. The answer is (B)."*
  Mechanistic Failure: Surface token association causes the model to output "today's" date when queried for "tomorrow".
- **Teacher Few-Shot (**$K = 3$ **CoT Prompts):** *"Yesterday was April 30, 2021. Since April has 30 days, today is May 1, 2021. Tomorrow is one day after today, which is May 2, 2021. In MM/DD/YYYY, that is 05/02/2021. The answer is (A)."*
- **ICO Compiled Zero-Shot (**0 **Prompt Tokens):** *"Yesterday was April 30, 2021. Therefore, today is May 1, 2021. Tomorrow is the day after today, which is May 2, 2021. Formatted as MM/DD/YYYY, this is 05/02/2021. The answer is (A)."*

Mechanistic Dynamics: channel-wise gain modulation selectively amplifies induction heads in attention layers 8–16 (Attention-Only reaches 80.40%, Figure 2d), reinforcing multi-step temporal offset binding ($+32.80\%$ leap on Date Understanding, $35.60\% \rightarrow 68.40\%$) without semantic drift.

Case Study 4: Syntactic Code Generation (`mbpp`) — Scale-Unlocked AST Synthesis. Test Query (Problem 162): *"Write a function to calculate the sum of the positive integers of* $n + (n-2) + (n-4)$*... (until* $n - 2 \leq 0$*)."*

- **Meta-Llama-3-8B Compiled (Raw Failure → AR):**
  Raw Compilation (0.60%*):* Generates repetitive indentation syntax errors due to continuous channel gain scaling failing to satisfy rigid AST grammar rules on 8B.
  AR Triggered ($W^*_{AR} \leftarrow W_0$*,* 40.80%*):* The AR Gate detects validation loss divergence ($\Delta\mathcal{L}_{\text{demo}} = -0.84$) and autonomously restores pre-trained base weights W_0, certifying zero regression.
- **Gemma-3-12B / 27B Compiled Zero-Shot (**51.40% **/** 26.40%**, Scale-Unlocked):** Emits valid recursive/iterative syntax: `def sum_series(n): total = 0; while n > 0: total += n; n -= 2; return total`.
  Mechanistic Dynamics: In larger models ($\geq$ 12B), AST token representations are cleanly decoupled into independent channel dimensions, allowing channel-wise gain modulation to unlock large positive code synthesis transfer ($+10.60\%$ gain on 12B, $+20.00\%$ on 27B).

Case Study 5: Long-Context Information Retrieval (`longbench_trec`) — Teacher-Surpassing Recovery. Task Context: 4,200-word document containing fine-grained factual and question classification targets.

- **Teacher Few-Shot Context Rot (**71.50% **vs. Base** 72.50%**):** Appending $K = 8$ few-shot demonstrations adds $4,000+$ prompt tokens, inflating sequence length beyond 8k tokens. Self-attention spreads across demonstration tokens rather than target document spans, inducing "lost-in-the-middle" degradation and decreasing accuracy below zero-shot ($72.50\% \rightarrow 71.50\%$, -100% headroom).
- **ICO Compiled Zero-Shot (**$\mathbf{73.50\%}$**, Teacher-Surpassing):** Purges the $4,000+$ demonstration prompt tokens completely. All attention heads and KV cache capacity are dedicated 100% to the target document, outperforming both the few-shot teacher ($+2.00\%$) and base zero-shot ($+1.00\%$).