CONTINUOUS-TIME MOTION PLANNING AND STEERING WITH GLOBAL CONVERGENCE

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ABSTRACT

Motion planning is central to robotics, and it is increasingly used to correct the motions proposed by learned planners and policies. Both require constraints to hold at every instant of the motion, not only at sampled times. To our knowledge, no algorithm is known to provably reach a globally optimal motion once a smoothness cost is combined with such constraints. We cast the problem as a single nonconvex program over polynomial curves. It is elementary to state, yet nonconvex. Robots avoid one another and static or moving obstacles, and a learned proposal is steered by keeping the motion close to it. Each constraint becomes one polynomial that must stay nonnegative over the whole motion. We propose GlobMP (Global Motion Planning), a gradient projection method that alternates cost reduction with feasibility restoration. Under mild assumptions, we prove that, from a fine grid start, it converges globally. We demonstrate its effectiveness and superior performance over strong baselines in real and synthetic robotics experiments. These include synthetic teams of up to 200 robots, multi-robot planning on the Robotarium testbed and an extended benchmark, maze navigation, and steering a tactile-reactive policy on a bimanual humanoid.

1 INTRODUCTION

Motion planning is a fundamental problem in robotics: it asks for a trajectory that connects a given initial state to a goal while satisfying motion constraints, such as avoiding collisions (LaValle, 1998). It underlies navigation (Kuffner & LaValle, 2000), manipulation (Sundaralingam & Hermans, 2019; Gao et al., 2024; Amice et al., 2024), multi-robot coordination (Li et al., 2021; Tang et al., 2025; Shaoul et al., 2025), and, increasingly, the *steering* of learned planners and policies (Janner et al., 2022; Carvalho et al., 2023; Chi et al., 2023; Giaretta et al., 2026).

Motion planners approach this problem in several ways, each with a gap. Sampling-based planners explore many routes (Karaman & Frazzoli, 2011), yet smoothing their waypoints can introduce collisions; optimization-based planners improve a motion locally (Schulman et al., 2014; Zucker et al., 2013; Tedrake & the Drake Development Team, 2019), with TrajOpt also supporting continuous collision checks (Schulman et al., 2014). Constraints enforced only at sampled times do not ensure feasibility between them (Hargraves & Paris, 1987). Convex-region planners search a decomposition of free space (Deits & Tedrake, 2015a; Marcucci et al., 2023), and semidefinite relaxations certify a polynomial motion as globally optimal only when the relaxation is tight (Graesdal et al., 2026). Even shortest-path complexity depends on the geometry: polynomial-time in the real-RAM model among planar discs (Chang et al., 2005), NP-hard among polyhedra in $\mathbb{R}^3$ (Canny & Reif, 1987), and open for disjoint unit balls in $\mathbb{R}^3$ (Goodman et al., 2017).

Our setting is a point robot in $\mathbb{R}^n$ that travels between prescribed boundary states around obstacles while minimizing a smoothness cost; a learned planner is steered by keeping the robot near its proposal. Although elementary to state, this setting already contains the core difficulty of avoiding collisions, yet, to our knowledge, no algorithm is known to converge to its global optimum once a derivative cost such as acceleration or snap is combined with collision constraints on the whole time interval. We introduce GlobMP and prove that its costs converge to the global optimum under three mild assumptions.

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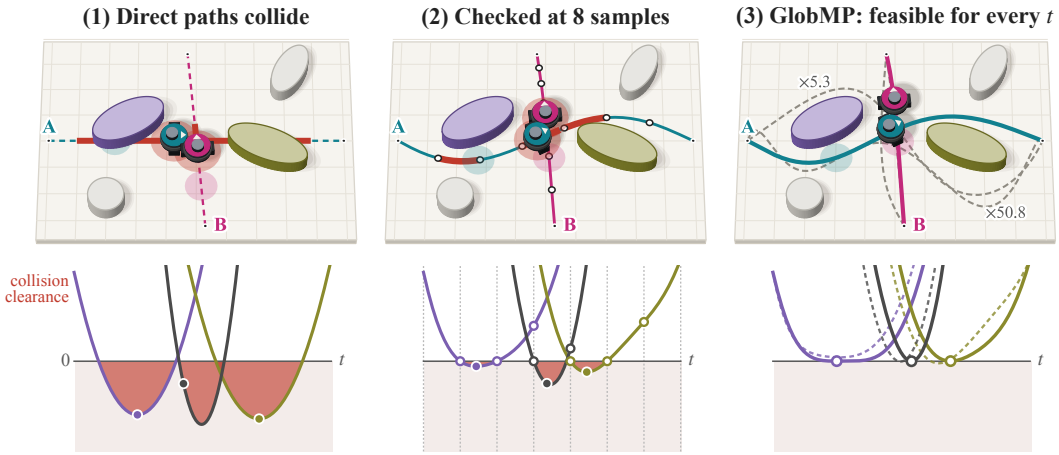


Figure 1: Two robots, A and B, planned jointly among ellipsoidal obstacles; each obstacle and the other robot enter (P_∞) as a collision constraint (ellipses through the weighted norm of Appendix A2.1). Top: each robot's path in its colour (A teal, B magenta), with both robots drawn at the time of the dark-grey A−B mark and a faded disc where each was earlier; red marks a collision. Bottom: robot A's clearance to each coloured obstacle and the A−B clearance (dark grey) over time; below zero is a collision. (1) The direct (dashed) paths collide. (2) Checking the constraints at 8 sample times (open dots) misses collisions between them. (3) The final solid curves keep every clearance nonnegative on $[0,1]$; coloured dashed: an earlier descent step; grey dashed: numerically refined alternatives, with cost relative to GlobMP's.

GlobMP combines exact whole-interval constraints, gradient-based refinement and a systematic search over initial curves. We consider

$$
\begin{aligned}
&\min_{\boldsymbol{\gamma}\in\mathscr{H}^k} && \int_0^1 \left\|\boldsymbol{\gamma}^{(k)}(t)\right\|^2 dt && && \text{smoothness} \\
&\text{subject to} && \left\|\boldsymbol{\gamma}(t)-\mathbf{c}_j(t)\right\| \geq r_j, && \forall t\in[0,1],\ j=1,\dots,p, && \text{collision} \\
&&& \left\|\boldsymbol{\gamma}(t)-\boldsymbol{\rho}(t)\right\| \leq \bar{r}, && \forall t\in[0,1], && \text{steering} \\
&&& \boldsymbol{\gamma}^{(s)}(0)=\boldsymbol{\gamma}_0^{(s)},\ \boldsymbol{\gamma}^{(s)}(1)=\boldsymbol{\gamma}_1^{(s)}, && s=0,\dots,\ell, && \text{boundary}
\end{aligned}
\qquad (P_\infty)
$$

Here $\boldsymbol{\gamma}:[0,1]\to\mathbb{R}^n$ is the robot's trajectory, with time normalized to $[0,1]$, and $\mathscr{H}^k = \mathscr{H}^k([0,1];\mathbb{R}^n)$ denotes the Sobolev space of curves whose derivatives through order k are square-integrable: the objective is finite exactly on $\mathscr{H}^k$, and its curves have $k-1$ continuous derivatives, so the constraints and boundary values are defined at every instant. The objective measures smoothness through the kth derivative: $k=2$ penalizes acceleration and $k=4$ snap (Mellinger & Kumar, 2011). The collision constraints keep the robot at distance at least $r_j>0$ from each of p obstacle centres $\mathbf{c}_j(t)$, which may move, and the steering constraint keeps it within $\bar{r}>0$ of a reference $\boldsymbol{\rho}(t)$, both at every instant. The boundary conditions fix the position and its derivatives through order $\ell<k$ at both ends. Unlike Graesdal et al. (2026), whose spheres are static, we let the centres move in time; this single extension covers several robots, each a moving centre for the others, and steering, where the robot must stay near the moving centre $\boldsymbol{\rho}(t)$.

Several robots are planned together by stacking their curves into one $\boldsymbol{\gamma}$ and keeping robots a and b apart by $\|\boldsymbol{\gamma}_a(t)-\boldsymbol{\gamma}_b(t)\|\geq r_a+r_b$ (Shaoul et al., 2025). The reference $\boldsymbol{\rho}$ can be the waypoints of a learned planner (Janner et al., 2022) or the action chunk of a policy (Chi et al., 2023), which the steering constraint corrects rather than replaces; with $\bar{r}=\infty$ it is dropped and (P_∞) is plain motion planning. The polynomial curves of §2 may fix boundary orders above ℓ. Ellipsoids, walls, corridors and other variants are in Appendix A2.1.

Our contributions address two difficulties of (P_∞): constraints must hold at every instant, and improving one feasible curve need not find the best route.

C1. We enforce collision constraints *exactly in continuous time*. At a fixed polynomial degree, each constraint becomes a univariate polynomial that must be nonnegative on $[0,1]$ (Powers &

Reznick, 2000), so it holds between sample times too; collision, robot separation and steering share one block formulation (§2).

C2. We propose *GlobMP*, a gradient projection algorithm that alternates a cost step with feasibility restoration (§2). Cone projections give coefficient corrections for violated blocks, which Jacobians turn into control-point updates, block by block and with the boundary states kept.

C3. We prove *global convergence* (Theorem 1). Under mild assumptions, GlobMP keeps every curve feasible without increasing its cost, and from a fine grid start it reaches the cheapest curves of N polynomial pieces, which approach the best continuous-time motion as N grows.

C4. We *deploy* GlobMP on real robots and show superior performance over strong baselines: multi-robot teams on the Robotarium testbed (§4.2), and steering state-of-the-art expert policies on a bimanual humanoid with dexterous hands in tactile manipulation tasks (§4.3). The appendix adds an extended multi-robot benchmark and maze navigation (Appendix A4).

2 Polynomial formulation and gradient projection

We optimize the control points of a polynomial curve, alternating a gradient step on the cost with restoration of feasibility; because the curve is a polynomial, each constraint is checked over the whole motion. We represent the curve by a polynomial of degree d (piecewise when the horizon is split) in the *Bernstein basis* $[\mathbf{b}_d(t)]_i = \binom{d}{i} t^i (1-t)^{d-i}$, $i = 0, \dots, d$, whose weights are nonnegative and sum to one on $[0,1]$:

$$\boldsymbol{\gamma}(t) = \boldsymbol{\Gamma}\mathbf{b}_d(t), \qquad \boldsymbol{\Gamma} \in \mathbb{R}^{n\times(d+1)}. \tag{1a}$$

The columns of $\boldsymbol{\Gamma}$ are the *control points*, whose convex hull contains the curve. Substituting this representation into the objective gives

$$\mathrm{cost}(\boldsymbol{\Gamma}) := \int_0^1 \left\|\boldsymbol{\Gamma}\mathbf{b}_d^{(k)}(t)\right\|^2 dt = \mathrm{tr}(\mathbf{Q}\boldsymbol{\Gamma}^\top\boldsymbol{\Gamma}), \tag{1b}$$

where tr is the trace and $\mathbf{Q} := \int_0^1 \mathbf{b}_d^{(k)}\mathbf{b}_d^{(k)\top} dt$ is precomputed exactly, so each cost step uses a matrix gradient without reevaluating the integral. Since $\mathbf{Q} \succeq 0$ (positive semidefinite), the objective is convex; the constraints carry the nonconvexity.

Write constraint j as $g_j(\boldsymbol{\gamma}(t), t) \geq 0$, where $g_j(\mathbf{x}, t) := \|\mathbf{x} - \mathbf{c}_j(t)\|^2 - r_j^2$ for the collision constraints, $j \leq p$ (for several robots, the robot's own position $\mathbf{x}_a$ replaces $\mathbf{x}$; each robot pair is a collision block too, with $\mathbf{x}_a - \mathbf{x}_b$ and $r_a + r_b$ replacing $\mathbf{x} - \mathbf{c}_j$ and r_j, and from here on p counts the obstacle and pair blocks), and $g_{p+1}(\mathbf{x}, t) := \bar{r}^2 - \|\mathbf{x} - \boldsymbol{\rho}(t)\|^2$ for the steering constraint when $\bar{r} < \infty$. For polynomial obstacle centres and a polynomial reference, the value along the curve is a polynomial in time,

$$g_j^{\boldsymbol{\Gamma}}(t) := g_j\big(\boldsymbol{\Gamma}\mathbf{b}_d(t), t\big) = \boldsymbol{\pi}_j(\boldsymbol{\Gamma})^\top \mathbf{b}_{m_j}(t), \tag{1c}$$

with degree bound $m_j = 2d$, or $2\max(d, d_j)$ for a centre moving on a polynomial path of degree d_j (the reference plays the role of a centre for $j = p+1$); piecewise-polynomial data are handled interval by interval (Appendix A3.3). Its coefficient vector $\boldsymbol{\pi}_j(\boldsymbol{\Gamma})$ is quadratic in $\boldsymbol{\Gamma}$. The constraint holds at every instant exactly when this vector belongs to the closed convex cone

$$\mathscr{K}_j := \{\boldsymbol{\zeta} \in \mathbb{R}^{m_j+1} : \boldsymbol{\zeta}^\top \mathbf{b}_{m_j}(t) \geq 0 \text{ for every } t \in [0,1]\}. \tag{1d}$$

This tests the entire polynomial: $(2t-1)^2$ has Bernstein coefficients $(1, -1, 1)$ yet is nonnegative, so a coefficient-sign test would wrongly reject it. The coefficients are quadratic in $\boldsymbol{\Gamma}$, so the set of control points that satisfy a block need not be convex. Integral budgets, an optional extension beyond (P_∞), fit the same notation as scalar blocks,

$$\pi_j(\boldsymbol{\Gamma}) := \int_0^1 \psi_j\big(\boldsymbol{\Gamma}\mathbf{b}_d(t), t\big)\, dt \in \mathscr{K}_j := \mathbb{R}_+, \tag{1e}$$

where ψ_j encodes the signed budget margin. Let $\bar{p}$ denote the number of pointwise blocks ($\bar{p} = p+1$ with the steering block, $\bar{p} = p$ without); the q budgets are blocks $j = \bar{p}+1, \dots, \bar{p}+q$. We collect the constraints as

$$\boldsymbol{\pi}(\boldsymbol{\Gamma}) := \big(\boldsymbol{\pi}_1(\boldsymbol{\Gamma}), \dots, \boldsymbol{\pi}_{\bar{p}}(\boldsymbol{\Gamma}), \pi_{\bar{p}+1}(\boldsymbol{\Gamma}), \dots, \pi_{\bar{p}+q}(\boldsymbol{\Gamma})\big), \qquad \mathscr{K} := \mathscr{K}_1 \times \cdots \times \mathscr{K}_{\bar{p}} \times \mathbb{R}_+^q. \tag{1f}$$

The degree-d problem is then

$$\inf_{\mathbf{\Gamma}\in\mathbb{R}^{n\times(d+1)}} \operatorname{cost}(\mathbf{\Gamma}) \quad \text{s.t.} \quad \boldsymbol{\pi}(\mathbf{\Gamma}) \in \mathscr{K}, \qquad \mathbf{\Gamma}\mathbf{A} = \mathbf{B}, \tag{1g}$$

The boundary conditions are linear equations $\mathbf{\Gamma}\mathbf{A} = \mathbf{B}$ in the control points; let $\mathscr{A}_{\text{bc}} := \{\mathbf{\Gamma} : \mathbf{\Gamma}\mathbf{A} = \mathbf{B}\}$ denote the control points that satisfy them. We fix one such point $\mathbf{\Gamma}_{\text{aff}}$ and allow only changes that keep the equations. When $2(\ell+1) \le d$, every $\mathbf{\Gamma} \in \mathscr{A}_{\text{bc}}$ is $\mathbf{\Gamma} = \mathbf{\Gamma}_{\text{aff}} + \mathbf{\Gamma}_{\text{free}}\mathbf{N}^\top$, where the columns of $\mathbf{N}$ are an orthonormal basis of the null space of $\mathbf{A}^\top$ and $\mathbf{\Gamma}_{\text{free}} \in \mathbb{R}^{n\times d_f}$, $d_f = d+1-2(\ell+1)$, collects the free control points (Lemma 3); distances between free and full control points agree. In these free coordinates

$$\begin{aligned} &\inf_{\mathbf{\Gamma}_{\text{free}}} \quad \operatorname{cost}\big(\mathbf{\Gamma}_{\text{aff}} + \mathbf{\Gamma}_{\text{free}}\mathbf{N}^\top\big) \\ &\text{subject to} \quad \mathbf{\Gamma}_{\text{free}} \in \mathscr{F} := \Big\{\mathbf{\Gamma}_{\text{free}} : \ \boldsymbol{\pi}\big(\mathbf{\Gamma}_{\text{aff}} + \mathbf{\Gamma}_{\text{free}}\mathbf{N}^\top\big) \in \mathscr{K}\Big\}. \end{aligned} \tag{P}$$

Every reconstructed curve meets the boundary conditions; for piecewise curves the junction conditions join the linear equations. Fixing the degree restricts the curves but neither discretizes time nor weakens the whole-interval constraints.

Write $\boldsymbol{\pi}_j(\mathbf{\Gamma}_{\text{free}})$ for $\boldsymbol{\pi}_j(\mathbf{\Gamma}_{\text{aff}} + \mathbf{\Gamma}_{\text{free}}\mathbf{N}^\top)$. Restoration, Algorithm 1, takes gradient steps on the summed squared distances of the blocks to their cones, where $P_{\mathscr{K}_j}$ maps a vector to its nearest point in $\mathscr{K}_j$,

$$V(\mathbf{\Gamma}_{\text{free}}) := \sum_{j=1}^{\bar{p}+q} \big\| \boldsymbol{\pi}_j(\mathbf{\Gamma}_{\text{free}}) - P_{\mathscr{K}_j}\big(\boldsymbol{\pi}_j(\mathbf{\Gamma}_{\text{free}})\big) \big\|^2. \tag{2}$$

The squared distance to a closed convex cone is differentiable, with gradient twice the residual $\mathbf{e}_j = \boldsymbol{\pi}_j - P_{\mathscr{K}_j}(\boldsymbol{\pi}_j)$, which vanishes exactly when block j is satisfied. The chain rule gives $2D\boldsymbol{\pi}_j^\top \mathbf{e}_j$, where $D\boldsymbol{\pi}_j$ is the Jacobian in free coordinates and the gradient is reshaped as a control-point matrix. Only violated blocks contribute (lines 2–6). Algorithm 2 alternates a cost step along $-2\mathbf{\Gamma}\mathbf{Q}\mathbf{N}$ with restoration (lines 4–6), and rebuilding $\mathbf{\Gamma}$ from the free coordinates preserves the boundary states. $P_{\mathscr{F}}$ denotes Algorithm 1, which need not return an exactly feasible or nearest curve in finitely many steps; the theory analyzes the refinement below.

Each cone projection is a small convex problem. Evaluating a polynomial at time t is an inner product of its coefficients with $\mathbf{b}_{m_j}(t)$, and the projection adds to $\boldsymbol{\pi}_j$ the nonnegative combination $\boldsymbol{\omega}$ of vectors $\mathbf{b}_{m_j}(t)$, $t \in [0,1]$, that is closest to $-\boldsymbol{\pi}_j$ (Moreau's decomposition). We build $\boldsymbol{\omega}$ greedily. Starting from no selected times, let $\boldsymbol{\xi} = -\boldsymbol{\pi}_j - \boldsymbol{\omega}$ denote what is left to correct; each step adds the time

$$t_* \in \operatorname*{arg\,max}_{t\in[0,1]} \frac{\langle \boldsymbol{\xi}, \mathbf{b}_{m_j}(t)\rangle}{\|\mathbf{b}_{m_j}(t)\|}, \tag{3}$$

where the corrected polynomial is most negative relative to its scale, and then refits nonnegative weights on all selected times (Algorithm 3; Locatello et al., 2017), with error bounds in Appendix A3.1.4; scalar budgets use $a \mapsto \max\{a, 0\}$. The selected times locate the violation rather than fix a list of checks, and blocks are projected independently, so the work per iteration grows linearly with the number of blocks at fixed degree and cone-solve budget.

For the analysis we use a feasible version of the repair step. Let z denote the free control points $\mathbf{\Gamma}_{\text{free}}$ stacked into one vector and $f(z)$ the cost; we keep the notation $\boldsymbol{\pi}_j(z)$ and $\mathscr{F}$ for the blocks and the feasible set, and write $D\boldsymbol{\pi}_j(z)$ for the Jacobian (matrix of first derivatives) of block j. Each obstacle margin is convex in the control points, so its linearization at any z lies below it at every instant. For a sphere the linearized constraint is a half-space that excludes the whole obstacle, so each step keeps every robot behind a moving plane that separates it from each obstacle and from each other robot. Let $\mathscr{D}$ denote the points that satisfy the steering block exactly and the optional budgets; the steering set is convex, since a ball is convex and the curve depends affinely on the free control points; the budgets are assumed convex ($\mathscr{D}$ is all points when neither is present). Let

$$\mathscr{C}(z) := \big\{w \in \mathscr{D} : \ \boldsymbol{\pi}_j(z) + D\boldsymbol{\pi}_j(z)(w - z) \in \mathscr{K}_j, \quad j = 1, \dots, p\big\}. \tag{4}$$

Then $\mathscr{C}(z)$ is convex and contained in $\mathscr{F}$, and it contains z when z is feasible. Line 6 of Algorithm 2 becomes the convex solve

$$z^+ = \operatorname*{arg\,min}_{w\in\mathscr{C}(z)} \tfrac{1}{2}\|w - (z - \eta\nabla f(z))\|^2. \tag{5}$$

Algorithm 1 Feasibility restoration for (P)

Require: free control points $\mathbf{\Gamma}_{\text{free}}$, blocks $\boldsymbol{\pi}_j$, tolerance tol, step η_{proj}, iterations T_{proj}

for $\nu = 1, \dots, T_{\text{proj}}$ **do**
  $\mathbf{e}_j \leftarrow (I - P_{\mathscr{K}_j})\,\boldsymbol{\pi}_j(\mathbf{\Gamma}_{\text{free}})$ *all blocks*
  if $\max_j \|\mathbf{e}_j\|^2 \le \text{tol}$ **then break**
  $\mathscr{V} \leftarrow \{j : \mathbf{e}_j \neq 0\}$ *violated blocks*
  $\mathbf{\Delta} \leftarrow 2\sum_{j\in\mathscr{V}} D\boldsymbol{\pi}_j^{\top}\mathbf{e}_j$ *merit gradient*
  $\mathbf{\Gamma}_{\text{free}} \leftarrow \mathbf{\Gamma}_{\text{free}} - \eta_{\text{proj}}\mathbf{\Delta}$ *restoration step*
end for
return $\mathbf{\Gamma}_{\text{free}}$ *candidate*

Algorithm 2 Gradient projection for (P)

Require: anchor $\mathbf{\Gamma}_{\text{aff}}$ and basis $\mathbf{N}$, start $\mathbf{\Gamma}_{\text{free}}$, step η, iterations T

$\mathbf{\Gamma}_{\text{free}} \leftarrow P_{\mathscr{F}}(\mathbf{\Gamma}_{\text{free}})$ *restore start*
for $\nu = 1, \dots, T$ **do**
  $\mathbf{\Gamma} \leftarrow \mathbf{\Gamma}_{\text{aff}} + \mathbf{\Gamma}_{\text{free}}\mathbf{N}^{\top}$ *reconstruct curve*
  $\mathbf{\Delta} \leftarrow 2\mathbf{\Gamma}\mathbf{Q}\mathbf{N}$ *cost gradient*
  $\mathbf{Y} \leftarrow \mathbf{\Gamma}_{\text{free}} - \eta\,\mathbf{\Delta}$ *cost step*
  $\mathbf{\Gamma}_{\text{free}} \leftarrow P_{\mathscr{F}}(\mathbf{Y})$ *restore, Alg. 1*
end for
return $\mathbf{\Gamma}_{\text{free}}$ *candidate*

Every iterate is feasible, and $\mathscr{C}(z)$ is rebuilt after each step, so this is not a convex reformulation of (P); it needs a feasible start and an exact solve. In the analysis, GlobMP starts this refinement from the cheapest point of a grid of free control points, each tested exactly over the whole interval. For (P_∞) it refines the curve by adding pieces (Theorem 1) or raising the degree (Algorithm 4).

3 Theory

We state one theorem, for (P_∞); its fixed-degree counterpart and all proofs are in Appendix A3. Let $E_\infty^\star$ denote the optimal value of (P_∞).

Assumption A (problem data)**.** *The cost order is $k \ge 1$ and the endpoints fix derivatives of orders $0, \dots, \ell$ with $\ell < k$; the obstacle centres and the reference are continuous and piecewise polynomial in t; the endpoints are strictly feasible; and (P_∞) has a feasible curve.*

Assumption B (enough boundary conditions)**.** *The two endpoints fix at least as many values per coordinate as the order of the cost: $2(\ell+1) \ge k$.*

Assumption C (free configurations)**.** *Let $g_j(\mathbf{x},t) = \|\mathbf{x} - \mathbf{c}_j(t)\|^2 - r_j^2$ for collisions, $j \le p$, and $g_{p+1}(\mathbf{x},t) = \bar{r}^2 - \|\mathbf{x} - \boldsymbol{\rho}(t)\|^2$ for steering. At every time and feasible position, some $\mathbf{v}$ increases every active constraint: $\nabla_{\mathbf{x}} g_j(\mathbf{x},t)^{\top}\mathbf{v} > 0$ whenever $g_j(\mathbf{x},t) = 0$.*

A change of the curve that keeps the boundary values and has zero kth-derivative cost is a polynomial of degree below k with a root of order $\ell+1$ at each endpoint, which is zero under Assumption B: minimum velocity ($k = 1$) needs only the endpoints, minimum snap ($k = 4$) also the velocities. Hence f is strongly convex, with curvature at most $\beta = 2\lambda_{\max}(\mathbf{H})$, $\mathbf{H} = \mathbf{N}^{\top}\mathbf{Q}\mathbf{N}$. Assumption C is the Mangasarian–Fromovitz condition on configurations; it rules out jamming and is used only within distance 1 of curves of cost at most $E_\infty^\star + 1$. See Appendix A3.3.

Let $\mathscr{S}_N$ denote the curves of N polynomial pieces of degree $2k-1$ on equal intervals, joined with $k-1$ continuous derivatives. Let z_h denote a cheapest feasible point of the grid of spacing h in their free control points, inside a box containing a global minimizer over $\mathscr{S}_N$ in its interior; grid points are checked exactly by root isolation, so the start is feasible.

Theorem 1 (global convergence for (P_∞))**.** *Under Assumptions A, B and C, the following hold.*

(i) There is N_0 such that, for every $N \ge N_0$, GlobMP over $\mathscr{S}_N$ with the repair (5), a step $0 < \eta < 2/\beta$ and the start z_h with h small enough produces curves that are feasible at every instant, with nonincreasing cost, and all its limit points are global minimizers over $\mathscr{S}_N$.

(ii) For $N \ge N_0$, the optimal cost $E_N^\star$ over $\mathscr{S}_N$ satisfies $E_\infty^\star \le E_N^\star \le E_\infty^\star + CN^{-(k-1/2)}$ with C independent of N, and as $N \to \infty$ subsequences of these minimizers converge uniformly to global minimizers of (P_∞).

The proof (Appendix A3.2) has three steps.

1. *Feasible descent.* Collision blocks are convex in position, so linearizations lie below them and $\mathscr{C}(z)$ is a convex feasible set; projecting onto it lowers f by at least $(1/\eta - \beta/2)\|z^{\nu+1} - z^{\nu}\|^2$.

2. *Global limits at fixed N.* Interpolating a direction that frees the robots at every instant (Assumption C) frees every block, so limits are KKT points; the problem over $\mathscr{S}_N$ is semialgebraic, so these take finitely many cost values (Bolte et al., 2007), and a start z_h cheaper than every non-optimal value can only reach the cheapest curves.
3. *Refinement.* Pushing an optimal curve of (P_∞) off the obstacles and interpolating it with N pieces gives a feasible curve in $\mathscr{S}_N$ within $CN^{-(k-1/2)}$ of $E^\star_\infty$.

Each step explains a design choice of Algorithm 2: the exact repair (5) keeps every iterate feasible, which sampled checks cannot (Figure 1), and convexity in the position makes it a convex projection, so the cost descends without penalties.

Corollary 2 (ε-optimal plans in continuous time)**.** *Under the assumptions of Theorem 1, let $\varepsilon > 0$ and $N \geq \max\{N_0, (2C/\varepsilon)^{1/(k-1/2)}\}$. Every curve that GlobMP produces over $\mathscr{S}_N$ is feasible for (P_∞), and, started from z_h, after finitely many iterations its cost is at most $E^\star_\infty + \varepsilon$.*

A run can be stopped at any iteration with a curve safe at every instant, and accuracy ε is reached with order ε^{-2}, $\varepsilon^{-2/3}$ and $\varepsilon^{-2/7}$ pieces for velocity, acceleration and snap ($k = 1, 2, 4$; proof in Appendix A3.2).

4 Experiments

We evaluate GlobMP on synthetic teams of 2 to 200 robots among spheres (§4.1), a multi-robot benchmark on the Robotarium testbed (§4.2) and tactile-reactive manipulation (§4.3), restoring with Algorithm 1 from heuristic seeds (route classes, prioritized plans, the executing curve).

4.1 Synthetic experiments

The synthetic study (Problem 1) has robots exchange places in pairs past static spheres, under the minimum-snap cost with piecewise degree-9 polynomials; each pair or sphere can be passed on either side, so an instance has many route classes. A curve is *feasible* if it makes no contact at any t; GlobMP keeps a 0.5 cm margin. GlobMP runs Algorithms 1–2 from one waypoint path per route class and keeps the cheapest feasible result. The baselines are RRT-Connect (Kuffner & LaValle, 2000), RRT* (Karaman & Frazzoli, 2011) and GCS (Marcucci et al., 2023) on IRIS regions (Deits & Tedrake, 2015a) in the joint configuration space, whose paths are fitted by the cheapest collision-free smooth curve of the same class (Richter et al., 2016), and SDP-MP of Graesdal et al. (2026) on the same program (Appendix A4.2).

Each baseline imposes the constraints off the curve it returns (along a sampled path, inside convex regions, or on a relaxed variable), keeps centimetres to decimetres of slack, and fails more often as robots are added; SDP-MP collides or times out on every instance (Figure 2a,b). GlobMP is feasible at every t on every instance and ends exactly at the margin. Feasible baseline curves cost from about twice to hundreds of times more, with a gap that grows with the number of robots (Figure 2c); where SDP-MP returns a lower bound, GlobMP is within 6% of it at the median. This class ($\ell = k$) and its seeds lie outside Theorem 1, so the lowest cost found is not a proven optimum. From 2 to 200 robots GlobMP is feasible at every t on every seed, at 200 robots (82,000 blocks) in about two minutes on CPU and 28 s on one GPU, setup included (Figure 3a,b), with time per iteration linear in the number of blocks (Figure 3c); the baselines are feasible on at most 2 of 5 seeds at eight robots and on none from 16.

4.2 Multi-robot motion planning

Multi-robot motion planning coordinates the simultaneous motion of several robots toward assigned goals while avoiding collisions with obstacles and with one another. We evaluate GlobMP on five multi-robot environments, `Empty`, `Conveyor`, `Cross-Room`, `Drop-Region` and `Corridor-Bay`, from Shaoul et al. (2025) (`Cross-Room` and `Corridor-Bay` reconstructed from Liang et al. (2025)), following the test-time velocity constraint and evaluation protocol of Giaretta et al. (2026) on the Georgia Tech Robotarium (Wilson et al., 2020), with teams of $N_a \in \{3, 6, 9, 12\}$ robots. The environments are fitted to the 3.2×2.0 m Robotarium arena, with the common velocity limit selected for the Robotarium setting and robot radius $r = 0.13$ m. For

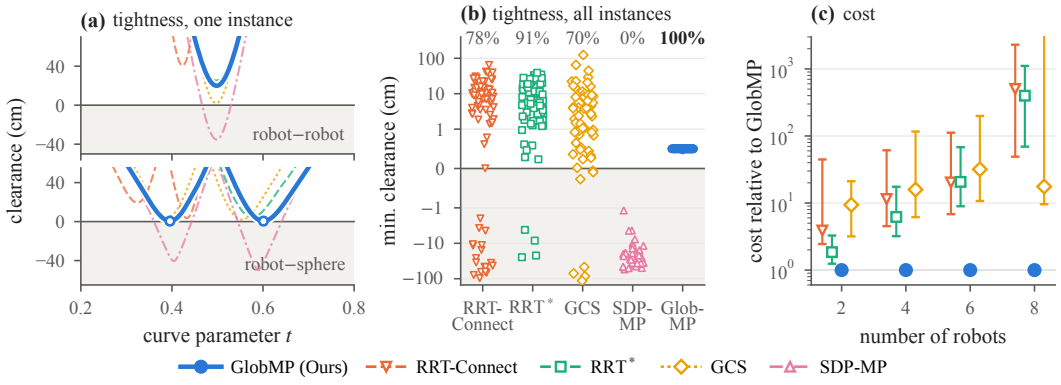


Figure 2: Tightness and cost, 20 instances each with 2, 4, 6 and 8 robots. (a) Clearance along the curve on one 2-robot instance, between the robots (top) and to the spheres (bottom). (b) Minimum clearance of every returned curve, with the share of returned curves feasible at every t. (c) Cost of feasible curves relative to GlobMP's, median and interquartile range.

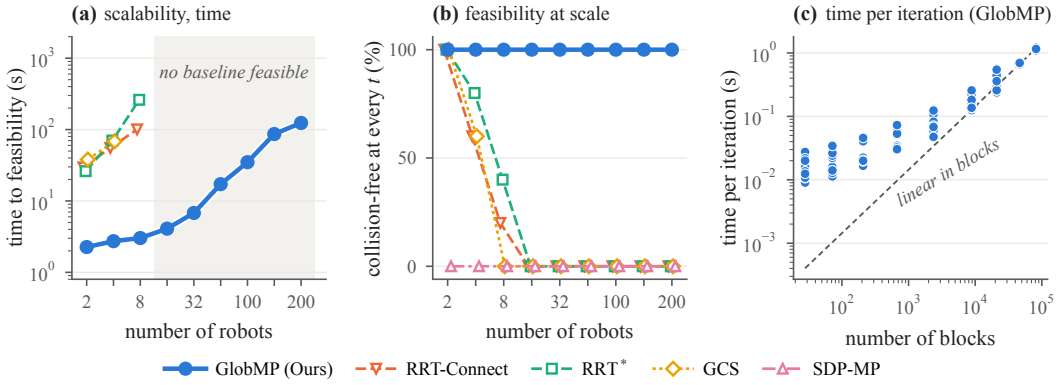


Figure 3: Scalability, 2 to 200 robots, 10 seeds per size for GlobMP and 5 for the baselines. (a) Time to a feasible trajectory, setup included. (b) Collision-free at every t. (c) GlobMP's time per iteration.

each (map, team-size) pair, we use ten start–goal instances: nine sampled from the benchmark distribution and one antipodal "ring" instance that forces the team to traverse the whole workspace. Hardware details of the Robotarium setup are provided in Appendix A4.1.

GlobMP needs no training: at test time it solves each instance directly as one (P_∞), with the robots' curves stacked, a collision constraint at radius $2r$ for every pair and affine blocks for boxes (Appendix A2.1.1), so its plan is collision-free at every t.

The baselines are CD, PCD-LB and PCD-SHD (Luan et al., 2026), Final Proj. and DiRecT (Giaretta et al., 2026) and MMD-CBS (Shaoul et al., 2025). Each method is evaluated on the same start–goal instances. The sampling baselines draw $B = 64$ candidates, whereas GlobMP returns one trajectory.

Figure 4 reports the success of a single returned plan across the five environments and team sizes. On `Conveyor` and `Cross-Room`, baseline performance drops rapidly as the number of robots increases: every baseline reaches zero on `Conveyor` at $N_a = 12$ and stays at or below 0.30 on `Cross-Room` from $N_a = 9$, whereas GlobMP's single plan succeeds on every instance except two of the ten on `Conveyor` at $N_a = 12$. The stronger projection baselines remain more reliable on `Empty`, `Drop-Region`, and `Corridor-Bay`, but most also degrade with team size. Figure 5 presents representative 12-robot ring trajectories on `Cross-Room` and `Conveyor`. Appendix A4.4 extends the evaluation to eight environments, teams of up to 21 robots, three velocity caps, and a larger set of start–goal instances.

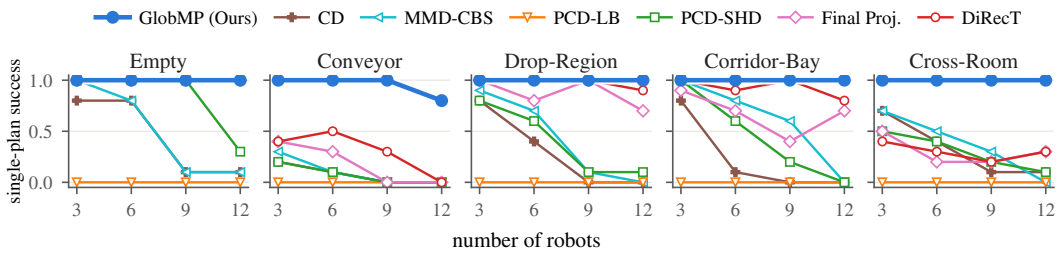


Figure 4: Success of a *single* returned plan on the Robotarium arena for $N_a \in \{3, 6, 9, 12\}$ agents, one panel per map; for GlobMP, the one plan it returns, and for a sampling baseline, the first of its 64 candidates. PCD-LB is at zero throughout: its log-barrier pushes agents into the arena boundary.

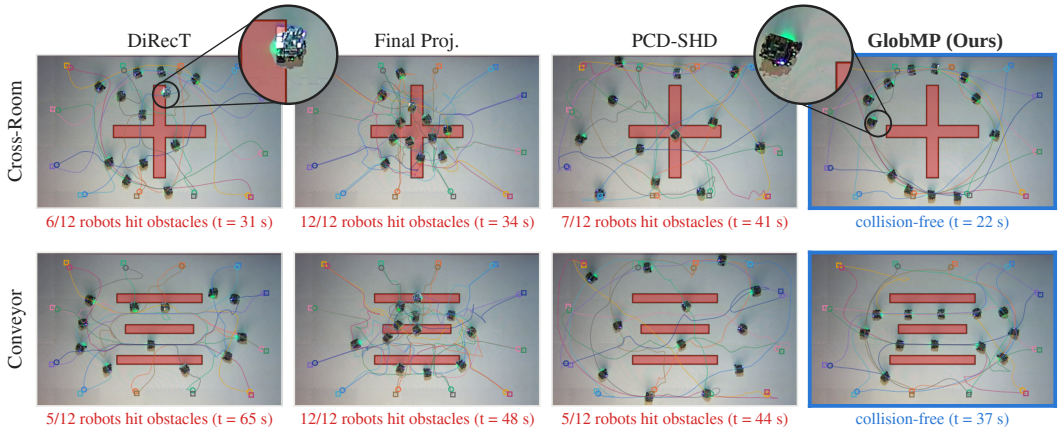


Figure 5: Executed trajectories for the 12-robot ring instance on `Cross-Room` and `Conveyor`, on the Robotarium arena. Obstacles are overlaid; circles magnify highlighted regions.

4.3 Steering tactile-reactive dexterous policies

A Dexmate Vega-1 with two XHAND1 hands with fingertip force sensing (Figure 7) performs four bimanual tasks learned from teleoperation (Figure 6a). The expert policies are two state-of-the-art vision-language-action models, GR00T N1.7 (Bjorck et al., 2025) and $\pi_{0.5}$ (Physical Intelligence et al., 2025), each with and without tactile input. Both command absolute joint targets for both arms and hands ($n = 38$) at 30 Hz in chunks of 16 executed steps.

Deployed, the policies fail geometrically (they replay memorised targets), temporally (consecutive chunks disagree by up to 0.5 rad) and at contact, mostly because of observations outside the training distribution, which no motion constraint can observe; so we separate *deciding* which constraints apply from *enforcing* them (Figure 6b). A *meta-reasoner*, the vision-language model Claude Opus 5.5 (Anthropic, 2026), defines the constraints once, at reset, from the images, the task and the measured state, for example the keep-out, keep-in and grip of Figure 6b. A *certifier* then solves (P) per chunk, with the policy's chunk as the reference $\boldsymbol{\rho}$, so the executed curve meets every active constraint at every t. The meta-reasoner only chooses constraints and never solves them, so a wrong choice changes *which* feasible curve is returned, never *whether* it is feasible, unlike model-written programs (Huang et al., 2024). If the policy slows, a demonstrated time law rescales the reference (Kant & Zucker, 1986).

The baselines are the expert policies alone, each with and without touch, on the same four tasks, and every condition runs 10 real trials per task.

Steering matches or raises every task (Figure 6c): it turns the tissue, which the policies alone rarely complete, into a mostly successful task, and brings the handover to every trial. Touch pays off where the hand must regulate contact, as in placing the toy, pulling the tissue and stowing the mug, and

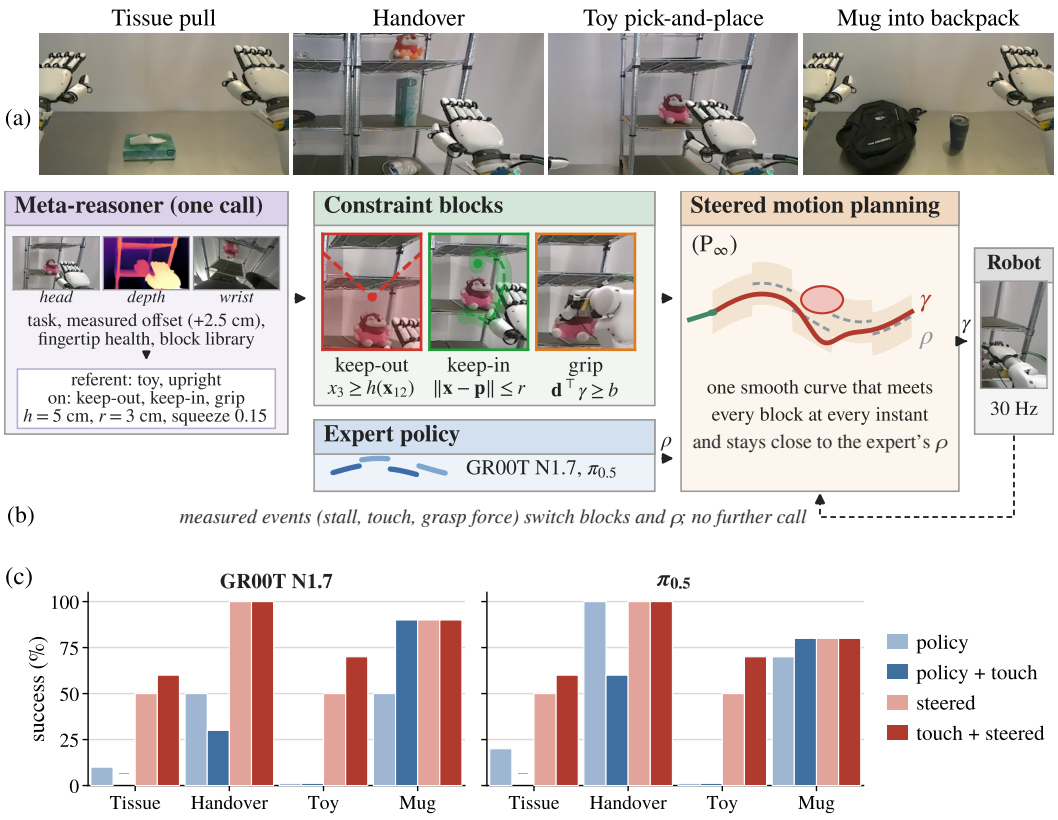


Figure 6: (a) The initial head-camera frame of each task. (b) Steered motion planning on one real toy trial: one meta-reasoner call at reset sets the active blocks and their parameters; (P) takes the policy's chunk as $\boldsymbol{\rho}$ and returns a curve that is smooth, collision-free and within $\bar{r}$ of $\boldsymbol{\rho}$ at every t; measured events then switch blocks. (c) Real-robot success of GR00T and $\pi_{0.5}$ per task (10 trials per bar): the policy without and with touch input, and steered without and with touch; dash: not run.

adds little where the task is guided by vision, as in the handover, where it even hurts the policies alone. In a closed-loop simulator replaying failed real tissue deployments, the pinch lands where and when demonstrated only with steering.

5 Conclusion

Polynomial trajectories turn continuous-time planning among robots and static or moving obstacles into a finite-dimensional problem that enforces every collision constraint at every instant, with planning and steering in one program. With a convex restoration step, GlobMP from a fine grid approaches the cheapest curves of each fine enough spline class, within $O(N^{-(k-1/2)})$ of the continuous-time optimum under enough boundary conditions, strictly feasible endpoints and no jammed configuration. The experiments use heuristic seeds and first-order restoration, which this guarantee does not cover and whose rate remains open.

AI use statement

Following the ICLR 2027 policy, we disclose the use of large language models (LLMs) to aid and polish the writing, editing the text for clarity and length; for retrieval and discovery, helping find related work and check citations; and to generate the synthetic datasets, whose instances are produced by LLM-assisted scripts from fixed seeds (Appendix A4.2). The authors take full responsibility for all content.

REPRODUCIBILITY STATEMENT

We will release all code: the GlobMP solver (restoration and projected gradient steps), the scripts and configurations for every experiment and figure, and the steering and deployment code for learned policies on the Vega-1 with XHAND1 hands (certifier, grounding, time law, policy servers), together with the teleoperation demonstrations and fine-tuned policy checkpoints of §4.3. All proofs and assumptions are in Appendix A3. Every proof was checked by hand by the authors, and exact-arithmetic checks of identities and counterexamples are in `theory_checks`. Experimental details and numerical checks, including precision limits, are documented in Appendix A4 and Appendix A4.5.

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Throughout, $\|\cdot\|$ without a subscript is the Euclidean norm of a vector (the Frobenius norm is written $\|\cdot\|_F$).

A1 ADDENDUM TO SECTION 1: RELATED WORK AND FIGURE 1

A1.1 EXTENDED RELATED WORK

Relation to existing formulations. Collocation replaces each block by inequalities at sampled times (Hargraves & Paris, 1987); the control-polygon rule asks every coefficient to be nonnegative (Gao et al., 2018), which is sufficient for $\mathscr{K}_j$ but conservative; convex decompositions (Deits &

Tedrake, 2015a; Marcucci et al., 2023) are exact for the decomposition rather than for the free space; semidefinite relaxations (Lasserre, 2001; Parrilo, 2003; Graesdal et al., 2026) certify a trajectory only when tight. We keep the original variables and free space, test the exact condition on each block, and reach global minimizers over each spline class, and the optimum of (P_∞) as the class is refined (Theorem 1).

Sampling-based planning and trajectory optimization. The two classical families both act on the nonconvex problem itself, and neither certifies global optimality. Sampling-based planners (LaValle, 1998; Karaman & Frazzoli, 2011) are probabilistically complete and, for RRT*, asymptotically optimal, but the guarantee is a limit statement: at any finite sample count nothing is certified about the returned path's optimality. Trajectory optimizers instead discretize and hand the result to local nonlinear programming, direct collocation (Hargraves & Paris, 1987), covariant gradient descent (Zucker et al., 2013), sequential convexification (Schulman et al., 2014), and differentiable collision geometry (Tracy et al., 2023), which is fast and widely deployed, but converges to a stationary point with no test for whether it is global. We keep this setting, and add a proof of global convergence.

Convex decomposition and graphs of convex sets. A second line makes the problem tractable by convexifying the *domain*. IRIS inflates large obstacle-free convex regions (Deits & Tedrake, 2015a;b); shortest-paths-in-graphs-of-convex-sets then plans over that decomposition as a single convex program (Marcucci et al., 2023), with subsequent work on scalability and tighter formulations (Marcucci et al., 2025; Werner et al., 2026; 2025), on space-time variants for multiple robots (Tang et al., 2025), and on using the convex solution to warm-start nonconvex trajectory optimization (von Wrangel & Tedrake, 2024). These methods are certifiably optimal *for the decomposition*: the regions are a modelling choice made before optimizing, and a curve is admissible only if it respects them. Our guarantee is stated on the original free space, with no decomposition.

Learning-based planners. Diffusion models have recently been trained to sample trajectories directly, for single (Carvalho et al., 2023) and multiple (Shaoul et al., 2025) robots, with constraints imposed during sampling by projection (Luan et al., 2026) or by receding-horizon denoising (Giaretta et al., 2026). These amortize planning into a fast forward pass and represent several routes, but they enforce feasibility only approximately at sampling time and produce no optimality certificate. We compare against them in §4; our contribution is a guarantee rather than a speedup, and the two can be combined (Appendix A2.2).

Convex relaxations of planning and control. Beyond Graesdal et al. (2026), several works apply semidefinite or moment-hierarchy relaxations (Shor, 1987; Lasserre, 2001; Parrilo, 2003) to planning and control problems and report tight solutions empirically, without analyzing when or why tightness holds. Teng et al. (2023) formulate kinodynamic motion planning on Lie groups as polynomial optimization and apply the moment hierarchy; Häring et al. (2024) study minimum-energy unicycle control with a semidefinite relaxation; Vega et al. (2025) apply the Shor relaxation to spacecraft maneuver planning with a spherical keep-out zone; Mahajan et al. (2026) use it for real-time predictive control around ball-shaped obstacles on embedded hardware. Control problems with free timing (Yang et al., 2025; Dong et al., 2026; Marcucci et al., 2024) and contact-rich planning (Graesdal et al., 2024; Kang et al., 2025; Wei & Dümbgen, 2026) have been relaxed in the same way. Outside planning, such relaxations underlie certifiable estimation, for example registration and SLAM (Yang et al., 2021; Rosen et al., 2019; Papalia et al., 2024), which we do not review. In contrast to the planning and control work above, which observes tightness empirically, we prove that a feasible descent method on the original nonconvex program, started from a fine grid, converges to global minimizers over each spline class, and to the optimal cost of the original program as the class is refined.

A1.2 Inexact sampled checks and the data for Figure 1

Sampling and control-polygon tests are not exact. *Sampling* t at a finite set is a strict *relaxation*. For any finite $S = \{t_1 < \cdots < t_M\} \subset [0,1]$, choosing $a < b$ inside any gap between consecutive samples makes $p(t) = (t-a)(t-b)$ (of degree $2 \le 2d$) nonnegative at every sample and negative in between, with $\min_{[0,1]} p = -\left(\frac{b-a}{2}\right)^2$. So the sampled feasible set is in general a proper superset of $\mathscr{F}$ at every M and every degree; no finite sample is ever exact. Structurally, each $\mathbf{b}_{2d}(t_0)$ is an extreme ray of $\mathscr{K}_{2d}^*$ (12): if $\mathbf{b}_{2d}(t_0) = x + y$ with $x, y \in \mathscr{K}_{2d}^*$, pairing with $(t-t_0)^2$ forces x's representing measure onto $\{t_0\}$, so a continuum of extreme rays would be replaced by finitely many.

The direction of the error is the unsafe one for a planner: the sampled problem gives a valid *lower* bound on the cost of (P), but the curve it returns may penetrate the obstacle, and the margin violation is only $\Theta(1/M^2)$: on a uniform grid a gap of width $h = 1/(M-1)$ admits violations approaching $\min_{[0,1]} p = -h^2/4$ above.

Requiring all Bernstein coefficients to be nonnegative is a strict *restriction*, and degree elevation does not close it. If $p \geq 0$ vanishes at some $t \in (0,1)$, then since every entry of $\mathbf{b}_D(t)$ is strictly positive, nonnegative coefficients would force $\mathbf{c}^\top \mathbf{b}_D(t) = 0$ with all terms nonnegative, hence $\mathbf{c} \equiv 0$ and $p \equiv 0$: no elevation degree ever works for a polynomial with an interior zero. This case arises here, because whenever a minimizer of (P) touches an obstacle at an interior time, its avoidance polynomial has an interior double root. The smallest Bernstein-D coefficient of such a polynomial stays negative and decays only like $-t^\star(1-t^\star)p''(t^\star)/(2D)$: for $p(t) = (t-\frac{1}{2})^2$ it is exactly $-1/\big(4(D-1)\big)$ for even D and $-1/(4D)$ for odd D, so no elevation is ever enough. Approaching tangency the required degree likewise diverges: for $p_\varepsilon(t) = (t-\frac{1}{2})^2 + \varepsilon$, so that $\min_{[0,1]} p_\varepsilon = \varepsilon$, the least D with all Bernstein-D coefficients nonnegative satisfies $D_{\min} = 1/(4\varepsilon) + O(1)$; in workspace terms $D_{\min} \approx \|\boldsymbol{\gamma}'\|^2 t^\star(1-t^\star)/(2r_j\delta)$ for clearance δ. The avoidance condition is therefore carried exactly, as written in (1d).

Data for Figure 1. The figure shows two robots, A (left to right) and B (bottom to top), planned jointly with one degree-6 Bernstein segment each; each robot is a moving centre for the other (§1). The obstacles are three ellipses and a circle, each enlarged by the robot radius 0.055 so that the constraint on the robot's centre keeps the whole robot clear, and the robots keep their centres at least 0.11 apart. The cost is the smoothness objective of (P_∞) with $k = 2$ (minimum acceleration), positions fixed at both ends ($\ell = 0$), and no steering constraint. Panel (2) imposes the constraints only at 8 sample times; panel (3) runs GlobMP with every constraint on all of $[0,1]$. The instance has eight route classes (A passes each of the two coloured ellipses on either side, and either robot crosses first); one opening curve per class, restored and then refined by Algorithm 2, gives feasible KKT curves in five of them, with costs 1, 1.2, 5.3, 5.6 and 51 times the cheapest; panel (3) shows the cheapest and, in grey, the routes that take a different path in space.

A2 Addendum to Section 2: Formulation and Algorithm

A2.1 Variants of the formulation

The experiments use four variants of the blocks of (P_∞). Each is convex in the position or has a convex feasible set imposed exactly, so Proposition 10 covers all four. Theorem 1 covers the position blocks (ellipsoids, robot pairs, boxes and walls); the speed limit constrains $\boldsymbol{\gamma}'$ and lies outside it.

Ellipsoidal obstacles. A weighted norm $\|\mathbf{x}\|_\mathbf{V} = (\mathbf{x}^\top \mathbf{V}\mathbf{x})^{1/2}$, $\mathbf{V} \succeq 0$, in the collision constraint, $\|\boldsymbol{\gamma}(t) - \mathbf{c}_j(t)\|_{\mathbf{V}_j} \geq r_j$, keeps every block quadratic; with $\mathbf{V}_j \succ 0$ it describes an ellipsoid.

Several robots. Several robots are planned jointly by stacking their curves into one $\boldsymbol{\gamma}$; let $\mathbf{E}_a$ select robot a's rows, so $\boldsymbol{\gamma}_a = \mathbf{E}_a\boldsymbol{\gamma}$. For a robot of radius r_a, a sphere of radius r and centre $\mathbf{m}_j$ is $\mathbf{V}_j = \mathbf{E}_a^\top \mathbf{E}_a$, $\mathbf{E}_a\mathbf{c}_j = \mathbf{m}_j$, $r_j = r + r_a$; two robots a, b are kept apart by $\mathbf{V}_j = \mathbf{E}_{ab}^\top \mathbf{E}_{ab}$ with $\mathbf{E}_{ab} = \mathbf{E}_a - \mathbf{E}_b$ and $r_j = r_a + r_b$, so that $\|\boldsymbol{\gamma}\|_{\mathbf{V}_j} = \|\boldsymbol{\gamma}_a - \boldsymbol{\gamma}_b\|$. The multi-robot experiments of §A4.4 use this separation.

Workspace box and speed limit. Box constraints use $x_i - \underline{x}_i \geq 0$ and $\overline{x}_i - x_i \geq 0$ (Appendix A4.5); a speed limit uses $(Tv_{\max})^2 - \|\boldsymbol{\gamma}'(t)\|^2 \geq 0$, where T is the trajectory duration, so the physical velocity is $\boldsymbol{\gamma}'/T$. Both are affine or concave quadratic, so their feasible sets are convex.

A2.1.1 Half-space and corridor constraints

Confining the curve to one side of a wall (a workspace bound, or a face of a corridor) is the constraint $\mathbf{a}_w^\top \boldsymbol{\gamma}(t) - \xi_w \geq 0$ for all t. The polynomial $\mathbf{a}_w^\top \boldsymbol{\gamma}(t) - \xi_w$ has degree d, so its Bernstein coefficient vector $\boldsymbol{\Gamma}^\top \mathbf{a}_w - \xi_w \mathbf{u}$ is raised to degree $2d$ by the elevation map $\mathbf{W} \in \mathbb{R}^{(2d+1)\times(d+1)}$, $\mathbf{W}_{ki} = \binom{d}{i}\binom{d}{k-i}\big/\binom{2d}{k}$:

$$\boldsymbol{\pi}_w(\boldsymbol{\Gamma}) \;:=\; \mathbf{W}\big(\boldsymbol{\Gamma}^\top \mathbf{a}_w - \xi_w \mathbf{u}\big) \;\in\; \mathscr{K}. \tag{6}$$

Elevation rewrites one polynomial in a higher basis and changes nothing else, so (6) holds exactly when the wall is respected at every t, and the block enters the same cone $\mathscr{K}$ as the collision blocks. It is affine in $\mathbf{\Gamma}$, so its linearization is exact. An obstacle block is a squared length and a wall block a length, so the implementation divides each obstacle block by r_j^2 and each wall block by the workspace scale; since $\mathscr{K}$ is a cone, this changes the numerics but not the feasible set.

A2.1.2 Reference motions from learned policies

The reference $\boldsymbol{\rho}$ of (P_∞) is a motion supplied by a learned planner or policy (one fixed proposal per solve), a demonstration or a classical planner. For positions $y_0,\dots,y_L \in \mathbb{R}^n$ at normalized times $0 = t_0 < \cdots < t_L = 1$, we use the piecewise-linear interpolant

$$\boldsymbol{\rho}(t) = \frac{t_{i+1}-t}{t_{i+1}-t_i}\, y_i + \frac{t-t_i}{t_{i+1}-t_i}\, y_{i+1}, \qquad t \in [t_i, t_{i+1}], \quad i = 0,\dots,L-1. \tag{7}$$

It need not have the derivatives required of $\boldsymbol{\gamma}$, and it enters only the steering constraint, so the cost is unchanged. Relative action commands are first composed from the current configuration into absolute waypoints.

A2.2 Steering a generative planner

Learned planners propose motion as waypoints. Diffusion and flow-matching planners (Janner et al., 2022; Carvalho et al., 2023; Shaoul et al., 2025; Giaretta et al., 2026) denoise an array $\mathbf{y} = (y_0,\dots,y_L)$ of positions at fixed times, one model per map, with start and goal written into the array at every step. Constraints enter either as gradient guidance, which does not enforce them, or as a projection of the waypoints, which enforces them at the $L+1$ sampled times only; Section 4.1 measures what happens between samples. We use (P) as that projection, so the object the sampler carries is a polynomial trajectory that satisfies every block at every t, and the generator is left untouched.

Given waypoints $\mathbf{y}$, the reference $\boldsymbol{\rho}$ is their piecewise-linear interpolant (7), and the steering constraint of (P_∞), $\|\boldsymbol{\gamma}(t) - \boldsymbol{\rho}(t)\| \le \bar{r}$ for every t, keeps the curve near the proposal. Nothing else changes: Algorithms 1–2 run as in §2.

A3 Addendum to Section 3: Theory and proofs

Appendix A3.1 sets up the chart, the matrices and the compactness facts used throughout. Appendix A3.2 proves Theorem 1.

A3.1 Preliminaries

A3.1.1 Boundary chart

Lemma 3. *If $\mathbf{A}$ has full column rank, then $\mathbf{\Gamma}_{\mathrm{aff}} = \mathbf{B}(\mathbf{A}^\top\mathbf{A})^{-1}\mathbf{A}^\top$ belongs to $\mathscr{A}_{\mathrm{bc}}$. If $\mathbf{N}$ is an orthonormal basis for $\ker\mathbf{A}^\top$, then $Z \longmapsto \mathbf{\Gamma}_{\mathrm{aff}} + Z\mathbf{N}^\top$ is an affine isometry from $\mathbb{R}^{n\times d_f}$ onto $\mathscr{A}_{\mathrm{bc}}$, with inverse $\mathbf{\Gamma} \longmapsto (\mathbf{\Gamma} - \mathbf{\Gamma}_{\mathrm{aff}})\mathbf{N}$.*

Proof of Lemma 3. Full column rank makes $\mathbf{A}^\top\mathbf{A}$ invertible, and multiplying the displayed anchor by $\mathbf{A}$ gives $\mathbf{B}$. Any difference of two solutions has rows in $\ker\mathbf{A}^\top$ and thus uniquely equals $Z\mathbf{N}^\top$. The inverse formula follows from $\mathbf{N}^\top\mathbf{N} = I$. Finally $\|Z\mathbf{N}^\top\|_F^2 = \mathrm{tr}(Z^\top Z) = \|Z\|_F^2$. The map and its inverse are continuous, so relative local minima and set membership transfer between the two coordinates. □

A3.1.2 Matrices, cone and bounds

Polynomial cost, boundary, and constraint matrices. The costs we use are all of the quadratic form

$$\mathrm{cost}(\mathbf{\Gamma}) = \mathrm{tr}(\mathbf{Q}\mathbf{\Gamma}^\top\mathbf{\Gamma}) - 2\,\mathrm{tr}(\mathbf{L}^\top\mathbf{\Gamma}) + c_0. \tag{8}$$

The main-text cost is the case $\mathbf{L} = 0$, $c_0 = 0$; the linear and constant terms cover tracking costs, and the gradient is $2(\mathbf{\Gamma Q} - \mathbf{L})\mathbf{N}$; the constraint blocks do not depend on the cost. For reference fitting, define $\mathbf{G}_d = \int_0^1 \mathbf{b}_d(t)\mathbf{b}_d(t)^\top dt$ and $\mathbf{R}_d = \int_0^1 \boldsymbol{\rho}(t)\mathbf{b}_d(t)^\top dt$. Then (8) uses $\mathbf{Q} = \frac{1}{2}\mathbf{G}_d$, $\mathbf{L} = \frac{1}{2}\mathbf{R}_d$, and $c_0 = \frac{1}{2}\int_0^1 \|\boldsymbol{\rho}(t)\|^2 dt$. The reference need only be square-integrable. For the integrated squared-derivative objective in (P_∞), let $\mathbf{D}_k$ be the matrix with $\mathbf{b}_d^{(k)} = \mathbf{D}_k\mathbf{b}_{d-k}$ and use $\mathbf{Q} = \mathbf{G}^{(k)} := \mathbf{D}_k\mathbf{G}_{d-k}\mathbf{D}_k^\top$, $\mathbf{L} = 0$, $c_0 = 0$. The endpoint evaluation matrix is

$$\mathbf{\Gamma A} = \mathbf{B}, \qquad \mathbf{A} = [\,\mathbf{b}_d(0)\;\mathbf{b}_d(1)\;\cdots\;\mathbf{b}_d^{(\ell)}(0)\;\mathbf{b}_d^{(\ell)}(1)\,] \in \mathbb{R}^{(d+1)\times 2(\ell+1)}. \tag{9}$$

The columns of $\mathbf{B}$ give the prescribed values in the same order. A scalar integral budget has coefficient map $\pi_E(\mathbf{\Gamma}) = E_{\max} - \mathrm{tr}(\mathbf{G}^{(k)}\mathbf{\Gamma}^\top\mathbf{\Gamma})$ and cone $\mathbb{R}_+$.

For the signed quadratic derivative blocks used in the examples, let $0 \le i_j \le k$, $\mathbf{V}_j \succeq 0$, $\bar{d}_j = d - i_j$, and $\mathbf{D}_0 = \mathbf{I}$. Then

$$\mathbf{M}_j(\mathbf{\Gamma}) = \sigma_j\big(\mathbf{\Gamma}_j^\top\mathbf{V}_j\mathbf{\Gamma}_j - r_j^2\mathbf{u}\mathbf{u}^\top\big), \qquad \mathbf{\Gamma}_j = \mathbf{\Gamma D}_{i_j} - \mathbf{C}_j, \quad \sigma_j \in \{-1,+1\}, \tag{10}$$

where $\mathbf{u}$ is the vector of ones. Let S_e denote the linear map that sends a symmetric $(e+1)\times(e+1)$ matrix $\mathbf{M}$ to the degree-$2e$ Bernstein coefficients of $\mathbf{b}_e(t)^\top\mathbf{M}\mathbf{b}_e(t)$. Then

$$g_j^{\mathbf{\Gamma}}(t) = \mathbf{b}_{\bar{d}_j}(t)^\top\mathbf{M}_j(\mathbf{\Gamma})\mathbf{b}_{\bar{d}_j}(t), \qquad \boldsymbol{\pi}_j(\mathbf{\Gamma}) = S_{\bar{d}_j}[\mathbf{M}_j(\mathbf{\Gamma})]. \tag{11}$$

Here $m_j = 2\bar{d}_j$. If the centre has higher degree, first elevate both coefficient representations to a common degree and use the corresponding linear derivative/elevation map in place of $\mathbf{D}_{i_j}$. Affine box blocks are degree d with affine coefficient maps.

Properties of the cone. All four properties of $\mathscr{K}_m$ for any degree $m \ge 0$ are immediate from (1d). It is an intersection of closed homogeneous half-spaces, hence a *closed convex cone*. It is *pointed*, since $\boldsymbol{\pi}, -\boldsymbol{\pi} \in \mathscr{K}_m$ forces the polynomial to vanish. It has *nonempty interior*, since $\mathbf{b}_m$ is a nonnegative partition of unity, so $|p_{\boldsymbol{\zeta}}(t)| \le \|\boldsymbol{\zeta}\|_\infty \le \|\boldsymbol{\zeta}\|$ and any $\boldsymbol{\pi}$ with $\min_{[0,1]} p_{\boldsymbol{\pi}} = \mu > 0$ carries the open μ-ball. And it is *semialgebraic*, by Tarski–Seidenberg applied to $\forall t\,(0 \le t \le 1 \to p_{\boldsymbol{\pi}}(t) \ge 0)$.

The *dual cone*, the set of vectors with nonnegative inner product with every element of $\mathscr{K}_m$, is

$$\mathscr{K}_m^* = \mathrm{cone}\{\mathbf{b}_m(t) : t \in [0,1]\}, \tag{12}$$

the nonnegative combinations of the evaluation vectors used in §2. This holds because $\mathscr{K}_m$ is by definition the dual of that conic hull, the hull is closed (each element needs at most $m+1$ generators by Carathéodory, and $\mathbf{u}^\top\mathbf{b}_m(t) \equiv 1$ bounds the total weight, so the generating data lie in a compact set), and the bipolar theorem (a closed convex cone is the dual of its own dual) applies.

Lipschitz bound for the restoration merit. For general C^2 coefficient maps on a compact convex region $\mathscr{R}$, set $\mathbf{e}_j(z) = \boldsymbol{\pi}_j(z) - P_{\mathscr{K}_j}(\boldsymbol{\pi}_j(z))$. Projection residuals are nonexpansive and $0 \in \mathscr{K}_j$, hence, with $B_{\pi,j}$ and $B_{J,j}$ bounds on $\|\boldsymbol{\pi}_j\|$ and $\|D\boldsymbol{\pi}_j\|$ over $\mathscr{R}$ and $L_{J,j}$ a Lipschitz constant of $D\boldsymbol{\pi}_j$ (computed below for unit-ball exclusions), $\|\mathbf{e}_j(z)\| \le B_{\pi,j}$ and $\|\mathbf{e}_j(z) - \mathbf{e}_j(y)\| \le B_{J,j}\|z - y\|$. The gradient $2D\boldsymbol{\pi}_j(z)^\top\mathbf{e}_j(z)$ therefore has Lipschitz constant at most $2B_{J,j}^2 + 2L_{J,j}B_{\pi,j}$, and summing over blocks bounds the full merit; the step scale of Appendix A3.1.4 uses the larger, also valid, constant with $4B_{J,j}^2$. A descent step must stay in the region; this gives descent of the violation merit, not exact feasibility. The remaining calculation specializes to unit-ball position exclusions.

$L_{J,j}$, the Lipschitz constant of the Jacobian $D\boldsymbol{\pi}_j$ of §2 as a function of the base point, is fixed and $\mathscr{R}$-independent. By the exact quadratic expansion of the coefficient map, the Jacobian of $\boldsymbol{\pi}_j$ at base point $\mathbf{\Gamma}$, applied to a direction $\mathbf{\Delta}$, is $J_j(\mathbf{\Gamma})[\mathbf{\Delta}] = S_d(\mathbf{\Delta}^\top\mathbf{\Gamma}_j + \mathbf{\Gamma}_j^\top\mathbf{\Delta})$. For two base points $\mathbf{\Gamma}^{(1)}, \mathbf{\Gamma}^{(2)} \in \mathscr{A}_{\mathrm{bc}}$, writing $\mathbf{F} := \mathbf{\Gamma}^{(1)} - \mathbf{\Gamma}^{(2)}$ (so $\mathbf{\Gamma}_j^{(1)} - \mathbf{\Gamma}_j^{(2)} = \mathbf{F}$ as well, the obstacle-centering shift being a constant offset),

$$\big(J_j(\mathbf{\Gamma}^{(1)}) - J_j(\mathbf{\Gamma}^{(2)})\big)[\mathbf{\Delta}] \;=\; S_d\big(\mathbf{\Delta}^\top\mathbf{F} + \mathbf{F}^\top\mathbf{\Delta}\big),$$

linear in $\mathbf{\Delta}$ and in $\mathbf{F}$. Since S_d is a fixed linear map, this gives $\|J_j(\mathbf{\Gamma}^{(1)}) - J_j(\mathbf{\Gamma}^{(2)})\|_{\mathrm{op}} \le 2\|S_d\|_{\mathrm{op}}\|\mathbf{F}\|$: J_j is Lipschitz on all of $\mathscr{A}_{\mathrm{bc}}$ with constant $2\|S_d\|_{\mathrm{op}}$, a fixed number depending only on d. Restricting to the free coordinate via Lemma 3's chart $\mathbf{Z} \mapsto \mathbf{\Gamma}_{\mathrm{aff}} + \mathbf{Z}\mathbf{N}^\top$ composes this bound with the fixed linear map $\mathbf{N}$, giving $L_{J,j} := 2\|S_d\|_{\mathrm{op}}\|\mathbf{N}\|_{\mathrm{op}}^2$: an explicit constant, fixed once d, ℓ (hence $\mathbf{N}$) are fixed, and independent of any compact set $\mathscr{R}$.

Quadratic structure and curvature. For matrices $\mathbf{X}, \mathbf{Y}$ of the same shape, $\langle \mathbf{X}, \mathbf{Y} \rangle := \mathrm{tr}(\mathbf{X}^\top \mathbf{Y})$ is the Frobenius inner product.

Lemma 4. *Fix the chart anchor $\mathbf{\Gamma}_{\mathrm{aff}} \in \mathscr{A}_{\mathrm{bc}}$ and write $\mathbf{\Gamma} = \mathbf{\Gamma}_{\mathrm{aff}} + \mathbf{Z}\mathbf{N}^\top$ (Lemma 3). Then, exactly, with no remainder,*

$$\mathrm{cost}(\mathbf{\Gamma}_{\mathrm{aff}} + \mathbf{Z}\mathbf{N}^\top) \;=\; \mathrm{cost}(\mathbf{\Gamma}_{\mathrm{aff}}) \;+\; 2\big\langle (\mathbf{\Gamma}_{\mathrm{aff}}\mathbf{Q} - \mathbf{L})\mathbf{N}, \mathbf{Z} \big\rangle \;+\; \mathrm{tr}\big(\mathbf{H}\mathbf{Z}^\top \mathbf{Z}\big), \qquad \mathbf{H} \;:=\; \mathbf{N}^\top \mathbf{Q}\mathbf{N} \;\in\; \mathbb{S}^{d_f}.$$

Proof of Lemma 4. Substitute $\mathbf{\Gamma} = \mathbf{\Gamma}_{\mathrm{aff}} + \mathbf{Z}\mathbf{N}^\top$ into (8). Bilinearity and cyclicity of the trace give the linear term $2\langle (\mathbf{\Gamma}_{\mathrm{aff}}\mathbf{Q} - \mathbf{L})\mathbf{N}, \mathbf{Z} \rangle$ and the quadratic term $\mathrm{tr}(\mathbf{H}\mathbf{Z}^\top \mathbf{Z})$. This proves the identity for both tracking and planning. □

Lemma 5. *For $\mathbf{Q} \succeq 0$ and a full-column-rank $\mathbf{N}$, $\mathbf{H} = \mathbf{N}^\top \mathbf{Q}\mathbf{N} \succ 0$ if and only if $\ker \mathbf{Q} \cap \mathrm{range}(\mathbf{N}) = \{0\}$. For the derivative objective in (P_∞) with the endpoint data (9), $\mathbf{H} \succ 0$ under Assumption B.*

Proof of Lemma 5. For a nonzero v, $(\mathbf{N}v)^\top \mathbf{Q}(\mathbf{N}v) > 0$ precisely when $\ker \mathbf{Q} \cap \mathrm{range}(\mathbf{N}) = \{0\}$, since $\mathbf{Q} \succeq 0$ and $\mathbf{N}$ has full column rank. This holds for tracking because $\mathbf{G}_d \succ 0$. For minimum-derivative planning, $\ker \mathbf{G}^{(k)}$ consists of polynomials of degree below k. A nonzero polynomial vanishing with its derivatives of orders $0, \dots, \ell$ at both endpoints has degree at least $2(\ell+1)$, so $k \le 2(\ell+1)$ suffices. □

Hence the constants

$$\alpha \;:=\; 2\lambda_{\min}(\mathbf{H}) \;>\; 0, \qquad \beta \;:=\; 2\lambda_{\max}(\mathbf{H}) \;<\; \infty \tag{13}$$

are well defined, and by Lemma 4 they bound the curvature on all of $\mathscr{A}_{\mathrm{bc}}$, with no remainder term. For tracking with orthonormal $\mathbf{N}$, $\lambda_{\min}(\mathbf{H}) \ge \frac{1}{2}\lambda_{\min}(\mathbf{G}_d) > 0$.

Compactness and existence on each component.

Lemma 6. *For any $\bar{\mathbf{\Gamma}} \in \mathscr{A}_{\mathrm{bc}}$, writing $\mathbf{\Gamma} = \mathbf{\Gamma}_{\mathrm{aff}} + \mathbf{Z}\mathbf{N}^\top$ in the chart of Lemma 3, $\mathscr{E}(\bar{\mathbf{\Gamma}}) := \{\mathbf{\Gamma} \in \mathscr{A}_{\mathrm{bc}} : \mathrm{cost}(\mathbf{\Gamma}) \le \mathrm{cost}(\bar{\mathbf{\Gamma}})\}$ is, under $\mathbf{H} \succ 0$ (Lemma 5), an ellipsoid in $\mathbf{Z}$-coordinates centred at the unconstrained minimizer $\mathbf{Z}^\star = -(\mathbf{\Gamma}_{\mathrm{aff}}\mathbf{Q} - \mathbf{L})\mathbf{N}\mathbf{H}^{-1}$ of cost on $\mathscr{A}_{\mathrm{bc}}$, hence a compact convex subset of $\mathscr{A}_{\mathrm{bc}}$.*

Proof of Lemma 6. Write $\mathbf{C} := (\mathbf{\Gamma}_{\mathrm{aff}}\mathbf{Q} - \mathbf{L})\mathbf{N}$ and $\mathbf{Z}^\star := -\mathbf{C}\mathbf{H}^{-1}$. Since $\mathbf{H}$ is symmetric, $\mathrm{tr}(\mathbf{H}\mathbf{Z}^\top \mathbf{Z}^\star) = \mathrm{tr}(\mathbf{H}\mathbf{Z}^{\star\top} \mathbf{Z}) = -\langle \mathbf{C}, \mathbf{Z} \rangle$, so completing the square in Lemma 4 gives

$$\mathrm{cost}(\mathbf{\Gamma}_{\mathrm{aff}} + \mathbf{Z}\mathbf{N}^\top) = \mathrm{cost}(\mathbf{\Gamma}_{\mathrm{aff}}) - \mathrm{tr}(\mathbf{H}\mathbf{Z}^{\star\top}\mathbf{Z}^\star) + \mathrm{tr}\big(\mathbf{H}(\mathbf{Z} - \mathbf{Z}^\star)^\top (\mathbf{Z} - \mathbf{Z}^\star)\big).$$

Hence $\mathscr{E}(\bar{\mathbf{\Gamma}})$ is the image under $\mathbf{Z} \mapsto \mathbf{\Gamma}_{\mathrm{aff}} + \mathbf{Z}\mathbf{N}^\top$ of $\{\mathbf{Z} : \mathrm{tr}(\mathbf{H}(\mathbf{Z} - \mathbf{Z}^\star)^\top (\mathbf{Z} - \mathbf{Z}^\star)) \le c\}$ with $c = \mathrm{cost}(\bar{\mathbf{\Gamma}}) - \mathrm{cost}(\mathbf{\Gamma}_{\mathrm{aff}}) + \mathrm{tr}(\mathbf{H}\mathbf{Z}^{\star\top}\mathbf{Z}^\star) \ge 0$, since $\bar{\mathbf{\Gamma}}$ lies in it. For $\mathbf{H} \succ 0$ this set is an ellipsoid centred at $\mathbf{Z}^\star$, compact and convex, and so is its affine image; $\bar{\mathbf{\Gamma}}$ enters only through c. □

Lemma 7. *If the coefficient maps are semialgebraic (in particular polynomial), the feasible set $\mathscr{A}_{\mathrm{bc}} \cap \bigcap_j \boldsymbol{\pi}_j^{-1}(\mathscr{K}_j)$ ($\mathscr{F}$ in the coordinates $\mathbf{\Gamma}$) has finitely many connected components $\mathscr{F}_1, \dots, \mathscr{F}_{\bar{c}}$, each clopen in $\mathscr{F}$ and closed in $\mathbb{R}^{n \times (d+1)}$; for any component $\mathscr{F}_c$ and compact $S \subset \mathscr{F}_c$, $d_0(S) := \mathrm{dist}(S, \mathscr{F} \setminus \mathscr{F}_c) > 0$ (or $+\infty$ if $\mathscr{F}_c = \mathscr{F}$).*

Proof of Lemma 7. $\mathscr{K}_j$ is a closed semialgebraic convex cone and each $\boldsymbol{\pi}_j$ is semialgebraic under this lemma's hypothesis, so $\mathscr{F} = \mathscr{A}_{\mathrm{bc}} \cap \bigcap_j \boldsymbol{\pi}_j^{-1}(\mathscr{K}_j)$ is closed and semialgebraic. A closed semialgebraic subset of a finite-dimensional Euclidean space has finitely many connected components, each itself closed and semialgebraic (Bochnak et al., 1998). With finitely many components $\mathscr{F}_1, \dots, \mathscr{F}_{\bar{c}}$, each is also open in $\mathscr{F}$ (its complement in $\mathscr{F}$ is a finite union of the other closed components, hence closed), so each is clopen in $\mathscr{F}$ and closed in $\mathbb{R}^{n \times (d+1)}$.

Fix $\mathscr{F}_c$ and let $S \subset \mathscr{F}_c$ be compact. Then $\mathscr{F} \setminus \mathscr{F}_c$ (a finite union of closed sets) is closed and disjoint from S, and the distance to a closed set attains a positive minimum on a compact set disjoint from it, so $d_0(S) := \mathrm{dist}(S, \mathscr{F} \setminus \mathscr{F}_c) > 0$; if $\mathscr{F}_c = \mathscr{F}$, set $d_0(S) = +\infty$. □

Lemma 8. *Under $\mathbf{H} \succ 0$, for every connected component $\mathscr{F}_c$ of $\mathscr{F}$,* $\arg\min_{\mathscr{F}_c}$ cost *is attained (possibly non-uniquely); fix any such minimizer $\mathbf{\Gamma}_c^\star$.*

Proof of Lemma 8. Fix any $\bar{\mathbf{\Gamma}} \in \mathscr{F}_c$ and let $\mathscr{E}(\bar{\mathbf{\Gamma}})$ be its ellipsoid (Lemma 6), compact since $\mathbf{H} \succ 0$. Continuity of the coefficient maps makes $\mathscr{F}$ closed, and every connected component of a closed set is closed. Thus $\mathscr{F}_c \cap \mathscr{E}(\bar{\mathbf{\Gamma}})$ is compact (a closed subset of the compact $\mathscr{E}(\bar{\mathbf{\Gamma}})$) and nonempty ($\bar{\mathbf{\Gamma}}$ lies in both); cost is continuous, so by Weierstrass it attains a minimum over $\mathscr{F}_c \cap \mathscr{E}(\bar{\mathbf{\Gamma}})$ at some $\mathbf{\Gamma}_c^\star$. Every $\mathbf{\Gamma} \in \mathscr{F}_c \setminus \mathscr{E}(\bar{\mathbf{\Gamma}})$ has $\mathrm{cost}(\mathbf{\Gamma}) > \mathrm{cost}(\bar{\mathbf{\Gamma}}) \geq \mathrm{cost}(\mathbf{\Gamma}_c^\star)$, so $\mathbf{\Gamma}_c^\star$ minimizes cost over all of $\mathscr{F}_c$. □

Cumulative constraints and derivative-cost curvature. The finite-dimensional reduction of (P_∞) also admits scalar budget blocks. Write $z = \mathrm{vec}(\mathbf{\Gamma}_{\mathrm{free}})$ and let $\pi_j(z)$ denote a budget block (1e). Its cone is $\mathbb{R}_+$, so its contribution to the restoration merit and gradient is

$$V_j(z) = \min\{\pi_j(z), 0\}^2, \qquad \nabla V_j(z) = 2\min\{\pi_j(z), 0\}\nabla \pi_j(z).$$

These formulas do not require the integrand to be polynomial in time. For a general integrand, differentiation under the integral is justified, for example, if it is measurable in time and C^2 in the trajectory arguments almost everywhere, with its value and first two composed z-derivatives dominated by integrable functions on each compact set of free coordinates.

For a single segment of degree $d \geq k$ with only start and goal fixed, the objective of (P_∞) has $\mathbf{H} \succ 0$ if and only if $k \leq 2$ (the case $\ell = 0$ of Assumption B): a free perturbation has zero derivative cost exactly when its degree is below k, and with $p(0) = p(1) = 0$ the first such nonzero polynomial is $t(1-t)$, of degree 2. When the cost has free zero-curvature directions, compactness must be checked separately. Additional endpoint derivatives can remove these directions as in Lemma 9; adding them changes the boundary data of the displayed problem.

A3.1.3 The structural assumption

Assumption D (finite-dimensional representation)**.** *$d \geq k$ and $2(\ell+1) \leq d$, and every pointwise coefficient map and scalar cumulative map is C^2 on an open neighborhood of the regions used in the analysis.*

The quadratic obstacle, separation and budget blocks satisfy the smoothness part. The degree condition makes $\mathbf{A}$ of full column rank, since its columns are independent Hermite interpolation conditions (Stoer & Bulirsch, 2002); $\mathbf{N}$ is an orthonormal basis of $\ker \mathbf{A}^\top$, and $\mathbf{Q} \succeq 0$ always. Under Assumption D, Assumption B is equivalent to $\mathbf{H} \succ 0$, with $\mathbf{H}$ as in Lemma 4 (Lemma 9).

Write P_r for scalar polynomials of degree at most r, with $P_{-1} = \{0\}$, and $\mathrm{coeff}(p)$ for their Bernstein-d coefficient vector.

Lemma 9 (kernels)**.** *For $0 \leq k \leq d$ and the endpoint matrix in (9),*

$$\ker \mathbf{G}^{(k)} = \mathrm{coeff}(P_{k-1}), \qquad \ker \mathbf{A}^\top = \mathrm{coeff}\big([t(1-t)]^{\ell+1} P_{d-2(\ell+1)}\big).$$

Under Assumption D, the latter has dimension $d_f = d + 1 - 2(\ell+1)$.

Proof of Lemma 9. For a coefficient vector c representing p, $c^\top \mathbf{G}^{(k)} c = \int_0^1 |p^{(k)}(t)|^2 dt$. This vanishes exactly when the polynomial $p^{(k)}$ vanishes identically, i.e. $p \in P_{k-1}$. Also $\mathbf{A}^\top c = 0$ precisely when the first $\ell+1$ endpoint evaluations vanish at both ends. Polynomial factorization makes this equivalent to divisibility by $t^{\ell+1}(1-t)^{\ell+1}$. Multiplication by this nonzero polynomial is injective, giving the stated dimension. □

A3.1.4 Step size and the cone solve

Step size. A valid conservative step scale, provided the step stays in a compact convex region $\mathscr{R}$, is

$$L_{\mathrm{proj}} = \sum_j (4B_{J,j}^2 + 2L_{J,j} B_{\pi,j}), \qquad \eta_{\mathrm{proj}} = 1/L_{\mathrm{proj}},$$

where $B_{J,j} = \max_{\mathscr{R}} \|D\boldsymbol{\pi}_j\|_{\mathrm{op}}$, $B_{\pi,j} = \max_{\mathscr{R}} \|\boldsymbol{\pi}_j\|$, and $L_{J,j}$ is the Lipschitz constant of the Jacobian $D\boldsymbol{\pi}_j$ of §2.

Algorithm 3 Approximate projection onto $\mathscr{K}_m$ by fully corrective nonnegative matching pursuit

Require: $\boldsymbol{\pi} \in \mathbb{R}^{m+1}$, degree $m \geq 0$, threshold $\varepsilon \geq 0$, iteration budget T_{cone}

if $\boldsymbol{\pi} = 0$ **then return** 0
$\boldsymbol{y} \leftarrow -\boldsymbol{\pi}/\|\boldsymbol{\pi}\|$, $S \leftarrow \emptyset$, $\boldsymbol{\omega} \leftarrow 0$, $\mathbf{v} \leftarrow \boldsymbol{y}$
for $\nu = 1, \ldots, T_{\text{cone}}$ **do**
  $t_\nu \leftarrow \arg\max_{t \in [0,1]} \langle \mathbf{v}, \mathbf{b}_m(t) \rangle / \|\mathbf{b}_m(t)\|$
  if $\langle \mathbf{v}, \mathbf{b}_m(t_\nu) \rangle / \|\mathbf{b}_m(t_\nu)\| \leq \varepsilon$ **then break**
  $S \leftarrow S \cup \{t_\nu\}$
  $w \leftarrow \arg\min_{w_t \geq 0, t \in S} \| \sum_{t \in S} w_t \mathbf{b}_m(t) - \boldsymbol{y} \|^2$
  $\boldsymbol{\omega} \leftarrow \sum_{t \in S} w_t \mathbf{b}_m(t)$, $\mathbf{v} \leftarrow \boldsymbol{y} - \boldsymbol{\omega}$
end for
return $\boldsymbol{\pi} + \|\boldsymbol{\pi}\| \boldsymbol{\omega}$

On a compact convex region containing both endpoints of an inner step, the Lipschitz bound of Appendix A3.1.2 gives

$$V(z - \eta \nabla V(z)) \leq V(z) - \frac{\eta}{2} \|\nabla V(z)\|^2,$$

for $0 < \eta \leq 1/L_{\text{proj}}$. The step must be kept in the region, which compactness does not do by itself, and constants taken over a ball around the unknown minimizer are not computable. Algorithm 1 keeps every nonzero residual, so omitted blocks have zero gradient and every update is a step on the full merit $V(z) = \sum_j \operatorname{dist}^2(\boldsymbol{\pi}_j(z), \mathscr{K}_j)$, the merit (2) in the coordinates z. Under the region and exact-projection conditions above, telescoping T executed steps gives

$$\min_{0 \leq \nu < T} \|\nabla V(z_\nu)\|^2 \leq \frac{2V(z_0)}{\eta T}.$$

The bound covers scalar cumulative blocks and does not need $\mathbf{H} \succ 0$. It gives a small merit gradient, not feasibility: for the scalar block $\pi(z) = z^2 - 1 \in \mathbb{R}_+$, $V(z) = (1 - z^2)^2$ near zero and $V'(0) = 0$ although $V(0) = 1$.

The cone solve and its accuracy. Algorithm 3 approximates the projection onto $\mathscr{K}_m$ for any degree $m \geq 0$, with $m = m_j$ for block j. By Moreau's identity it projects $-\boldsymbol{\pi}$ onto the dual cone $\mathscr{K}_m^*$ with the atoms $\mathbf{b}_m(t)$, normalized by $\|\boldsymbol{\pi}\|$, and adds the result to $\boldsymbol{\pi}$; T_{cone} is a numerical budget.

The normalized correlation gives the best decrease along a single nonnegative ray. It is not a polynomial in t: if $\mu(t) = \langle \mathbf{v}, \mathbf{b}_m(t) \rangle$ and $\|\mathbf{b}_m(t)\|^2 > 0$, its interior stationary points satisfy $2\mu' \|\mathbf{b}_m\|^2 - \mu(\|\mathbf{b}_m\|^2)' = 0$. Evaluate these roots and the endpoints; an identically zero derivative means the correlation is constant.

A direct rate argument specializes the conic pursuit framework of Locatello et al. (2017). Let $\boldsymbol{\omega}_\nu$ be an exact fully corrective iterate, $\boldsymbol{\omega}_* = P_{\mathscr{K}_m^*}(\boldsymbol{y})$, and $\theta_\nu = \frac{1}{2}\|\boldsymbol{y} - \boldsymbol{\omega}_\nu\|^2 - \frac{1}{2}\|\boldsymbol{y} - \boldsymbol{\omega}_*\|^2$, which is nonnegative because $\boldsymbol{\omega}_*$ is the nearest point of the cone. For the normalized atoms $a_t = \mathbf{b}_m(t)/\|\mathbf{b}_m(t)\|$, nonnegativity of their entries gives $\mathbf{u}^\top a_t \geq 1$. Since $\|\boldsymbol{\omega}_*\| \leq \|\boldsymbol{y}\| = 1$, it has an atomic representation of total weight at most $\sqrt{m+1}$. Exact nonnegative least squares gives $\langle \boldsymbol{y} - \boldsymbol{\omega}_\nu, \boldsymbol{\omega}_\nu \rangle = 0$. Convexity then bounds θ_ν by $\sqrt{m+1}$ times the largest positive atom correlation. Adding that unit atom with its optimal nonnegative weight decreases the objective by half the squared correlation; full correction does at least as well. Hence

$$\theta_{\nu+1} \leq \theta_\nu - \frac{\theta_\nu^2}{2(m+1)}, \qquad \theta_\nu \leq \frac{2(m+1)}{\nu + 4(m+1)},$$

using $\theta_0 \leq 1/2$, for exact maximization and weight solves without a stopping threshold; it gives neither a linear rate nor finite termination.

If the correlation test terminates at threshold ε, then $\theta_\nu \leq \sqrt{m+1}\,\varepsilon$. Strong convexity of the squared-distance objective gives projection error at most $\|\boldsymbol{\pi}\| \sqrt{2\sqrt{m+1}\,\varepsilon}$ for the returned vector. Its polynomial values are at least $-\|\boldsymbol{\pi}\|\varepsilon$, since that vector is $-\|\boldsymbol{\pi}\|\mathbf{v}$. For $\varepsilon > 0$ these bound the error rather than give exact membership, and stopping on the budget gives neither bound. Proposition 10 and Theorem 1 assume an exact solve of the convex problem (5), which is not this cone projection; Algorithm 3 plays no role there.

A3.2 Proof of Theorem 1

Throughout this subsection, the blocks are polynomial in $\mathbf{\Gamma}$, as the sphere, pair and affine blocks are, and z stands for $\mathbf{\Gamma}_{\text{free}}$. It uses the boundary chart (Lemma 3) and the compact cost sublevel sets (Lemma 6) of Appendix A3.1, and the push maps that move a curve off the obstacles (Lemma 12).

A3.2.1 Fixed degree

Let $E^\star$ denote the optimum of (P), whose collision blocks ($j \le p$) are polynomial and convex in the position and whose steering block and budgets have convex feasible sets. Let z_h be a cheapest point of the feasible grid $\mathscr{F} \cap h\mathbb{Z}^{nd_f} \cap \mathscr{B}$ (in the rational chart of free control points described after the grid part below), for a box $\mathscr{B}$ with a global minimizer of (P) in its interior; the start of Theorem 1 is this point for the class $\mathscr{S}_N$, where d_f counts the free control points per coordinate left by the boundary and junction equations. Theorem 1 rests on the following result at fixed degree, which uses a constraint qualification on the control points.

Assumption E (constraint qualification)**.** *For some $U > E^\star$, every $z \in \mathscr{F}$ with $f(z) \le U$ has a* $\mathbf{v}$ *with $\boldsymbol{\pi}(z) + D\boldsymbol{\pi}(z)\,\mathbf{v} \in \operatorname{int}\mathscr{K}$.*

Assumption E is Robinson's condition (Robinson, 1976): some change of the control points makes every block strictly feasible to first order. It is needed only on this sublevel set, and on fine splines it follows from Assumption C (Lemma 14).

Proposition 10 (GlobMP at fixed degree)**.** *Under Assumptions D, B and E (so that $\mathbf{H} \succ 0$), with $0 < \eta < 2/\beta$, the refinement (5) from a feasible start of cost at most U keeps every iterate feasible, decreases f by at least $(1/\eta - \beta/2)\|z^{\nu+1} - z^\nu\|^2$ per step, and every limit point is a KKT point of (P). Started from z_h with h small enough, it has $f(z^\nu) \downarrow E^\star$, and all its limit points are global minimizers of (P).*

The proof has a descent part and a grid part. With uniform feasible random starts, a run with globally minimizing limit points is obtained with probability one after finitely many starts (Corollary 11).

Proof of Proposition 10, descent part. *The inner set.* For a collision block ($j \le p$), $g_j(\cdot,t)$ is convex in the position, so for curves $\boldsymbol{\gamma}_z$ and $\boldsymbol{\gamma}_w$ with control points z and w, $g_j(\boldsymbol{\gamma}_w(t),t) \ge g_j(\boldsymbol{\gamma}_z(t),t) + \nabla_{\mathbf{x}} g_j(\boldsymbol{\gamma}_z(t),t)^\top(\boldsymbol{\gamma}_w(t) - \boldsymbol{\gamma}_z(t))$ for every t. The right side is the piecewise polynomial with coefficients $\boldsymbol{\pi}_j(z) + D\boldsymbol{\pi}_j(z)(w - z)$, since $\boldsymbol{\pi}_j$ is quadratic and its remainder is the nonnegative polynomial $\|\boldsymbol{\gamma}_w - \boldsymbol{\gamma}_z\|^2$ (for a robot pair, the squared norm of the difference between the two robots' changes). If these coefficients lie in $\mathscr{K}_j$, so do those of the left side. The steering block and the budgets are imposed exactly through $\mathscr{D}$, a closed convex set. Hence $\mathscr{C}(z) \subset \mathscr{F}$; it is closed and convex, and contains z when z is feasible.

Descent. Since $z^\nu \in \mathscr{C}(z^\nu)$, the projection inequality gives $\langle \nabla f(z^\nu), z^{\nu+1} - z^\nu \rangle \le -\|z^{\nu+1} - z^\nu\|^2/\eta$, and ∇f is β-Lipschitz, so $f(z^{\nu+1}) \le f(z^\nu) - c_\eta \|z^{\nu+1} - z^\nu\|^2$, with $c_\eta = 1/\eta - \beta/2 > 0$ as $\eta < 2/\beta$. Summing, $\sum_{\nu<T} \|z^{\nu+1} - z^\nu\|^2 \le (f(z^0) - f(z^T))/c_\eta \le (f(z^0) - E^\star)/c_\eta$. The iterates stay in the compact sublevel set of the strongly convex f, and the steps are square summable.

Limit points. Since the steps shrink to zero, a limit point is left in place by its own projected step, and such a point satisfies the first-order optimality conditions. Let $z^{\nu_i} \to \bar{z}$; then also $z^{\nu_i+1} \to \bar{z}$, and $\bar{z} \in \mathscr{F}$. By Assumption E some $\mathbf{v}$ has $\boldsymbol{\pi}(\bar{z}) + D\boldsymbol{\pi}(\bar{z})\mathbf{v} \in \operatorname{int}\mathscr{K}$, so for small $s > 0$ the point $w_0 = \bar{z} + s\mathbf{v}$ satisfies every constraint of $\mathscr{C}(\bar{z})$ strictly: the linearized blocks equal $(1-s)\boldsymbol{\pi}(\bar{z}) + s(\boldsymbol{\pi}(\bar{z}) + D\boldsymbol{\pi}(\bar{z})\mathbf{v})$, and the steering block and the budgets, imposed exactly, have the same first-order expansion up to $O(s^2)$; the set $\mathscr{D}$ does not depend on z. For $w \in \mathscr{C}(\bar{z})$ and $\theta \in (0,1]$, the point $(1-\theta)w + \theta w_0$ is strictly feasible for $\mathscr{C}(\bar{z})$, hence, by continuity of the linearization in z, lies in $\mathscr{C}(z^{\nu_i})$ for large i. The projection inequality at that point, in the limit $i \to \infty$ and then $\theta \to 0$, gives $\langle \nabla f(\bar{z}), w - \bar{z} \rangle \ge 0$ for every $w \in \mathscr{C}(\bar{z})$. So $\bar{z}$ minimizes f over $\mathscr{C}(\bar{z})$, whose constraints have the strictly feasible point w_0. The steering block and the budgets enter $\mathscr{C}(\bar{z})$ through $\mathscr{D}$, whose normal cone at $\bar{z}$ is $D\boldsymbol{\pi}_{\mathscr{D}}(\bar{z})^\top \mathscr{N}_{\mathscr{K}}(\boldsymbol{\pi}_{\mathscr{D}}(\bar{z}))$ for their blocks $\boldsymbol{\pi}_{\mathscr{D}}$, by Assumption E and Rockafellar & Wets (1998, Thm. 6.14). So the optimality conditions of this convex program involve only the values $\boldsymbol{\pi}(\bar{z})$ and the derivative $D\boldsymbol{\pi}(\bar{z})$ of the blocks of (P), and $\bar{z}$ is a KKT point of (P).

Proof of Proposition 10, grid part. We show (i) $f(z_h) \to E^\star$, with every limit point of z_h a global minimizer, and (ii) for small h the refinement from z_h has $f(z^\nu) \downarrow E^\star$ and only globally minimizing limit points; the descent part is proved above. Let $z^\star$ be a global minimizer in the interior of $\mathscr{B}$ and $\mathbf{v}$ the vector that Assumption E gives at $z^\star$. For $s \in (0,1)$, $\boldsymbol{\pi}(z^\star + s\mathbf{v}) = (1-s)\boldsymbol{\pi}(z^\star) + s\big(\boldsymbol{\pi}(z^\star) + D\boldsymbol{\pi}(z^\star)\mathbf{v}\big) + o(s)$. The first term lies in the cone $\mathscr{K}$; the second lies in $\mathscr{K}$ with a ball of radius sr_0 about it, for some $r_0 > 0$; so for small s the sum lies in $\operatorname{int}\mathscr{K}$. By continuity of $\boldsymbol{\pi}$ there is then $\delta > 0$ with $\overline{B}(z^\star + s\mathbf{v}, \delta) \subset \mathscr{F} \cap \mathscr{B}$. Given $\varepsilon > 0$, taking s and then δ small makes the cost below $E^\star + \varepsilon$ on that ball, and any sufficiently small h puts a grid point in it (the grid lies in a chart that is a linear bijection), so $E^\star \le f(z_h) < E^\star + \varepsilon$. This proves the first claim of (i); a limit point of z_h lies in the closed set $\mathscr{F} \cap \mathscr{B}$ and, by continuity, has cost $E^\star$.

For (ii) we use one fact about problems built from polynomials. Let $\varphi(z)$ denote $f(z)$ for feasible z and $+\infty$ otherwise. The set $\mathscr{F}$ is closed and *semialgebraic*, that is, described by finitely many polynomial equations and inequalities: $\boldsymbol{\pi}$ is polynomial, and the condition "for every $t \in [0,1]$" in each $\mathscr{K}_j$ can be rewritten without the quantifier by the Tarski–Seidenberg theorem. So φ is lower semicontinuous and semialgebraic, and continuous on its closed domain $\mathscr{F}$. For such a φ the critical points, roughly the points where the negative cost gradient points out of $\mathscr{F}$ (Clarke critical points), take only finitely many cost values (Bolte et al., 2007, Cor. 9(ii)). Let $\mathscr{N}_{\mathscr{K}}(\mathbf{y})$ denote the directions pointing out of the convex set $\mathscr{K}$ at $\mathbf{y}$ (its normal cone); a KKT point of (P) is a feasible z with $-\nabla f(z) \in D\boldsymbol{\pi}(z)^\top \mathscr{N}_{\mathscr{K}}(\boldsymbol{\pi}(z))$. It is such a critical point, since $D\boldsymbol{\pi}(z)^\top \mathscr{N}_{\mathscr{K}}(\boldsymbol{\pi}(z))$ lies in the normal cone of $\mathscr{F}$ at z (Rockafellar & Wets, 1998, Thm. 6.14). Let $\varepsilon_0 > 0$ be the smaller of $U - E^\star$ and the gap between $E^\star$ and the next critical value, and by (i) take h_0 with $f(z_h) < E^\star + \varepsilon_0$ for $h \le h_0$. By the descent part, the costs of GlobMP from z_h decrease to some $\bar{E} \le f(z_h)$, every limit point has cost $\bar{E}$ and is a KKT point, and $\bar{E} \ge E^\star$ because it is feasible. So $\bar{E}$ is a critical value in $[E^\star, E^\star + \varepsilon_0)$, hence $\bar{E} = E^\star$, and every limit point is a global minimizer. The iterates are bounded, so their distance to the compact set of global minimizers tends to zero.

The chart of Lemma 3 has irrational entries in general. The control points that the boundary conditions leave free form a rational affine chart of $\mathscr{A}_{\mathrm{bc}}$: the conditions fix the others in terms of these and the data. For a single segment, a move of the free coordinates moves $\boldsymbol{\Gamma}$ by the same amount in the Frobenius norm and the free coordinates have norm at most $\|\boldsymbol{\Gamma}\|_F$, which is all Proposition 15 uses of Lemma 3; for a spline class such as $\mathscr{S}_N$ the chart is still a linear bijection, which is all the grid part of Proposition 10 needs. In both cases the membership test is exact in rational arithmetic for rational data and breakpoints.

Corollary 11 (random starts)**.** *Under the hypotheses of Proposition 10, draw starts independently and uniformly from $\mathscr{F} \cap \mathscr{B}$, run the exact refinement (5) from each, and keep the cheapest limiting cost. The probability that this cost exceeds $E^\star$ after n_s starts is at most $(1-\vartheta)^{n_s}$ for some instance-dependent $\vartheta > 0$. Thus a run with globally minimizing limit points occurs after finitely many starts with probability one. Each run is understood in its infinite-iteration limit.*

Proof of Corollary 11. Step (i) of the grid part of the proof of Proposition 10 gives a ball in $\mathscr{F} \cap \mathscr{B}$ on which $f < E^\star + \varepsilon_0$; it has positive volume, and so does $\mathscr{F} \cap \mathscr{B}$, which contains it. Let ϑ be the ratio of the two volumes; membership in $\mathscr{F}$ is decided exactly, so uniform starts are drawn by rejection from $\mathscr{B}$. A start in the ball has cost below $E^\star + \varepsilon_0$, and step (ii) shows that GlobMP from it reaches only global minimizers; the starts are independent, so none of the first n_s lands in the ball with probability $(1-\vartheta)^{n_s}$.

The grid has $O((\operatorname{diam}\mathscr{B}/h)^{nd_f})$ points, so the proposition establishes global convergence, not an efficient search.

A3.2.2 Theorem 1: the continuous-time problem

Lemma 12 (pushing into the interior)**.** *Let $F := \{\mathbf{x} \in \mathbb{R}^n : g_j(\mathbf{x}) \ge 0 \ \forall j\}$, let every $\mathbf{x} \in F$ admit a $\mathbf{v}$ with $\nabla g_j(\mathbf{x}) \cdot \mathbf{v} > 0$ for all j with $g_j(\mathbf{x}) = 0$ (Assumption C at a fixed time), and let the start and goal positions be strictly feasible. For every compact $K \subset \mathbb{R}^n$ there are $\mu_0, G > 0$ and, for $0 < \mu \le \mu_0$, smooth maps Φ_μ of $\mathbb{R}^n$, equal to the identity near the start and goal positions, such that $g_j(\Phi_\mu(x)) \ge \mu$ for all j and $x \in F \cap K$, and $\|\Phi_\mu - \mathrm{id}\|_{C^i(K)} \le a_i \mu$ for every i, with a_i independent of μ. Every y with $g_j(y) \ge \mu$ for all j and y within distance 1 of K has $B(y, \mu/G) \subset F$.*

Proof. We glue the local directions of the Mangasarian–Fromovitz condition into one smooth field W and follow it for time μ: small clearances grow by about 2μ and points move $O(\mu)$. Let K' be the closed 1-neighbourhood of K. For $x \in F \cap K'$ let $J(x) = \{j : g_j(x) = 0\}$ and choose $\mathbf{v}_x$ with $\nabla g_j(x) \cdot \mathbf{v}_x > 0$ for $j \in J(x)$, scaled so that, on an open neighbourhood O_x of x, $\nabla g_j \cdot \mathbf{v}_x \geq 2$ for $j \in J(x)$ and $g_j \geq \kappa_x > 0$ for $j \notin J(x)$. Finitely many O_{x_i} cover $F \cap K'$; let $\kappa > 0$ be below every κ_{x_i} and below $\frac{1}{2}\min_j g_j$ on small balls about the start and goal positions. With a partition of unity $\sum_i \theta_i = 1$ on $F \cap K'$ subordinate to the cover and a smooth cutoff χ that vanishes on those balls and equals 1 where some $g_j < \kappa$, put $W = \chi \sum_i \theta_i \mathbf{v}_{x_i}$. If $y \in F \cap K'$ and $g_j(y) < \kappa$, then $j \in J(x_i)$ whenever $\theta_i(y) > 0$, so $\nabla g_j(y) \cdot W(y) \geq 2$. Let Φ_μ be the time-μ flow of W, $M = \sup|W|$, G a bound on $|\nabla g_j|$ on the closed 2-neighbourhood of K. Along a trajectory from $x \in F \cap K$, g_j increases at rate at least 2 while it is below κ and decreases at rate at most GM otherwise, so for $\mu \leq \mu_0 = \min\{\kappa/4, \kappa/(2GM), 1/M, G\}$ the trajectory stays in $F \cap K'$ and g_j stays at least $\min\{2\mu, \kappa/2\} \geq \mu$ at time μ. For the C^i bound, write $\Phi_\mu - \mathrm{id} = \int_0^\mu W \circ \Phi_s \, ds$; the derivatives of Φ_s up to order i are bounded on K uniformly in $s \leq \mu_0$, since they solve the flow equation differentiated in x, so the integrand is bounded in $C^i(K)$. Finally g_j is G-Lipschitz on the 2-neighbourhood of K, so $g_j(y) \geq \mu$ gives $g_j > 0$ on $B(y, \mu/G)$, as $\mu/G \leq 1$. □

Lemma 13 (rate in the number of pieces)**.** *Write $E_N^\star$ for the optimum over $\mathscr{S}_N$. Under the hypotheses of Theorem 1, there are C and N_0, depending on the instance, with $E_\infty^\star \leq E_N^\star \leq E_\infty^\star + CN^{-(k-1/2)}$ for all $N \geq N_0$.*

Proof of Lemma 13. We push an optimal curve inward by order μ and interpolate it with N pieces; interpolation never raises the cost and moves the curve by $O(N^{-(k-1/2)})$, which a push of that order absorbs.

Let $E(\boldsymbol{\gamma}) = \int_0^1 \|\boldsymbol{\gamma}^{(k)}\|^2 dt$ denote the cost of (P_∞). The lower bound holds because the class is contained in $\mathscr{H}^k$. Let $\boldsymbol{\gamma}^\star$ minimize (P_∞) (it exists by the existence step of the proof of Proposition 15) and let Φ_μ be the maps of Lemma 12, applied to every block, including the steering block. For moving centres and a moving reference, the same construction on the compact space-time set $\{(x,t) \in K' \times [0,1] : g_j(x,t) \geq 0 \ \forall j\}$, with K' the closed 1-neighbourhood of a compact set containing $\boldsymbol{\gamma}^\star([0,1])$, with the cutoff χ vanishing near $(\boldsymbol{\gamma}_0, 0)$ and $(\boldsymbol{\gamma}_1, 1)$, with a partition of unity in (x,t), gives a field $W(x,t)$ that is smooth in (x,t) (the partition of unity is smooth even where the centres are only continuous) and maps $\Phi_\mu(\cdot,t)$ that push at each instant; the extra chain-rule terms, derivatives of Φ_μ in t, are $O(\mu)$. The maps are the identity near the endpoints, give every clearance at least μ, and satisfy $\|\Phi_\mu - \mathrm{id}\|_{C^k} \leq a_k \mu$ on a compact set containing the curve. Put $\boldsymbol{\gamma}_\mu(t) = \Phi_\mu(\boldsymbol{\gamma}^\star(t), t)$. By the chain rule, $\boldsymbol{\gamma}_\mu^{(k)} - \boldsymbol{\gamma}^{\star(k)}$ is a sum of terms each containing a derivative of $\Phi_\mu - \mathrm{id}$ and bounded derivatives of $\boldsymbol{\gamma}^\star$ of order below k, times at most one factor $\boldsymbol{\gamma}^{\star(k)} \in L^2$; so $\|\boldsymbol{\gamma}_\mu^{(k)} - \boldsymbol{\gamma}^{\star(k)}\|_{L^2} \leq c_1 \mu$ and $E(\boldsymbol{\gamma}_\mu) \leq E_\infty^\star + c_2 \mu$. Every point of $\boldsymbol{\gamma}_\mu$ is at distance at least μ/G from the boundary of the feasible region, with G a Lipschitz bound of the constraint functions. Let $\mathbf{q}$ be the piecewise Hermite interpolant of $\boldsymbol{\gamma}_\mu$ of degree $2k-1$ at the $N+1$ nodes; it matches the states of order below k at every node, hence the boundary data, and lies in $\mathscr{S}_N$. On each piece, integrating by parts k times shows that $\boldsymbol{\delta} = (\boldsymbol{\gamma}_\mu - \mathbf{q})^{(k)}$ is orthogonal in L^2 to every polynomial of degree below k, so $\mathbf{q}^{(k)}$ is the L^2 projection $P_N \boldsymbol{\gamma}_\mu^{(k)}$ onto piecewise polynomials of degree below k, and $\|\boldsymbol{\delta}\|_{L^2} \leq \|\boldsymbol{\gamma}_\mu^{(k)}\|_{L^2}$. By Pythagoras, $E(\mathbf{q}) = \|P_N \boldsymbol{\gamma}_\mu^{(k)}\|^2 = E(\boldsymbol{\gamma}_\mu) - \|\boldsymbol{\delta}\|^2 \leq E(\boldsymbol{\gamma}_\mu)$: interpolation never increases the cost. The error $\boldsymbol{\gamma}_\mu - \mathbf{q}$ has zero states at the nodes, so Taylor's formula with integral remainder on each piece of length $1/N$ gives $\|\boldsymbol{\gamma}_\mu - \mathbf{q}\|_\infty \leq N^{-(k-1/2)} \|\boldsymbol{\delta}\|_{L^2} \leq c_3 N^{-(k-1/2)}$, where $c_3 = \sqrt{E_\infty^\star + c_2 \mu_0}$ bounds $\|\boldsymbol{\gamma}_\mu^{(k)}\|_{L^2}$ for every $\mu \leq \mu_0$. Take $\mu = 2Gc_3 N^{-(k-1/2)}$, below the admissible μ_0 for large N: then $\mathbf{q}$ stays within $\mu/(2G)$ of $\boldsymbol{\gamma}_\mu$, so it is feasible at every instant, and it costs at most $E_\infty^\star + 2c_2 G c_3 N^{-(k-1/2)}$.

Lemma 14 (Assumption C gives Assumption E)**.** *Under the hypotheses of Lemma 13, for every U and all sufficiently large N, Assumption E holds at every feasible curve of $\mathscr{S}_N$ with cost at most U.*

Proof of Lemma 14. Interpolating the pushing field along the curve gives, on short pieces, a change of the control points that increases every small clearance, which is Robinson's condition.

Let $W(\mathbf{x}, t)$ be the space-time field built from Lemma 12 in the proof of Lemma 13, on a compact set that contains every curve of cost at most U (such curves are bounded by the coercivity step of the

proof of Proposition 15, which uses Assumption B): it is of class C^k, vanishes near the endpoints, and has $\nabla_{\mathbf{x}} g_j \cdot W \geq 2$ wherever $g_j < \kappa$, for some $\kappa > 0$. Let $\boldsymbol{\gamma}$ be a feasible curve of the class with cost at most U, and let $\mathbf{v}$ be the piecewise Hermite interpolant of $t \mapsto W(\boldsymbol{\gamma}(t), t)$ at the nodes. Since $\boldsymbol{\gamma}$ starts and ends at points where W vanishes identically nearby, $\mathbf{v}$ has zero boundary states, so it is an admissible change of the control points. By the chain rule, $\|(W \circ \boldsymbol{\gamma})^{(k)}\|_{L^2}$ is bounded by a constant c_4 that depends only on U, so, as in the proof of Lemma 13, $\|\mathbf{v} - W \circ \boldsymbol{\gamma}\|_\infty \leq c_4 N^{-(k-1/2)}$, which is below $1/G$ for large N. Then $\nabla_{\mathbf{x}} g_j \cdot \mathbf{v} \geq 1$ wherever $g_j < \kappa$ along $\boldsymbol{\gamma}$, while $g_j \geq \kappa$ elsewhere; for small $s > 0$ the piecewise polynomial $g_j + s \nabla_{\mathbf{x}} g_j \cdot \mathbf{v}$, which has the coefficients $\boldsymbol{\pi}_j + D\boldsymbol{\pi}_j(s\mathbf{v})$, is therefore positive on all of $[0,1]$, that is, lies in $\operatorname{int} \mathscr{K}_j$. This is Assumption E at $\boldsymbol{\gamma}$.

The degree-refinement procedure. At level r, take degrees $d_r \uparrow \infty$ satisfying Assumption D, grid spacing $h_r = (2d_r)^{-(k+1)}/r$, and a free-coordinate box $\mathscr{B}_r$ in the free control points (the rational chart after the proof of Proposition 10), centred at zero with half-width $r\sqrt{d_r + 1}$. Write $d_f(r) = d_r + 1 - 2(\ell + 1)$. Degree elevation keeps the previous output without changing its curve or cost. Algorithm 2 may run for finitely many iterations, since a trial replaces its seed only when it is feasible and no more expensive. Membership in the grid candidate set is tested over the whole interval, exactly, interval by interval, for rational piecewise-polynomial data and breakpoints by Sturm sequences (an exact count of the real roots of a polynomial in an interval).

Algorithm 4 Degree and seed refinement for (P_∞)

Require: degrees d_r with charts $(\boldsymbol{\Gamma}_{\mathrm{aff}}^{(r)}, \mathbf{N}_r)$, spacings h_r, boxes $\mathscr{B}_r$; levels R; finite local budgets

```
Γ_best ← none
for r = 1, ..., R do
    G_r ← feasible degree-d_r curves from h_r Z^{n d_f(r)} ∩ B_r
    if Γ_best ≠ none then add its degree-d_r elevation to G_r
    if G_r = ∅ then continue
    Γ_seed ← arg min_{Γ ∈ G_r} cost(Γ)
    Γ_trial,free ← Algorithm 2 with chart (Γ_aff^(r), N_r), initialized at (Γ_seed − Γ_aff^(r)) N_r
    Γ_trial ← Γ_aff^(r) + Γ_trial,free N_r^⊤
    Γ_best ← cheaper feasible member of {Γ_seed, Γ_trial}
end for
return Γ_best                                   or no feasible curve found within the budget
```

Proposition 15 (degree refinement)**.** *Under Assumptions A, B and C, once initialized, the output costs of Algorithm 4 are nonincreasing and converge to $E_\infty^\star$, the optimum $E_d^\star$ of* (P) *at degree d decreases to $E_\infty^\star$, and $E_\infty^\star$ is attained. Every limit point of the output curves, weakly in $\mathscr{H}^k$ and uniformly with derivatives below k, is a global minimizer of* (P_∞).

Proof of Proposition 15. A smooth near-optimal curve with a positive margin, approximated by a Bernstein polynomial and then by a grid point, stays feasible and near-optimal, and the refinement can only improve on it. We prove the conclusions for constraints g_j and ψ_j that depend continuously on t and on derivatives of order below k, under a coercive cost and the approach from inside: for every $\varepsilon > 0$ some C^∞ curve with the boundary data costs at most $E_\infty^\star + \varepsilon$ and has every constraint bounded below by a positive number. The last step derives the approach from inside from the hypotheses of the theorem. As in the proof of Lemma 13, E denotes the cost.

Existence. A minimizing sequence in $\mathscr{H}^k$ is bounded by coercivity; a subsequence converges weakly in $\mathscr{H}^k$ and, with its derivatives below k, uniformly, so the limit keeps the boundary data and the constraints, which depend only on those derivatives, and E, a continuous convex quadratic, is weakly lower semicontinuous (the cost of a weak limit is at most the $\liminf$ of the costs). Coercivity under $2(\ell + 1) \geq k$: otherwise a sequence with bounded cost and $\|\boldsymbol{\gamma}_i\|_{\mathscr{H}^k} \to \infty$, normalized, has kth derivatives tending to zero, so by the compact embedding of $\mathscr{H}^k$ in $\mathscr{H}^{k-1}$ a subsequence converges in $\mathscr{H}^k$ to a polynomial of norm one, of degree below k, whose derivatives of orders $0, \ldots, \ell$ vanish at both ends, which is impossible. So $E_\infty^\star$ is attained over $\mathscr{H}^k$, and the approach from inside shows that the infimum over C^k is the same number.

Polynomial approximation. Fix $\varepsilon > 0$ and a C^∞ curve $\boldsymbol{\gamma}_\varepsilon$ from the hypothesis, with $E(\boldsymbol{\gamma}_\varepsilon) \le E^\star_\infty + \varepsilon$ and every constraint at least $\mu > 0$. By continuity there is $\rho > 0$ such that every curve with the boundary data within 2ρ of $\boldsymbol{\gamma}_\varepsilon$ in C^k is feasible and costs at most $E^\star_\infty + 2\varepsilon$. The Bernstein polynomials of $\boldsymbol{\gamma}_\varepsilon$ converge to it in C^k (Lorentz, 1986); adding the Hermite polynomial of degree $2\ell+1$ that corrects their boundary states, whose errors tend to zero, gives curves p_d of degree d with the boundary data and $\|p_d - \boldsymbol{\gamma}_\varepsilon\|_{C^k} \le \rho$ for $d \ge d(\varepsilon)$.

A grid point near p_d. For a degree-d curve with control points $\boldsymbol{\Gamma}$, $\boldsymbol{\gamma}^{(i)} = \frac{d!}{(d-i)!}\sum_l (\Delta^i \boldsymbol{\Gamma})_l\, [\mathbf{b}_{d-i}]_l$ with $\|(\Delta^i\boldsymbol{\Gamma})_l\| \le 2^i \|\boldsymbol{\Gamma}\|_F$ and weights summing to one, so $\|\boldsymbol{\gamma}^{(i)}\|_\infty \le (2d)^i \|\boldsymbol{\Gamma}\|_F$. For one segment the rational chart (after the proof of Proposition 10) is an isometry, so moving the free coordinates of p_{d_r} to a nearest point of $h_r\mathbb{Z}^{nd_f}$ moves the curve by at most $(2d_r)^k h_r \sqrt{n(d_r+1)}/2 \le \sqrt{n}/(2r)$ in C^k, which is below ρ for large r. The free coordinates of p_d have norm at most $\|\boldsymbol{\Gamma}(p_d)\|_F \le \sqrt{n(d+1)}\, C_\varepsilon$, since they are entries of $\boldsymbol{\Gamma}(p_d)$, and the Bernstein coefficients of p_d are bounded by some C_ε; so for large r the box contains the grid point. Hence for $r \ge r(\varepsilon)$ the seed costs at most $E^\star_\infty + 2\varepsilon$, and so does the output, while every output is feasible at degree d_r, so costs at least $E^\star_{d_r} \ge E^\star_\infty$. The previous output, written at the higher degree, is a candidate at each level, so the costs are nonincreasing, and $E^\star_d$ is nonincreasing in d and bounded above by the output costs.

Limit points. The outputs have bounded cost, hence are bounded in $\mathscr{H}^k$; a weak limit is feasible, as in the existence step, and costs at most $E^\star_\infty$.

Approach from inside. With position constraints satisfying the Mangasarian–Fromovitz condition at every t and strictly feasible endpoints, let $\boldsymbol{\gamma}^\star$ minimize over $\mathscr{H}^k$ and apply the maps Φ_μ of Lemma 12: $\Phi_\mu \circ \boldsymbol{\gamma}^\star$ keeps the boundary data, has every $g_j \ge \mu$, and its cost is at most $E^\star_\infty + c_2\mu$ by the chain-rule estimate in the proof of Lemma 13, which also covers moving centres with the curve $t \mapsto \Phi_\mu(\boldsymbol{\gamma}^\star(t), t)$. Smoothing it in $\mathscr{H}^k$ and restoring the boundary data by a small Hermite correction keeps the cost within ε and every $g_j \ge \mu/2$.

Proof of Theorem 1. The proof chains four results: on fine splines Assumption C gives Robinson's condition (Lemma 14); the fixed-degree result then makes GlobMP find the cheapest spline (Proposition 10, with $\mathbf{H} \succ 0$ on $\mathscr{S}_N$ because a spline with $k-1$ continuous derivatives and zero kth derivative is one polynomial of degree below k, which the boundary data remove under Assumption B); the rate lemma bounds how far its cost is from $E^\star_\infty$ (Lemma 13); and compactness passes to the limit $N \to \infty$. Existence of a global minimizer is the existence step of the proof of Proposition 15, which needs only a feasible curve and Assumption B. Let $U = E^\star_\infty + 1$; by Lemma 13, $E^\star_N < U$ for all large N. By Lemma 14, for large N Assumption E holds at every feasible curve of $\mathscr{S}_N$ of cost at most U, which contains the global minimizers over $\mathscr{S}_N$ and every limit point of GlobMP from a start of cost at most U, so Assumption E holds with this U. Assumption D enters the proofs of Proposition 10 and Corollary 11 only through $\mathbf{H} \succ 0$, shown above, and a linear chart of the boundary equations; on $\mathscr{S}_N$ the chart exists because the states through order $k-1$ at the $N+1$ nodes determine the spline. There $\beta = 2\lambda_{\max}(\mathbf{N}^\top \mathbf{Q} \mathbf{N})$, with $\mathbf{Q}$ the block-diagonal cost matrix of the pieces and $\mathbf{N}$ an orthonormal basis of the directions the boundary and junction equations leave free, and z_h is mapped to these coordinates before the refinement. Both results, applied over $\mathscr{S}_N$ with a box that contains a global minimizer in its interior, show that GlobMP from z_h has only global minimizers of $\mathscr{S}_N$ as limit points (with independent uniform starts, some run does so after finitely many starts, with probability one); these limit points are feasible at every instant and cost $E^\star_N$, which tends to $E^\star_\infty$ at the stated rate by Lemma 13. As $N \to \infty$ these costs are bounded, so by Assumption B the curves are bounded in $\mathscr{H}^k$; a subsequence converges weakly in $\mathscr{H}^k$ and uniformly with the derivatives below k, the limit is feasible and has the boundary data, and by weak lower semicontinuity its cost is at most $\lim E^\star_N = E^\star_\infty$, so it is a global minimizer of (P_∞).

Proof of Corollary 2. A curve of $\mathscr{S}_N$ that is feasible at every instant has the boundary data and meets every constraint of (P_∞), so every iterate is feasible for (P_∞) by part (i) of Theorem 1. From z_h, Proposition 10 gives $f(z^\nu) \downarrow E^\star_N$, so $f(z^\nu) \le E^\star_N + \varepsilon/2$ after finitely many iterations. By part (ii), $E^\star_N \le E^\star_\infty + C N^{-(k-1/2)} \le E^\star_\infty + \varepsilon/2$ for the stated N. Adding the two bounds proves the claim.

A3.3 Scope of Assumptions A–C

This subsection records which parts of the three assumptions the proof needs, which can be weakened, and why Assumption C cannot be dropped.

Piecewise-polynomial data. Assumption A asks for continuous, piecewise-polynomial obstacle centres and reference, which covers the piecewise-linear reference (7) of a learned policy. The proof uses polynomial data in two places only. The cone test stays exact for rational data and breakpoints: on each interval of a common partition of $[0,1]$ into the pieces of the curve and of the data, the block is a polynomial in t, and $\mathscr{K}_j$ becomes the product of the cones of these intervals, still closed and convex. The problem over $\mathscr{S}_N$ stays semialgebraic, which is all the finite-critical-values step needs. The push, the rate and Lemma 14 use only continuity of the centres and the reference: the field W of Lemma 12 is a smooth partition of unity times fixed vectors, and continuity of g_j and $\nabla_{\mathbf{x}} g_j$ in $(\mathbf{x},t)$ is all its construction needs.

Bounded curves instead of Assumption B. Assumption B enters only through boundedness: curves of bounded cost are bounded. The descent step uses the upper curvature β and not α. When the steering constraint is present ($\bar{r} < \infty$), every feasible curve has $\|\boldsymbol{\gamma}\|_\infty \le \|\boldsymbol{\rho}\|_\infty + \bar{r}$, and the coercivity argument of Proposition 15 applies unchanged: a normalized sequence with vanishing kth derivatives and vanishing sup norm cannot converge to a polynomial of norm one. So the conclusions of Theorem 1 hold with $\bar{r} < \infty$ in place of Assumption B, with f convex rather than strongly convex over $\mathscr{S}_N$ and compactness supplied by the bounded feasible set.

When Assumption C holds. It is needed only at feasible positions within distance 1 of curves of cost at most $E^\star_\infty + 1$. It always holds for robots without obstacles or steering: the direction $\mathbf{v} = \mathbf{x}$, which spreads every robot away from the origin, gives $\nabla g_j \cdot \mathbf{v} = 2\|\mathbf{x}_a - \mathbf{x}_b\|^2 > 0$ for every touching pair; spreading about the centre of a single ball obstacle does the same for that obstacle and any number of robots. For static constraints it holds for almost every choice of margins: give block j the radius $r_j + \delta_j$. For each set J of blocks, the map $\mathbf{x} \mapsto (g_j(\mathbf{x}))_{j\in J}$ is polynomial, so by Sard's theorem almost every value is regular, and at every configuration where exactly the blocks of J are active their gradients are then linearly independent, which implies Assumption C. There are finitely many sets J, so the exceptional margins $\boldsymbol{\delta}$ form a null set. For moving obstacles or a moving reference the same count suggests that, generically, the condition fails only at isolated points of space-time, and a cheapest curve may pass through one.

Assumption C cannot be dropped. The condition is needed by the algorithm, not only by the proof. Take a point robot in $\mathbb{R}^2$ and two discs of radius r centred at $(\pm r, 0)$, which touch at the origin, where Assumption C fails, and the cost $c\,x_2$ with $c > 0$ over $x_2 \ge -1$; the vertical line $x_1 = 0$ is feasible throughout. From $(0,y)$ with $0 < y \le \eta c$, the linearized set is $\{w : w_2 \ge y/2 + (r/y)|w_1|\}$, and the repair returns its apex $(0, y/2)$. The iterates halve their distance to the origin at every step and converge to it, a point that is not a KKT point, since the cost still decreases along the feasible line. A cheapest curve through a zero-width gap between two touching obstacles can stall GlobMP in the same way.

A4 Addendum to Section 4: Experiments

A4.1 Experimental setup

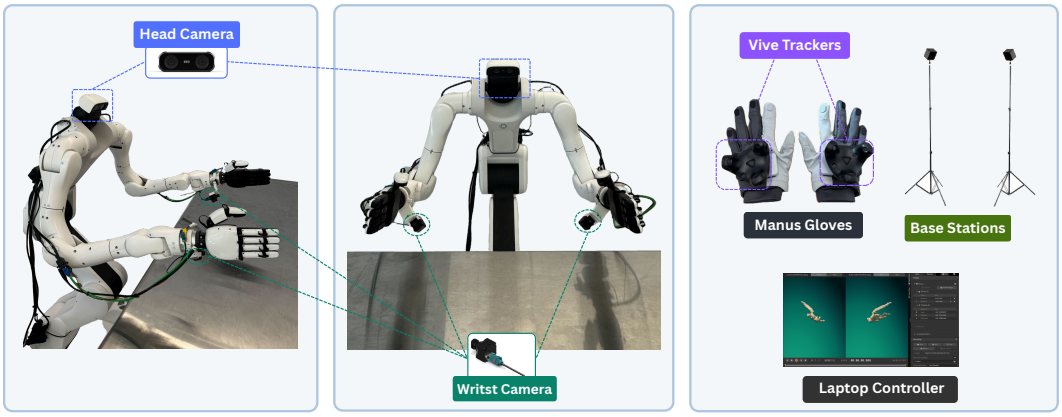
This subsection gives the hardware, robot platforms and software shared by all experiments. The later subsections are instances of (P), each a block list of (P_∞) solved by Algorithms 1–2 under the protocol of its source; every baseline number is our own reproduction.

Hardware. The maze baselines ran on a workstation with a 24-core Intel Core Ultra 9 285K at 3.7 GHz and one NVIDIA GeForce RTX 5090 (32 GB); the synthetic study of §4.1 and the multi-robot suite of Tables 3–10 ran on a shared node with two 32-core AMD EPYC 7502 CPUs and eight NVIDIA A100-PCIE GPUs (40 GB each), as did GlobMP's maze runs, so the maze times compare different machines. Training, sampling and model evaluation were performed on a single GPU. The RRT, GCS and SDP-MP baselines ran on CPU with each run pinned to fixed cores, and

Algorithms 1–2 ran on CPU or GPU as stated with each experiment (for the synthetic study, in Appendix A4.2).

Robot platforms. The bimanual demonstrations are collected on the setup of Figure 7, using a Dexmate Vega-1 with two XHAND1 dexterous hands. Two ZED X One S cameras are mounted at the wrists and one ZED X Mini camera is mounted on the head. Teleoperation uses Manus gloves for hand target gestures and HTC VIVE trackers for wrist target poses. Only the resulting trajectories enter this paper; none of the teleoperation hardware is used at inference.

The multi-robot experiments use the Georgia Tech Robotarium (Wilson et al., 2020), with its 3.2×2.0 m arena, GRITSBot geometry, and differential-drive dynamics. The benchmark environments of §4.2 are adapted to the Robotarium workspace, with their obstacle geometry represented virtually in the arena. The GRITSBots have physical radius 0.055 m, while planning uses an inflated radius $r = 0.13$ m to account for tracking error and the testbed's barrier-certificate safety layer as well as non-holonomic conversion of the trajectory plan. The planned trajectories are tracked through the Robotarium's single-integrator-to-unicycle controller, and an overhead camera records the executed motion. The virtual obstacles are overlaid from the corresponding map definitions, and clearance is evaluated from the robot trajectories tracked in the video after correcting for lens distortion. Across the four obstacle maps, no GlobMP robot comes closer than 0.06 m to an obstacle; the closest pass occurs on `Conveyor` and is shown in Figure 8.



(a) Robot Platform **(b)** operator Station

Figure 7: Robot system setup on the Dexmate Vega-1 bimanual robot and the XHAND1 dexterous hands. Two ZED X One S (wide view) cameras are mounted at the wrists, and one ZED X Mini camera is mounted on the head. For teleoperation we use Manus gloves to retrieve hand target gesture and HTC VIVE trackers for wrist target pose.

Software. All experiments are based on PyTorch 2.13 for automatic differentiation, JAX 0.6.2 for DiRecT's SCP–ADMM projector, and IPOPT through Pyomo for the augmented-Lagrangian projection, under an external wall-clock cap. The RRT variants run through OMPL and their steering functions through Drake, which supplies the reported conic and nonlinear solvers. The multi-robot baselines build on DiRecT (Giaretta et al., 2026) (MIT), itself an extension of PROJECTED COUPLED DIFFUSION (Luan et al., 2026) (MIT), MMD (Shaoul et al., 2025) (MIT) and MDOC (He et al., 2026) (MIT). Every learned baseline is trained from scratch under its source's protocol rather than scored from a released checkpoint.

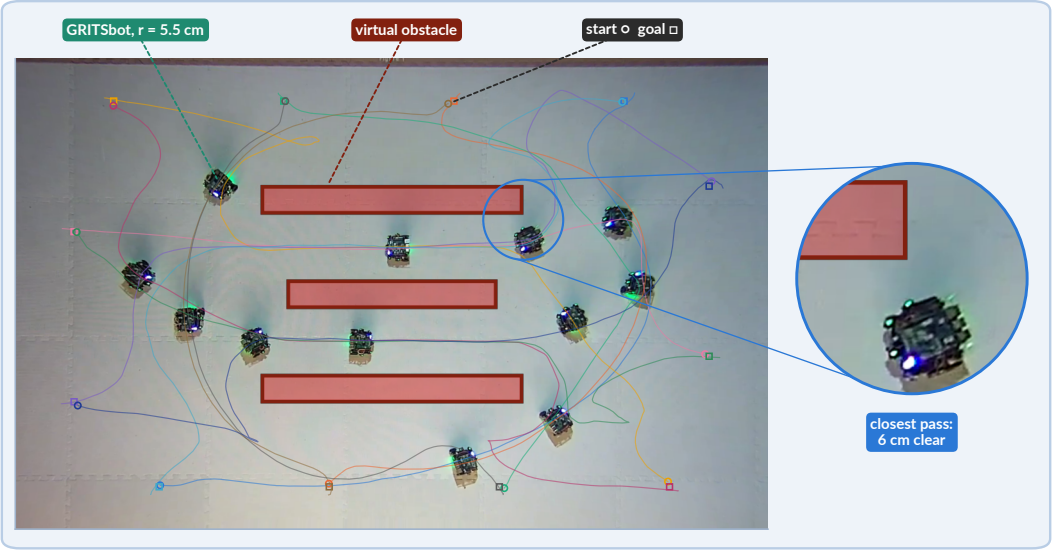


Figure 8: GlobMP on the Georgia Tech Robotarium: twelve GRITSBots on `Conveyor` in the 3.2×2.0 m arena, at $t = 25$ s. The paths the robots drove, starts (circles), goals (squares), and the virtual obstacles drawn from the map. The circle magnifies the run's closest pass: 6 cm between the robot's body and the wall.

A4.2 Synthetic experiments

The synthetic study of §4.1 tests continuous-time feasibility, the cost of the returned curve, and scalability on one family. Every instance is a minimum-derivative problem with snap cost $k = 4$, endpoint data $\ell = 4$, piecewise degree-9 polynomials, C^3 junctions and margin $\delta = 0.005$ m, so the piecewise conditions of Appendix A4.5 apply and the planning curve lies in the $\mathscr{H}^4$ class of (P_∞).

Problem 1 (robots exchanging places among spheres)**.** N_a robots, N_a even, are stacked into one curve $\boldsymbol{\gamma}$ with values in $\mathbb{R}^{2N_a}$ and form $N_a/2$ exchanging pairs: in pair i one robot moves from (x_0, y_i) to $(x_0 + D_i, y_i)$ and the other the reverse way along $y_i + o_i$, with $D_i \sim U(6,8)$ m and $o_i \sim U(0.3, 0.9) R_S$. Every pair of robots carries a separation block, with the other robot as centre ($\mathbf{c}_j = \boldsymbol{\gamma}_b$, $\mathbf{V}_j = \mathbf{I}$, $r_j = R_S = 1.2$ m), and every robot a ball block for each static sphere, with $\mathbf{c}_j$ its centre, $\mathbf{V}_j = \mathbf{I}$ and r_j its radius. Two spheres of radius $U(0.45, 0.65)$ m sit 0.3 to 0.9 radii below the first row, so the first robot's straight path enters both, and every start and goal clears them by 0.2 m. There is no workspace bound, so every inequality block is nonconvex. Each exchanging pair, and each sphere a straight path enters, is passed on one of two sides, which gives 8 to 64 route classes per instance.

The main comparison uses $N = 7$ pieces and 20 instances at each of $N_a = 2, 4, 6, 8$. The scaling study uses $N = 4$ pieces, rows spaced $3R_S$ apart with $o_i = 0.6R_S$, three spheres, and, from 2 to 200 robots, 10 seeds per size for GlobMP and 5 for the baselines.

Methods. GlobMP opens each route class with a waypoint path that passes every pair and every crossed sphere on the chosen side, runs Algorithms 1–2 and the finishing step of Appendix A4.5, and returns the cheapest feasible result over the classes. RRT-Connect and RRT* are the OMPL implementations (Kuffner & LaValle, 2000; Karaman & Frazzoli, 2011) in $\mathbb{R}^{2N_a}$ with the exact validity test (every pair at least R_S apart, every robot outside every sphere); they sample in the rectangle spanned by the instance plus 3 m, since a sampling planner needs a bounded region, and get a 60 s budget (600 s in the scaling study). Their path is collision-free as a polyline but is not a curve of the class, so the returned curve is the cheapest collision-free of several fits to it, each scored on the true geometry: the minimum-snap interpolant through the path at the segment breakpoints, which cuts corners; fits that trade snap against the squared distance to the path; and minimum-snap fits kept within a corridor around the path wherever the current fit collides, with the corridor width set by the path's own clearance (Richter et al., 2016), each on the path and on a shortcut version of

it. GCS (Marcucci et al., 2023) works in the same joint space, where both collision sets are convex: robot a inside a sphere is $\{q : \|q_a - \mathbf{c}\| \le r\}$ and a colliding pair is $\{q : \|q_a - q_b\| \le R_S\}$. IRIS (Deits & Tedrake, 2015a) grows regions from seeds every 0.5 m along an RRT-Connect path, Drake's GCS trajectory optimization finds the shortest path through them with Bézier order 5, falling back to 3 and 2 where its convex solve fails numerically, and its curve is fitted in the same way. SDP-MP, the semidefinite relaxation of Graesdal et al. (2026), is written on the same program in its own affine chart, so that the endpoint and junction conditions hold by construction; the per-segment form with lifted junctions stops with a numerical error in Clarabel from two segments on. The chart form is joint over all robots and segments, and each block must lie on each segment in the cone of polynomials nonnegative on $[0,1]$ of §2, written as sums of squares. On an obstacle-free instance it reproduces the exact minimum-snap optimum. Its rounded curve is the first-order part of the solution. The baselines run on CPU, one run per four pinned cores, and a run that returns nothing within one hour counts as a failure. GlobMP's route classes run on CPU in the same way at $N_a = 2, 4$ and on the first 10 instances at $N_a = 6$, and on A100 GPUs, four to ten runs per GPU, for the rest, so its times are totals of work rather than a like-for-like speed comparison.

A4.3 Maze navigation

We examine whether the proposed formulation remains effective in a nonconvex maze. We consider the safe maze navigation benchmark of Xiao et al. (2025), following the setting reported by Giaretta et al. (2026). A point robot traverses the D4RL `maze2d-large` environment (Fu et al., 2020) between fixed endpoints while avoiding keep-out regions placed within the maze corridors. The state contains planar position and velocity, and the action is a planar force. A rollout is considered safe if the executed trajectory remains outside every keep-out region. We evaluate two settings, `broad`, containing two keep-outs, and `narrow`, containing six.

This benchmark combines both constraint constructions used in this paper. The maze walls form a union of cells and therefore define a disjunctive free space that cannot be represented by a single membership in $\mathscr{K}$. A breadth-first search on the cell graph selects a corridor in $\approx$1 ms, after which the continuous trajectory is optimized and verified within the selected corridor. Each of the 12 corridor boxes along the route contributes four affine blocks (6) and one Bézier segment, with adjacent segments joined C^ℓ, resulting in 48 affine blocks. The keep-out regions contribute the nonlinear obstacle blocks. We use $k = \ell = 2$ with pieces of degree 9, so Assumption A ($\ell < k$) does not hold here. The maze geometry is not learned. The baselines rely on the diffusion prior to remain within the maze and explicitly constrain only the keep-outs, whereas our formulation enforces the corridor boundaries through affine blocks and does not use a learned prior.

Baselines. We compare against ten diffusion planners, nine of them constrained: *Diffuser* (Janner et al., 2022), *Classifier Guidance* (Dhariwal & Nichol, 2021), *Gradient Guidance* (Chung et al., 2023), *Projected Diffusion* (Christopher et al., 2024), *Primal–Dual* and *Augmented Lagrangian* (Zhang et al., 2025), the *SafeDiffuser* variants RoS, ReS, and TVS (Xiao et al., 2025), and *DiRecT* (Giaretta et al., 2026), under the configuration reported by Giaretta et al. (2026) for this task. Methods requiring a nonlinear solver use IPOPT (Wächter & Biegler, 2006). These methods explicitly constrain the keep-out regions but leave satisfaction of the maze walls to the learned prior, and none guarantees feasibility in continuous time. Their constraints are enforced pointwise during sampling or projection, so feasibility holds at the sampled states rather than along the whole curve.

Training and environment details. Each keep-out region is represented by an ellipse or superellipse centered at (c_x, c_y) with semi-axes (r_x, r_y) and order p,

$$\left|\frac{x-c_x}{r_x}\right|^p + \left|\frac{y-c_y}{r_y}\right|^p \ge 1. \tag{14}$$

The `broad` setting uses one region with $p = 2$ and one with $p = 4$, while all six keep-outs in `narrow` use $p = 4$. Each region straddles a wall stub and extends into the corridor, so every keep-out is active for any route connecting the fixed endpoints, as illustrated in Figure 9. A single continuous-time diffusion model with a temporal UNet is used by all baselines. It is trained for 500,000 steps with Adam, batch size 256, on the D4RL `maze2d-large-v1` dataset using min–max normalization. The learning rate is 10^{-3} with cosine annealing over the first 10,000 steps, the exponential moving average decay is 0.995, and the planning horizon is H=384. The model outputs a state–action trajectory. Dynamics are identified from the same dataset by ordinary least squares over 3,993,503 transitions and are imposed on the baselines as equality constraints. Because

Table 1: `Maze2D Broad`, in the format of Giaretta et al. (2026)'s Table 5. Mean ± standard deviation. The four *Gen.* columns score the generated trajectory. Best value(s) in bold.

Method	SR ↑	Viol. ↓	Score ↑	Time (s) ↓	Gen. SR ↑	Gen. Viol. ↓	Gen. AS ↓	Gen. CS ↓
Diffuser	0.000	45.920 ± 11.863	1.601 ± 0.025	0.316 ± 0.078	0.000 ± 0.000	49.040 ± 17.426	0.057 ± 0.005	0.636 ± 0.077
Classifier Guidance	0.030	37.480 ± 13.260	1.587 ± 0.164	0.654 ± 0.093	0.040 ± 0.197	35.760 ± 19.482	0.055 ± 0.005	0.619 ± 0.077
Gradient Guidance	0.040	25.520 ± 10.178	1.587 ± 0.164	3.637 ± 0.076	0.040 ± 0.197	10.220 ± 7.882	0.056 ± 0.005	0.636 ± 0.073
Projected Diffusion	0.220	42.000 ± 47.282	1.119 ± 0.708	46.398 ± 11.096	**1.000** ± 0.000	**0.000** ± 0.000	0.002 ± 0.000	0.011 ± 0.004
Primal–Dual	0.170	26.610 ± 31.734	0.267 ± 0.417	5.957 ± 0.083	**1.000** ± 0.000	**0.000** ± 0.000	21.299 ± 0.219	1.861 ± 0.015
Augmented Lagrangian	0.080	72.920 ± 92.751	−0.010 ± 0.145	5.676 ± 0.566	0.000 ± 0.000	91.010 ± 55.079	2.152 ± 0.974	1.407 ± 0.452
SafeDiffuser-RoS	0.040	66.260 ± 41.720	1.186 ± 0.533	148.967 ± 3.831	**1.000** ± 0.000	**0.000** ± 0.000	0.008 ± 0.007	0.010 ± 0.004
SafeDiffuser-ReS	0.520	9.690 ± 11.948	0.907 ± 0.815	486.568 ± 61.073	**1.000** ± 0.000	**0.000** ± 0.000	0.003 ± 0.003	0.015 ± 0.006
SafeDiffuser-TVS	0.040	62.240 ± 38.460	1.219 ± 0.561	128.758 ± 2.988	**1.000** ± 0.000	**0.000** ± 0.000	0.005 ± 0.004	0.010 ± 0.004
DiRecT	0.970	0.450 ± 3.806	1.500 ± 0.419	17.657 ± 0.629	**1.000** ± 0.000	**0.000** ± 0.000	0.002 ± 0.000	0.008 ± 0.002
GlobMP (Ours), disc cover	**1.000**	**0.000**	1.591	270.3	**1.000**	**0.000**	0.0009	0.0010
GlobMP (Ours), exact p=4	**1.000**	**0.000**	**1.629**	32.8	**1.000**	**0.000**	**0.0005**	**0.0001**

open-loop execution of the fitted linear model diverges over this horizon, all methods execute their planned trajectories using a proportional–derivative controller with $P = 5$ and $D = 1$.

Evaluation. We follow the protocol of Giaretta et al. (2026). Each baseline generates 100 i.i.d. trajectories conditioned on the same fixed endpoints, with each trajectory executed for at most 800 environment steps. We report *SR*, the fraction of collision-free rollouts; *Viol.*, the mean number of steps spent inside a keep-out; *Score*, the D4RL normalized return; and *Time*, the planning time. The *Gen.* columns evaluate the generated trajectory before execution and additionally report acceleration smoothness, *AS*, and curvature smoothness, *CS*. Baseline values are means over 100 samples, whereas our method is deterministic and reports a single trajectory.

Tables 1 and 2 reveal an important difference between planning and execution. In `Broad`, several baselines generate trajectories with *Gen. SR*= 1.00 but lose safety during execution. For example, DiRecT decreases from a generated safety rate of 1.00 to a rollout *SR* of 0.97, while the gap is substantially larger for several projection and guidance methods. Both of our formulations maintain *SR*= 1.00 with zero violations before and after execution. The disc-cover formulation attains a score of 1.591, while the exact $p = 4$ formulation improves the score to 1.629 and achieves the lowest *AS* and *CS*. Both satisfy the keep-out constraints for every t.

The distinction becomes more pronounced in `Narrow`, where feasibility is difficult already at the planning stage. Among the baselines, DiRecT is the only method with *Gen. SR*= 1.00, but its rollout safety decreases to 0.94. Our trajectory maintains *SR*= 1.00 with zero violations, achieves a score of 1.625 compared with 1.624 for DiRecT, and obtains the lowest *AS* and *CS*, at a planning time of 90.0 s compared with 98.093 s for DiRecT (different machines, Appendix A4.1). That time is not a single descent: Algorithm 2 is within 1% of its own final cost after 19 s and stops at 57 s (60 s cap), and an SQP polish then supplies the final 6% of the cost (30 s, plus 3 s for the finishing step); letting Algorithm 2 run to its own stopping rule instead (532 s) changes neither the cost nor the score. A 20 s cap gives 32.8 s on `Broad` against 17.7 s for DiRecT. On both maps the returned trajectory satisfies the keep-out constraints at every $t \in [0, 1]$, not only at the sampled states that the benchmark checks.

In Tables 1 and 2, most baselines lose safety between the generated plan and the executed rollout, whereas both GlobMP rows are at 1.00 on both. Figure 9 shows the trajectory we return next to its PD-tracked execution, and Figure 10 follows its keep-out clearance over time. The plan touches the enforced margin exactly at its active contacts (minimum clearance 3.1×10^{-7} on `Broad` and 3.7×10^{-10} on `Narrow`) and never crosses it by more than 10^{-7}. Tracking error takes the executed rollout up to 0.24 into the margin band, which is what the margin is for: the rollout stays outside every keep-out, and the benchmark counts zero violations.

Table 2: `Maze2D Narrow`, the six-keep-out setting. Mean ± standard deviation. The four *Gen.* columns score the generated trajectory. Best value(s) in bold.

Method	SR ↑	Viol. ↓	Score ↑	Time (s) ↓	Gen. SR ↑	Gen. Viol. ↓	Gen. AS ↓	Gen. CS ↓
Diffuser	0.000	103.940 ± 21.881	1.601 ± 0.025	0.321 ± 0.086	0.000 ± 0.000	101.230 ± 24.551	0.057 ± 0.005	0.636 ± 0.077
Classifier Guidance	0.000	85.280 ± 24.224	1.608 ± 0.022	0.695 ± 0.079	0.000 ± 0.000	76.120 ± 26.527	0.056 ± 0.005	0.629 ± 0.077
Gradient Guidance	0.000	51.300 ± 19.221	1.619 ± 0.029	3.608 ± 0.023	0.000 ± 0.000	23.390 ± 14.123	0.056 ± 0.005	0.647 ± 0.072
Projected Diffusion	0.780	37.290 ± 129.876	0.163 ± 0.459	112.918 ± 19.517	0.420 ± 0.496	17.630 ± 30.680	0.018 ± 0.039	0.033 ± 0.077
Primal–Dual	0.000	55.970 ± 49.478	0.268 ± 0.418	5.154 ± 0.464	0.910 ± 0.288	0.110 ± 0.373	21.306 ± 0.226	1.861 ± 0.014
Augmented Lagrangian	0.000	217.510 ± 159.431	0.156 ± 0.449	6.022 ± 0.122	0.000 ± 0.000	197.520 ± 45.739	5.516 ± 1.301	1.765 ± 0.040
SafeDiffuser-RoS	0.000	278.870 ± 209.870	0.735 ± 0.681	509.850 ± 19.929	0.000 ± 0.000	136.250 ± 39.029	0.419 ± 0.168	0.350 ± 0.148
SafeDiffuser-ReS	0.370	79.090 ± 161.395	1.155 ± 0.764	1423.534 ± 41.319	0.760 ± 0.429	6.320 ± 16.578	0.031 ± 0.084	0.078 ± 0.164
SafeDiffuser-TVS	0.000	315.040 ± 223.000	0.729 ± 0.694	384.860 ± 9.681	0.000 ± 0.000	35.460 ± 12.962	0.059 ± 0.026	0.038 ± 0.011
DiRecT	0.940	0.230 ± 1.038	1.624 ± 0.027	98.093 ± 5.047	**1.000** ± 0.000	**0.000** ± 0.000	0.002 ± 0.000	0.010 ± 0.006
GlobMP (Ours), exact p=4	**1.000**	**0.000**	**1.625**	90.0	**1.000**	**0.000**	**0.0006**	**0.0002**

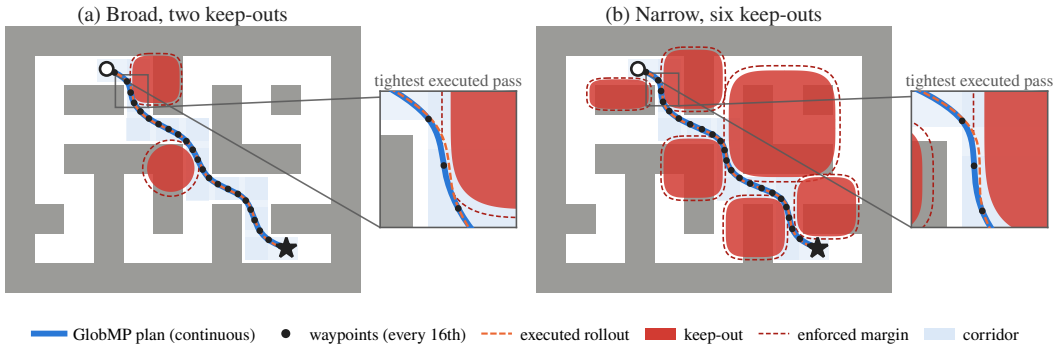


Figure 9: The trajectory GlobMP returns on both `maze2d-large` settings: the continuous plan, every 16th of the H=384 waypoints the benchmark receives, and the rollout executed by the benchmark's PD controller. The shaded band is the corridor fixed by breadth-first search, whose boxes enter as affine blocks. The side panels magnify the rollout's tightest pass: tracking error enters the enforced margin (dashed) but not the keep-out.

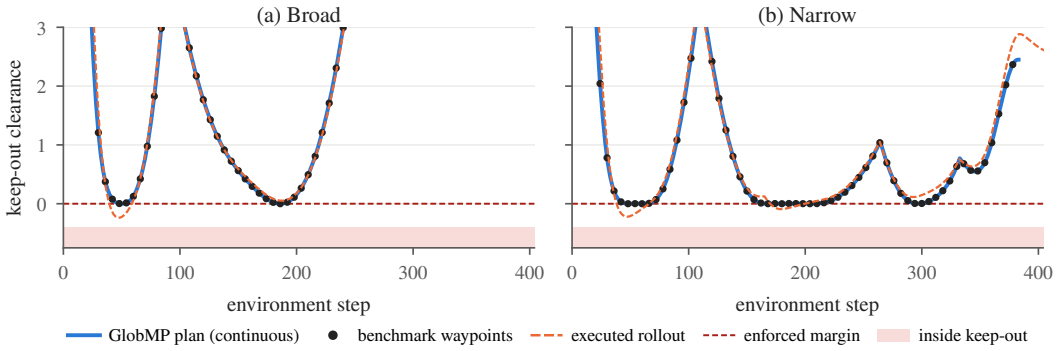


Figure 10: Keep-out clearance of the trajectory in Figure 9 over the episode: $\min_j \left(\left| \frac{x-c_x}{r_x} \right|^p + \left| \frac{y-c_y}{r_y} \right|^p \right) - (1+m)$ over the keep-outs j of (14), with enforced margin m=0.4. Zero is the enforced margin and the shaded band is the inside of a keep-out. The continuous plan reaches zero only at its active contacts; the executed rollout dips into the margin but stays outside every keep-out.

A4.4 Multi-robot motion planning

We extend the multi-robot evaluation of §4.2 to a broader benchmark with eight environments, team sizes $N_a \in \{3, 6, \ldots, 21\}$, and three test-time velocity caps. The benchmark follows Shaoul et al. (2025), with the velocity constraint of Luan et al. (2026) and the evaluation protocol of Giaretta et al. (2026). Six environments are from Shaoul et al. (2025); `Corridor-Bay` and `Cross-Room`

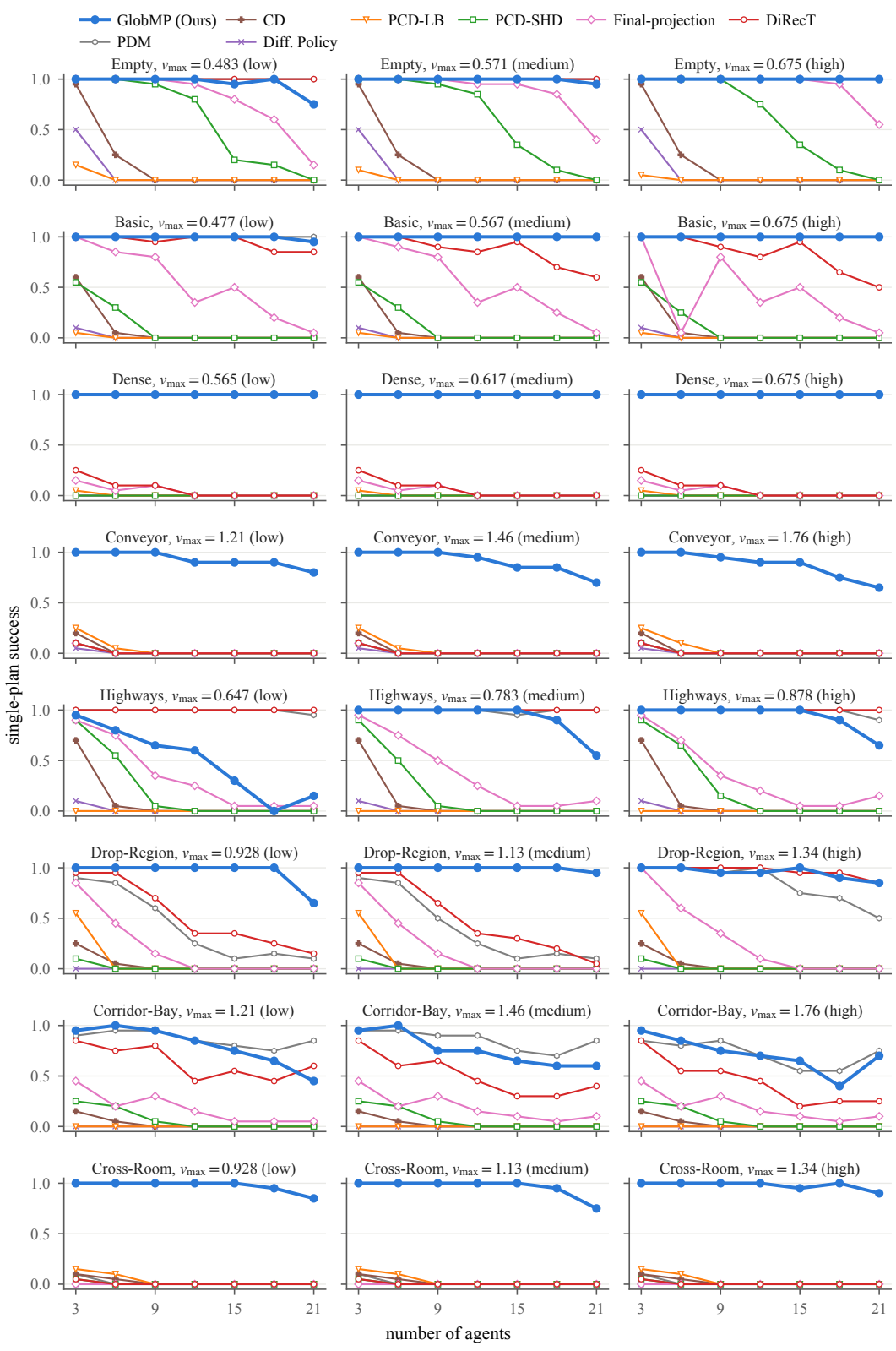


Figure 11: Single-plan success for every cell of the suite, one row per map and one column per velocity cap; Figure 13 averages the three caps.

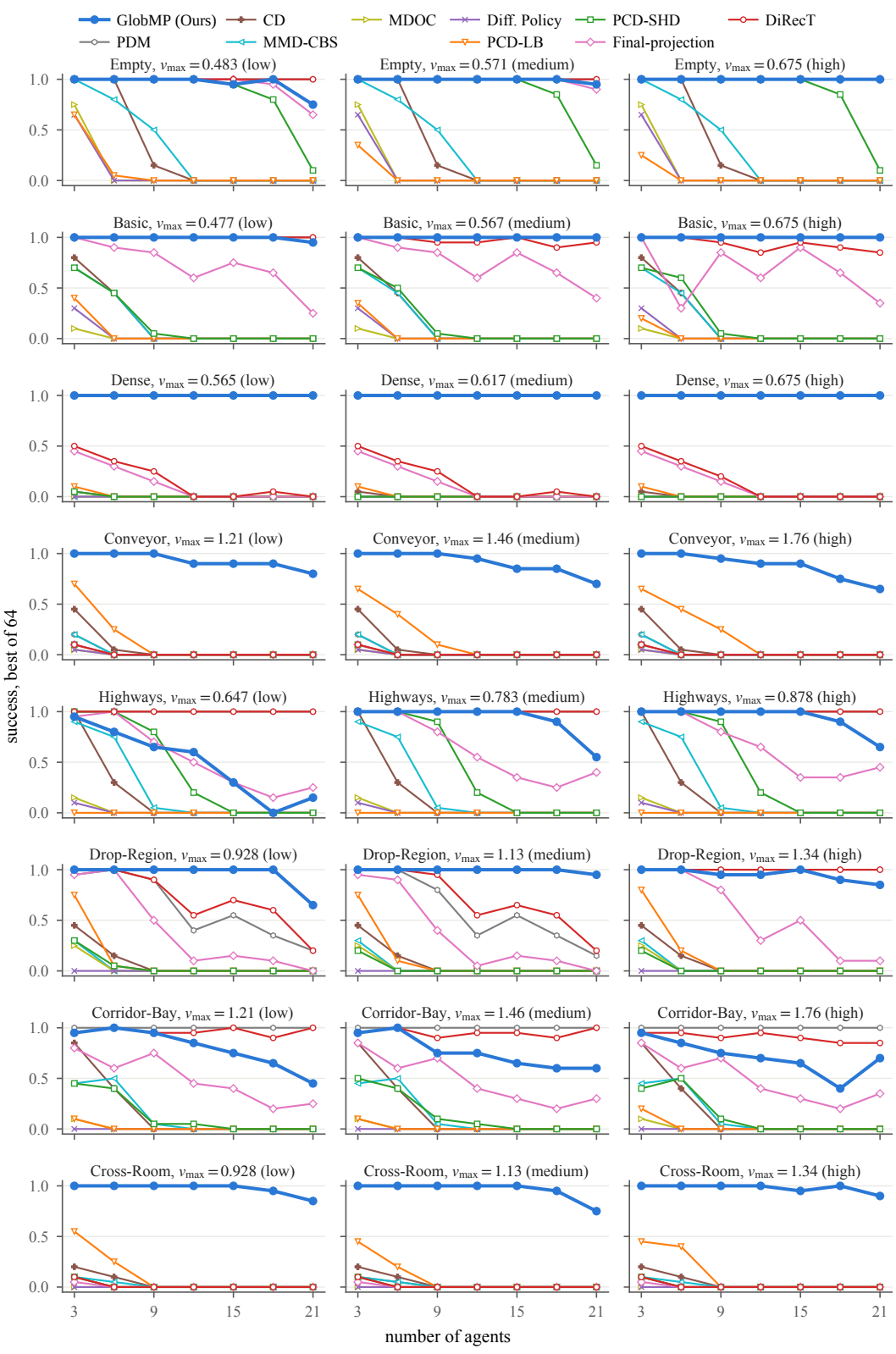


Figure 12: Best-of-64 success rate for $N_a \in \{3, 6, \ldots, 21\}$ agents on every cell of the suite, one row per map and one column per velocity cap: an instance counts as solved when any of the 64 candidates a baseline samples is collision free. This is the SR reported in Tables 3–10; our method still returns one trajectory.

Table 3: Multi robot planning performance on `Empty` for $N_a \in \{3, 6, \ldots, 21\}$ under the Low, Medium, and High velocity caps. *SR* and *CS* measure trajectory feasibility, *Coll* reports inter robot collisions, *DA* measures adherence to the environment specific motion pattern, and *Time* is the planning time per configuration. † marks methods that apply no velocity limit, so their three blocks report the same run; in addition, MMD-CBS returns a single plan per configuration rather than a batch of 64 candidates. A dash under *Coll* marks a count that is not commensurable with the others: MDOC and MMD-CBS count colliding time steps, where every other row counts agent pairs with any collision. MDOC's *CS* is omitted because its plans that do not cover every agent were counted as feasible. MMD-CBS plans under a 60 s budget per configuration and GlobMP (Ours) under 300 s per team. For GlobMP (Ours), *Time* is the median over the configurations and its subscript the standard deviation; for the baselines it is the mean, so the two are not directly comparable.

		Low ($v_{\max} = 0.483$)					Medium ($v_{\max} = 0.571$)					High ($v_{\max} = 0.675$)				
N_a	Method	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓
3	Diff. Policy†	65.0	48.2$_{\pm 46.5}$	0.67$_{\pm 0.77}$	0.999$_{\pm 0.001}$	0.5	65.0	48.2$_{\pm 46.5}$	0.67$_{\pm 0.77}$	0.999$_{\pm 0.001}$	0.3	65.0	48.2$_{\pm 46.5}$	0.67$_{\pm 0.77}$	0.999$_{\pm 0.001}$	0.5
	CD†	100.0	1.1$_{\pm 4.4}$	0.14$_{\pm 0.28}$	0.517$_{\pm 0.149}$	1.0$_{\pm 0.1}$	100.0	7.1$_{\pm 20.2}$	0.14$_{\pm 0.28}$	0.517$_{\pm 0.149}$	1.1$_{\pm 0.1}$	100.0	20.5$_{\pm 26.1}$	0.14$_{\pm 0.28}$	0.517$_{\pm 0.149}$	0.5$_{\pm 0.0}$
	MMD-CBS†	100.0	70.0$_{\pm 45.8}$	—	0.946$_{\pm 0.071}$	3.1$_{\pm 2.5}$	100.0	70.0$_{\pm 45.8}$	—	0.946$_{\pm 0.071}$	3.1$_{\pm 2.5}$	100.0	70.0$_{\pm 45.8}$	—	0.946$_{\pm 0.071}$	3.1$_{\pm 2.5}$
	MDOC†	75.0	—	—	0.572$_{\pm 0.400}$	39.0$_{\pm 6.1}$	75.0	—	—	0.572$_{\pm 0.400}$	34.8$_{\pm 4.3}$	75.0	—	—	0.572$_{\pm 0.400}$	39.6$_{\pm 4.8}$
	PCD-LB	65.0	12.3$_{\pm 20.2}$	0.02$_{\pm 0.05}$	0.325$_{\pm 0.135}$	2.8$_{\pm 0.8}$	35.0	4.6$_{\pm 11.9}$	0.01$_{\pm 0.02}$	0.316$_{\pm 0.133}$	4.2$_{\pm 0.9}$	25.0	3.3$_{\pm 9.1}$	0.01$_{\pm 0.01}$	0.307$_{\pm 0.130}$	3.2$_{\pm 0.6}$
	PCD-SHD	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.828$_{\pm 0.113}$	3.0$_{\pm 0.4}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.831$_{\pm 0.109}$	1.4$_{\pm 0.1}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.833$_{\pm 0.107}$	1.4$_{\pm 0.2}$
	Final Proj.	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.998**$_{\pm 0.003}$	3.7$_{\pm 0.3}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.998**$_{\pm 0.003}$	2.0$_{\pm 0.5}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.998**$_{\pm 0.003}$	3.7$_{\pm 0.5}$
	DiRecT	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.993$_{\pm 0.018}$	31.5$_{\pm 0.6}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.993$_{\pm 0.018}$	31.4$_{\pm 0.5}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.993$_{\pm 0.017}$	31.0$_{\pm 1.3}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.933$_{\pm 0.149}$	4.0$_{\pm 17.1}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.934$_{\pm 0.149}$	4.0$_{\pm 6.3}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.984$_{\pm 0.043}$	0.2$_{\pm 0.1}$
6	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	4.77$_{\pm 2.73}$	0.999$_{\pm 0.001}$	0.5	0.0	0.0$_{\pm 0.0}$	4.77$_{\pm 2.73}$	0.999$_{\pm 0.001}$	0.6	0.0	0.0$_{\pm 0.0}$	4.77$_{\pm 2.73}$	0.999$_{\pm 0.001}$	0.3
	CD†	100.0	0.0$_{\pm 0.0}$	0.96$_{\pm 0.85}$	0.582$_{\pm 0.079}$	1.1$_{\pm 0.1}$	100.0	0.0$_{\pm 0.0}$	0.96$_{\pm 0.85}$	0.582$_{\pm 0.079}$	0.9$_{\pm 0.3}$	100.0	0.2$_{\pm 0.7}$	0.96$_{\pm 0.85}$	0.582$_{\pm 0.079}$	1.1$_{\pm 0.1}$
	MMD-CBS†	80.0	15.0$_{\pm 35.7}$	—	0.879$_{\pm 0.065}$	30.3$_{\pm 21.2}$	80.0	15.0$_{\pm 35.7}$	—	0.879$_{\pm 0.065}$	30.3$_{\pm 21.2}$	80.0	15.0$_{\pm 35.7}$	—	0.879$_{\pm 0.065}$	30.3$_{\pm 21.2}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	57.6$_{\pm 4.8}$	0.0	—	—	0.000$_{\pm 0.000}$	58.2$_{\pm 4.8}$	0.0	—	—	0.000$_{\pm 0.000}$	67.8$_{\pm 5.2}$
	PCD-LB	5.0	0.1$_{\pm 0.3}$	1.01$_{\pm 0.73}$	0.299$_{\pm 0.050}$	3.8$_{\pm 0.7}$	0.0	0.0$_{\pm 0.0}$	1.04$_{\pm 0.72}$	0.300$_{\pm 0.056}$	5.8$_{\pm 1.1}$	0.0	0.0$_{\pm 0.0}$	1.08$_{\pm 0.69}$	0.302$_{\pm 0.060}$	2.3$_{\pm 0.3}$
	PCD-SHD	**100.0**	99.8$_{\pm 0.5}$	0.00$_{\pm 0.00}$	0.608$_{\pm 0.073}$	4.3$_{\pm 1.4}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.612$_{\pm 0.071}$	2.0$_{\pm 0.3}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.613$_{\pm 0.068}$	1.6$_{\pm 0.1}$
	Final Proj.	**100.0**	98.3$_{\pm 7.5}$	0.04$_{\pm 0.15}$	**0.995**$_{\pm 0.006}$	3.2$_{\pm 0.5}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.996**$_{\pm 0.005}$	1.8$_{\pm 0.5}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.996**$_{\pm 0.004}$	1.8$_{\pm 0.8}$
	DiRecT	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.985$_{\pm 0.017}$	27.6$_{\pm 3.6}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.985$_{\pm 0.016}$	24.6$_{\pm 5.9}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.985$_{\pm 0.016}$	24.0$_{\pm 6.4}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.975$_{\pm 0.090}$	4.4$_{\pm 22.0}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.976$_{\pm 0.090}$	4.3$_{\pm 4.2}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.960$_{\pm 0.089}$	0.2$_{\pm 0.1}$
9	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	9.22$_{\pm 3.01}$	0.999$_{\pm 0.001}$	0.6	0.0	0.0$_{\pm 0.0}$	9.22$_{\pm 3.01}$	0.999$_{\pm 0.001}$	0.6	0.0	0.0$_{\pm 0.0}$	9.22$_{\pm 3.01}$	0.999$_{\pm 0.001}$	0.6
	CD†	15.0	0.0$_{\pm 0.0}$	3.80$_{\pm 1.33}$	0.569$_{\pm 0.075}$	1.1$_{\pm 0.1}$	15.0	0.0$_{\pm 0.0}$	3.80$_{\pm 1.33}$	0.569$_{\pm 0.075}$	0.5$_{\pm 0.1}$	15.0	0.0$_{\pm 0.0}$	3.80$_{\pm 1.33}$	0.569$_{\pm 0.075}$	1.1$_{\pm 0.1}$
	MMD-CBS†	50.0	0.0$_{\pm 0.0}$	—	0.877$_{\pm 0.062}$	57.5$_{\pm 8.9}$	50.0	0.0$_{\pm 0.0}$	—	0.877$_{\pm 0.062}$	57.5$_{\pm 8.9}$	50.0	0.0$_{\pm 0.0}$	—	0.877$_{\pm 0.062}$	57.5$_{\pm 8.9}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	83.3$_{\pm 5.9}$	0.0	—	—	0.000$_{\pm 0.000}$	84.1$_{\pm 5.7}$	0.0	—	—	0.000$_{\pm 0.000}$	97.4$_{\pm 7.2}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	4.78$_{\pm 1.47}$	0.253$_{\pm 0.038}$	4.6$_{\pm 0.7}$	0.0	0.0$_{\pm 0.0}$	5.26$_{\pm 1.65}$	0.260$_{\pm 0.037}$	6.4$_{\pm 0.8}$	0.0	0.0$_{\pm 0.0}$	5.78$_{\pm 1.64}$	0.271$_{\pm 0.037}$	4.6$_{\pm 0.5}$
	PCD-SHD	**100.0**	94.8$_{\pm 8.7}$	0.05$_{\pm 0.09}$	0.547$_{\pm 0.052}$	5.4$_{\pm 0.9}$	**100.0**	96.2$_{\pm 5.0}$	0.04$_{\pm 0.05}$	0.557$_{\pm 0.052}$	2.2$_{\pm 0.2}$	**100.0**	97.5$_{\pm 3.8}$	0.02$_{\pm 0.04}$	0.562$_{\pm 0.052}$	1.9$_{\pm 0.1}$
	Final Proj.	**100.0**	99.4$_{\pm 2.1}$	0.01$_{\pm 0.04}$	**0.993**$_{\pm 0.006}$	4.1$_{\pm 0.5}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.995**$_{\pm 0.005}$	2.2$_{\pm 0.5}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.995**$_{\pm 0.004}$	2.1$_{\pm 0.8}$
	DiRecT	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.972$_{\pm 0.020}$	33.5$_{\pm 7.9}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.973$_{\pm 0.020}$	33.3$_{\pm 7.9}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.974$_{\pm 0.019}$	33.2$_{\pm 8.0}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.989$_{\pm 0.062}$	6.1$_{\pm 19.5}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.990$_{\pm 0.062}$	4.7$_{\pm 1.9}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.932$_{\pm 0.116}$	0.4$_{\pm 0.2}$
12	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	17.68$_{\pm 4.12}$	0.999$_{\pm 0.001}$	0.6	0.0	0.0$_{\pm 0.0}$	17.68$_{\pm 4.12}$	0.999$_{\pm 0.001}$	0.6	0.0	0.0$_{\pm 0.0}$	17.68$_{\pm 4.12}$	0.999$_{\pm 0.001}$	0.6
	CD†	0.0	0.0$_{\pm 0.0}$	8.90$_{\pm 2.05}$	0.555$_{\pm 0.045}$	0.5$_{\pm 0.0}$	0.0	0.0$_{\pm 0.0}$	8.90$_{\pm 2.05}$	0.555$_{\pm 0.045}$	1.1$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	8.90$_{\pm 2.05}$	0.555$_{\pm 0.045}$	0.5$_{\pm 0.0}$
	MMD-CBS†	0.0	0.0$_{\pm 0.0}$	—	0.961$_{\pm 0.022}$	67.5$_{\pm 4.9}$	0.0	0.0$_{\pm 0.0}$	—	0.961$_{\pm 0.022}$	67.5$_{\pm 4.9}$	0.0	0.0$_{\pm 0.0}$	—	0.961$_{\pm 0.022}$	67.5$_{\pm 4.9}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	110.0$_{\pm 8.6}$	0.0	—	—	0.000$_{\pm 0.000}$	111.2$_{\pm 8.5}$	0.0	—	—	0.000$_{\pm 0.000}$	128.9$_{\pm 10.2}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	11.68$_{\pm 1.53}$	0.259$_{\pm 0.038}$	5.8$_{\pm 0.8}$	0.0	0.0$_{\pm 0.0}$	13.11$_{\pm 1.52}$	0.265$_{\pm 0.038}$	8.0$_{\pm 1.0}$	0.0	0.0$_{\pm 0.0}$	14.36$_{\pm 1.63}$	0.279$_{\pm 0.040}$	3.1$_{\pm 0.4}$
	PCD-SHD	**100.0**	74.0$_{\pm 15.1}$	0.27$_{\pm 0.19}$	0.515$_{\pm 0.047}$	7.1$_{\pm 1.4}$	**100.0**	80.6$_{\pm 13.5}$	0.18$_{\pm 0.13}$	0.521$_{\pm 0.043}$	2.9$_{\pm 0.5}$	**100.0**	81.6$_{\pm 12.9}$	0.17$_{\pm 0.13}$	0.524$_{\pm 0.040}$	2.3$_{\pm 0.1}$
	Final Proj.	**100.0**	97.1$_{\pm 6.5}$	0.05$_{\pm 0.11}$	0.988$_{\pm 0.009}$	6.4$_{\pm 0.6}$	**100.0**	99.9$_{\pm 0.3}$	0.00$_{\pm 0.01}$	0.990$_{\pm 0.007}$	6.5$_{\pm 0.6}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.992**$_{\pm 0.006}$	3.0$_{\pm 0.8}$
	DiRecT	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.955$_{\pm 0.023}$	45.2$_{\pm 9.5}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.957$_{\pm 0.022}$	45.8$_{\pm 9.8}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.958$_{\pm 0.021}$	50.0$_{\pm 5.4}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.994**$_{\pm 0.024}$	9.7$_{\pm 31.5}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.996**$_{\pm 0.016}$	7.0$_{\pm 5.7}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.904$_{\pm 0.133}$	0.6$_{\pm 0.3}$
15	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	26.96$_{\pm 5.94}$	0.999$_{\pm 0.001}$	0.6	0.0	0.0$_{\pm 0.0}$	26.96$_{\pm 5.94}$	0.999$_{\pm 0.001}$	0.6	0.0	0.0$_{\pm 0.0}$	26.96$_{\pm 5.94}$	0.999$_{\pm 0.001}$	0.3
	CD†	0.0	0.0$_{\pm 0.0}$	15.53$_{\pm 2.54}$	0.587$_{\pm 0.059}$	1.2$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	15.53$_{\pm 2.54}$	0.587$_{\pm 0.059}$	0.9$_{\pm 0.3}$	0.0	0.0$_{\pm 0.0}$	15.53$_{\pm 2.54}$	0.587$_{\pm 0.059}$	1.2$_{\pm 0.1}$
	MMD-CBS†	0.0	0.0$_{\pm 0.0}$	—	0.989$_{\pm 0.013}$	74.4$_{\pm 9.3}$	0.0	0.0$_{\pm 0.0}$	—	0.989$_{\pm 0.013}$	74.4$_{\pm 9.3}$	0.0	0.0$_{\pm 0.0}$	—	0.989$_{\pm 0.013}$	74.4$_{\pm 9.3}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	139.2$_{\pm 10.6}$	0.0	—	—	0.000$_{\pm 0.000}$	139.0$_{\pm 10.6}$	0.0	—	—	0.000$_{\pm 0.000}$	153.1$_{\pm 12.1}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	20.89$_{\pm 2.48}$	0.280$_{\pm 0.038}$	6.8$_{\pm 0.9}$	0.0	0.0$_{\pm 0.0}$	23.55$_{\pm 2.73}$	0.284$_{\pm 0.037}$	9.2$_{\pm 0.7}$	0.0	0.0$_{\pm 0.0}$	26.34$_{\pm 2.90}$	0.300$_{\pm 0.037}$	6.8$_{\pm 0.4}$
	PCD-SHD	95.0	33.7$_{\pm 20.2}$	0.99$_{\pm 0.52}$	0.506$_{\pm 0.028}$	9.5$_{\pm 2.0}$	**100.0**	37.2$_{\pm 16.9}$	0.79$_{\pm 0.44}$	0.517$_{\pm 0.026}$	2.7$_{\pm 0.3}$	**100.0**	37.5$_{\pm 18.1}$	0.73$_{\pm 0.43}$	0.523$_{\pm 0.027}$	2.7$_{\pm 0.1}$
	Final Proj.	**100.0**	82.8$_{\pm 26.1}$	0.60$_{\pm 1.61}$	0.975$_{\pm 0.013}$	5.2$_{\pm 0.4}$	**100.0**	94.7$_{\pm 13.7}$	0.13$_{\pm 0.38}$	0.979$_{\pm 0.011}$	8.5$_{\pm 2.0}$	**100.0**	98.9$_{\pm 3.4}$	0.02$_{\pm 0.07}$	**0.982**$_{\pm 0.010}$	4.2$_{\pm 0.8}$
	DiRecT	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.926$_{\pm 0.028}$	61.9$_{\pm 11.6}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.930$_{\pm 0.025}$	68.1$_{\pm 0.6}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.932$_{\pm 0.024}$	46.4$_{\pm 18.9}$
	GlobMP (Ours)	95.0	95.0$_{\pm 21.8}$	1.65$_{\pm 7.19}$	**0.983**$_{\pm 0.043}$	24.5$_{\pm 61.2}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.987**$_{\pm 0.033}$	14.6$_{\pm 40.9}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.905$_{\pm 0.134}$	1.1$_{\pm 1.6}$
18	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	39.06$_{\pm 6.29}$	0.999$_{\pm 0.001}$	0.7	0.0	0.0$_{\pm 0.0}$	39.06$_{\pm 6.29}$	0.999$_{\pm 0.001}$	0.7	0.0	0.0$_{\pm 0.0}$	39.06$_{\pm 6.29}$	0.999$_{\pm 0.001}$	0.7
	CD†	0.0	0.0$_{\pm 0.0}$	22.94$_{\pm 1.70}$	0.565$_{\pm 0.049}$	1.2$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	22.94$_{\pm 1.70}$	0.565$_{\pm 0.049}$	0.6$_{\pm 0.0}$	0.0	0.0$_{\pm 0.0}$	22.94$_{\pm 1.70}$	0.565$_{\pm 0.049}$	1.2$_{\pm 0.1}$
	MMD-CBS†	0.0	0.0$_{\pm 0.0}$	—	0.996$_{\pm 0.009}$	81.6$_{\pm 17.1}$	0.0	0.0$_{\pm 0.0}$	—	0.996$_{\pm 0.009}$	81.6$_{\pm 17.1}$	0.0	0.0$_{\pm 0.0}$	—	0.996$_{\pm 0.009}$	81.6$_{\pm 17.1}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	171.7$_{\pm 13.7}$	0.0	—	—	0.000$_{\pm 0.000}$	172.6$_{\pm 14.8}$	0.0	—	—	0.000$_{\pm 0.000}$	185.6$_{\pm 17.7}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	29.66$_{\pm 2.84}$	0.271$_{\pm 0.038}$	8.1$_{\pm 0.9}$	0.0	0.0$_{\pm 0.0}$	33.68$_{\pm 2.86}$	0.276$_{\pm 0.037}$	11.0$_{\pm 1.2}$	0.0	0.0$_{\pm 0.0}$	38.10$_{\pm 2.86}$	0.289$_{\pm 0.037}$	4.1$_{\pm 0.5}$
	PCD-SHD	80.0	7.1$_{\pm 7.0}$	2.21$_{\pm 0.82}$	0.485$_{\pm 0.034}$	5.2$_{\pm 1.0}$	85.0	11.2$_{\pm 9.5}$	1.83$_{\pm 0.78}$	0.494$_{\pm 0.035}$	3.3$_{\pm 0.5}$	85.0	9.9$_{\pm 8.8}$	1.69$_{\pm 0.73}$	0.502$_{\pm 0.036}$	3.0$_{\pm 0.2}$
	Final Proj.	95.0	67.3$_{\pm 36.5}$	1.60$_{\pm 2.70}$	0.966$_{\pm 0.015}$	13.6$_{\pm 0.6}$	**100.0**	85.0$_{\pm 28.5}$	0.45$_{\pm 0.95}$	0.971$_{\pm 0.012}$	10.2$_{\pm 3.0}$	**100.0**	95.4$_{\pm 11.3}$	0.08$_{\pm 0.24}$	**0.975**$_{\pm 0.011}$	5.8$_{\pm 0.8}$
	DiRecT	**100.0**	99.1$_{\pm 4.1}$	0.00$_{\pm 0.00}$	0.901$_{\pm 0.025}$	38.0$_{\pm 0.6}$	**100.0**	99.1$_{\pm 3.7}$	0.00$_{\pm 0.00}$	0.907$_{\pm 0.024}$	38.2$_{\pm 0.6}$	**100.0**	99.2$_{\pm 3.4}$	**0.00**$_{\pm 0.00}$	0.911$_{\pm 0.023}$	38.2$_{\pm 0.6}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.977**$_{\pm 0.047}$	27.5$_{\pm 51.1}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.984**$_{\pm 0.034}$	15.0$_{\pm 25.0}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.876$_{\pm 0.161}$	1.6$_{\pm 1.9}$
21	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	60.02$_{\pm 7.50}$	0.999$_{\pm 0.001}$	0.7	0.0	0.0$_{\pm 0.0}$	60.02$_{\pm 7.50}$	0.999$_{\pm 0.001}$	0.3	0.0	0.0$_{\pm 0.0}$	60.02$_{\pm 7.50}$	0.999$_{\pm 0.001}$	0.7
	CD†	0.0	0.0$_{\pm 0.0}$	35.83$_{\pm 3.71}$	0.587$_{\pm 0.041}$	1.3$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	35.83$_{\pm 3.71}$	0.587$_{\pm 0.041}$	1.2$_{\pm 0.0}$	0.0	0.0$_{\pm 0.0}$	35.83$_{\pm 3.71}$	0.587$_{\pm 0.041}$	1.3$_{\pm 0.1}$
	MMD-CBS†	0.0	0.0$_{\pm 0.0}$	—	1.000$_{\pm 0.000}$	82.4$_{\pm 9.9}$	0.0	0.0$_{\pm 0.0}$	—	1.000$_{\pm 0.000}$	82.4$_{\pm 9.9}$	0.0	0.0$_{\pm 0.0}$	—	1.000$_{\pm 0.000}$	82.4$_{\pm 9.9}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	193.5$_{\pm 25.2}$	0.0	—	—	0.000$_{\pm 0.000}$	192.8$_{\pm 25.2}$	0.0	—	—	0.000$_{\pm 0.000}$	217.6$_{\pm 36.3}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	45.06$_{\pm 2.97}$	0.255$_{\pm 0.026}$	9.9$_{\pm 1.5}$	0.0	0.0$_{\pm 0.0}$	51.43$_{\pm 2.86}$	0.260$_{\pm 0.022}$	13.2$_{\pm 1.8}$	0.0	0.0$_{\pm 0.0}$	57.66$_{\pm 3.16}$	0.273$_{\pm 0.020}$	9.3$_{\pm 0.8}$
	PCD-SHD	10.0	0.2$_{\pm 0.7}$	5.13$_{\pm 1.31}$	0.467$_{\pm 0.028}$	6.8$_{\pm 1.5}$	15.0	0.2$_{\pm 0.6}$	4.10$_{\pm 1.05}$	0.480$_{\pm 0.024}$	4.3$_{\pm 0.6}$	10.0	0.4$_{\pm 1.2}$	3.64$_{\pm 0.92}$	0.491$_{\pm 0.023}$	3.6$_{\pm 0.2}$
	Final Proj.	65.0	19.2$_{\pm 27.0}$	7.83$_{\pm 6.74}$	0.940$_{\pm 0.020}$	10.8$_{\pm 2.2}$	90.0	45.0$_{\pm 35.2}$	3.88$_{\pm 4.47}$	0.948$_{\pm 0.018}$	14.5$_{\pm 4.3}$	**100.0**	62.4$_{\pm 34.9}$	1.70$_{\pm 2.11}$	**0.954**$_{\pm 0.016}$	7.9$_{\pm 0.8}$
	DiRecT	**100.0**	**99.8**$_{\pm 0.7}$	**0.00**$_{\pm 0.00}$	0.857$_{\pm 0.033}$	99.1$_{\pm 31.9}$	**100.0**	**99.9**$_{\pm 0.3}$	**0.00**$_{\pm 0.00}$	0.867$_{\pm 0.031}$	91.9$_{\pm 33.8}$	**100.0**	99.9$_{\pm 0.3}$	**0.00**$_{\pm 0.00}$	0.873$_{\pm 0.028}$	85.6$_{\pm 34.6}$
	GlobMP (Ours)	75.0	75.0$_{\pm 43.3}$	17.15$_{\pm 30.27}$	**0.972**$_{\pm 0.055}$	72.7$_{\pm 81.0}$	95.0	95.0$_{\pm 21.8}$	4.50$_{\pm 19.62}$	**0.972**$_{\pm 0.050}$	31.2$_{\pm 46.6}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.839$_{\pm 0.185}$	3.1$_{\pm 3.8}$

Table 4: Multi robot planning performance on `Basic` for $N_a \in \{3, 6, \ldots, 21\}$ under the Low, Medium, and High velocity caps. *SR* and *CS* measure trajectory feasibility, *Coll* reports inter robot collisions, *DA* measures adherence to the environment specific motion pattern, and *Time* is the planning time per configuration. † marks methods that apply no velocity limit, so their three blocks report the same run; in addition, MMD-CBS returns a single plan per configuration rather than a batch of 64 candidates. A dash under *Coll* marks a count that is not commensurable with the others: MDOC and MMD-CBS count colliding time steps, where every other row counts agent pairs with any collision. MDOC's *CS* is omitted because its plans that do not cover every agent were counted as feasible. MMD-CBS plans under a 60 s budget per configuration and GlobMP (Ours) under 300 s per team. For GlobMP (Ours), *Time* is the median over the configurations and its subscript the standard deviation; for the baselines it is the mean, so the two are not directly comparable.

		Low ($v_{\max} = 0.477$)					Medium ($v_{\max} = 0.567$)					High ($v_{\max} = 0.675$)				
N_a	Method	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓
3	Diff. Policy†	30.0	13.0$_{\pm 29.6}$	0.35$_{\pm 0.64}$	0.998$_{\pm 0.001}$	0.3	30.0	13.0$_{\pm 29.6}$	0.35$_{\pm 0.64}$	0.998$_{\pm 0.001}$	0.3	30.0	13.0$_{\pm 29.6}$	0.35$_{\pm 0.64}$	0.998$_{\pm 0.001}$	0.3
	CD†	80.0	0.0$_{\pm 0.0}$	0.05$_{\pm 0.08}$	0.611$_{\pm 0.138}$	1.1$_{\pm 0.0}$	80.0	0.0$_{\pm 0.0}$	0.05$_{\pm 0.08}$	0.611$_{\pm 0.138}$	0.4$_{\pm 0.0}$	80.0	0.4$_{\pm 1.0}$	0.05$_{\pm 0.08}$	0.611$_{\pm 0.138}$	1.1$_{\pm 0.1}$
	MMD-CBS†	70.0	35.0$_{\pm 47.7}$	—	0.895$_{\pm 0.113}$	2.5$_{\pm 1.7}$	70.0	50.0$_{\pm 50.0}$	—	0.895$_{\pm 0.113}$	2.5$_{\pm 1.7}$	70.0	55.0$_{\pm 49.7}$	—	0.895$_{\pm 0.113}$	2.5$_{\pm 1.7}$
	MDOC†	10.0	—	—	0.090$_{\pm 0.269}$	33.4$_{\pm 4.5}$	10.0	—	—	0.090$_{\pm 0.269}$	33.6$_{\pm 4.4}$	10.0	—	—	0.090$_{\pm 0.269}$	33.5$_{\pm 4.6}$
	PCD-LB	40.0	5.2$_{\pm 10.1}$	0.01$_{\pm 0.03}$	0.361$_{\pm 0.114}$	5.2$_{\pm 1.6}$	35.0	2.2$_{\pm 3.6}$	0.01$_{\pm 0.03}$	0.349$_{\pm 0.112}$	4.4$_{\pm 0.6}$	20.0	2.1$_{\pm 6.1}$	0.01$_{\pm 0.03}$	0.336$_{\pm 0.106}$	3.6$_{\pm 0.4}$
	PCD-SHD	70.0	61.2$_{\pm 43.9}$	0.06$_{\pm 0.14}$	0.847$_{\pm 0.120}$	3.1$_{\pm 0.2}$	70.0	61.1$_{\pm 43.9}$	0.06$_{\pm 0.14}$	0.848$_{\pm 0.118}$	1.5$_{\pm 0.1}$	70.0	60.7$_{\pm 43.8}$	0.05$_{\pm 0.12}$	0.849$_{\pm 0.118}$	3.0$_{\pm 0.1}$
	Final Proj.	**100.0**	95.1$_{\pm 21.5}$	**0.00**$_{\pm 0.00}$	**0.964**$_{\pm 0.050}$	4.0$_{\pm 0.7}$	**100.0**	95.1$_{\pm 21.5}$	**0.00**$_{\pm 0.00}$	**0.969**$_{\pm 0.043}$	4.0$_{\pm 0.8}$	**100.0**	95.1$_{\pm 21.5}$	**0.00**$_{\pm 0.00}$	0.973$_{\pm 0.037}$	2.2$_{\pm 0.8}$
	DiRecT	**100.0**	99.8$_{\pm 1.0}$	**0.00**$_{\pm 0.00}$	0.925$_{\pm 0.098}$	26.3$_{\pm 6.4}$	**100.0**	95.6$_{\pm 19.1}$	**0.00**$_{\pm 0.00}$	0.930$_{\pm 0.093}$	26.8$_{\pm 6.5}$	**100.0**	95.5$_{\pm 19.4}$	**0.00**$_{\pm 0.00}$	0.931$_{\pm 0.092}$	27.7$_{\pm 7.3}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.878$_{\pm 0.169}$	2.1$_{\pm 2.0}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.851$_{\pm 0.184}$	0.2$_{\pm 2.0}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.987**$_{\pm 0.026}$	0.0$_{\pm 0.1}$
6	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	3.63$_{\pm 1.90}$	0.998$_{\pm 0.001}$	0.3	0.0	0.0$_{\pm 0.0}$	3.63$_{\pm 1.90}$	0.998$_{\pm 0.001}$	0.4	0.0	0.0$_{\pm 0.0}$	3.63$_{\pm 1.90}$	0.998$_{\pm 0.001}$	0.5
	CD†	45.0	0.0$_{\pm 0.0}$	1.14$_{\pm 0.58}$	0.633$_{\pm 0.079}$	1.1$_{\pm 0.1}$	45.0	0.0$_{\pm 0.0}$	1.14$_{\pm 0.58}$	0.633$_{\pm 0.079}$	1.2$_{\pm 0.0}$	45.0	0.0$_{\pm 0.0}$	1.14$_{\pm 0.58}$	0.633$_{\pm 0.079}$	1.1$_{\pm 0.1}$
	MMD-CBS†	45.0	0.0$_{\pm 0.0}$	—	0.797$_{\pm 0.102}$	9.3$_{\pm 9.6}$	45.0	5.0$_{\pm 21.8}$	—	0.797$_{\pm 0.102}$	9.3$_{\pm 9.6}$	45.0	10.0$_{\pm 30.0}$	—	0.797$_{\pm 0.102}$	9.3$_{\pm 9.6}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	90.6$_{\pm 21.6}$	0.0	—	—	0.000$_{\pm 0.000}$	69.7$_{\pm 5.3}$	0.0	—	—	0.000$_{\pm 0.000}$	77.3$_{\pm 14.4}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	0.69$_{\pm 0.50}$	0.360$_{\pm 0.064}$	6.4$_{\pm 0.9}$	0.0	0.0$_{\pm 0.0}$	0.72$_{\pm 0.44}$	0.347$_{\pm 0.056}$	4.9$_{\pm 0.5}$	0.0	0.0$_{\pm 0.0}$	0.78$_{\pm 0.43}$	0.336$_{\pm 0.051}$	1.8$_{\pm 0.1}$
	PCD-SHD	45.0	22.2$_{\pm 33.5}$	0.46$_{\pm 0.35}$	0.773$_{\pm 0.084}$	3.3$_{\pm 0.2}$	50.0	21.4$_{\pm 33.1}$	0.45$_{\pm 0.33}$	0.777$_{\pm 0.082}$	3.2$_{\pm 0.1}$	60.0	20.8$_{\pm 31.9}$	0.44$_{\pm 0.31}$	0.780$_{\pm 0.079}$	3.1$_{\pm 0.1}$
	Final Proj.	90.0	85.6$_{\pm 32.3}$	0.09$_{\pm 0.33}$	**0.959**$_{\pm 0.031}$	4.3$_{\pm 0.8}$	90.0	88.6$_{\pm 29.9}$	0.02$_{\pm 0.06}$	**0.965**$_{\pm 0.026}$	4.4$_{\pm 0.7}$	30.0	9.1$_{\pm 22.6}$	0.08$_{\pm 0.11}$	**0.983**$_{\pm 0.011}$	4.2$_{\pm 0.6}$
	DiRecT	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.919$_{\pm 0.059}$	30.6$_{\pm 7.9}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.921$_{\pm 0.058}$	29.4$_{\pm 7.8}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.922$_{\pm 0.057}$	30.9$_{\pm 8.8}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.932$_{\pm 0.141}$	4.2$_{\pm 18.1}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.934$_{\pm 0.137}$	4.2$_{\pm 5.0}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.980$_{\pm 0.041}$	0.2$_{\pm 0.1}$
9	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	6.58$_{\pm 2.27}$	0.999$_{\pm 0.001}$	0.3	0.0	0.0$_{\pm 0.0}$	6.58$_{\pm 2.27}$	0.999$_{\pm 0.001}$	0.3	0.0	0.0$_{\pm 0.0}$	6.58$_{\pm 2.27}$	0.999$_{\pm 0.001}$	0.3
	CD†	0.0	0.0$_{\pm 0.0}$	2.86$_{\pm 1.16}$	0.630$_{\pm 0.065}$	0.5$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	2.86$_{\pm 1.16}$	0.630$_{\pm 0.065}$	1.2$_{\pm 0.0}$	0.0	0.0$_{\pm 0.0}$	2.86$_{\pm 1.16}$	0.630$_{\pm 0.065}$	1.1$_{\pm 0.1}$
	MMD-CBS†	0.0	0.0$_{\pm 0.0}$	—	0.756$_{\pm 0.082}$	15.1$_{\pm 23.5}$	0.0	0.0$_{\pm 0.0}$	—	0.756$_{\pm 0.082}$	15.1$_{\pm 23.5}$	0.0	0.0$_{\pm 0.0}$	—	0.756$_{\pm 0.082}$	15.1$_{\pm 23.5}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	92.2$_{\pm 17.8}$	0.0	—	—	0.000$_{\pm 0.000}$	81.2$_{\pm 17.7}$	0.0	—	—	0.000$_{\pm 0.000}$	74.6$_{\pm 5.1}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	2.85$_{\pm 1.07}$	0.365$_{\pm 0.072}$	7.4$_{\pm 0.5}$	0.0	0.0$_{\pm 0.0}$	3.06$_{\pm 1.03}$	0.365$_{\pm 0.065}$	5.6$_{\pm 0.4}$	0.0	0.0$_{\pm 0.0}$	3.27$_{\pm 0.98}$	0.361$_{\pm 0.061}$	4.4$_{\pm 0.3}$
	PCD-SHD	5.0	0.3$_{\pm 1.4}$	0.81$_{\pm 0.55}$	0.735$_{\pm 0.061}$	3.6$_{\pm 0.1}$	5.0	0.5$_{\pm 2.0}$	0.81$_{\pm 0.53}$	0.738$_{\pm 0.060}$	3.2$_{\pm 0.6}$	5.0	0.4$_{\pm 1.7}$	0.84$_{\pm 0.50}$	0.740$_{\pm 0.058}$	2.8$_{\pm 0.8}$
	Final Proj.	85.0	76.3$_{\pm 38.4}$	0.11$_{\pm 0.24}$	0.949$_{\pm 0.027}$	5.7$_{\pm 0.7}$	85.0	78.4$_{\pm 38.7}$	0.06$_{\pm 0.21}$	0.957$_{\pm 0.023}$	3.1$_{\pm 0.7}$	85.0	79.0$_{\pm 38.6}$	0.05$_{\pm 0.20}$	0.962$_{\pm 0.020}$	5.8$_{\pm 0.7}$
	DiRecT	**100.0**	99.0$_{\pm 4.4}$	0.01$_{\pm 0.04}$	0.903$_{\pm 0.050}$	46.6$_{\pm 13.9}$	95.0	93.9$_{\pm 22.0}$	0.01$_{\pm 0.04}$	0.902$_{\pm 0.051}$	47.1$_{\pm 14.1}$	95.0	94.0$_{\pm 21.9}$	0.01$_{\pm 0.04}$	0.907$_{\pm 0.046}$	42.6$_{\pm 12.5}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.969**$_{\pm 0.100}$	4.4$_{\pm 8.8}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.969**$_{\pm 0.099}$	4.3$_{\pm 1.2}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.972**$_{\pm 0.055}$	0.3$_{\pm 0.1}$
12	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	13.12$_{\pm 3.83}$	0.999$_{\pm 0.001}$	0.3	0.0	0.0$_{\pm 0.0}$	13.12$_{\pm 3.83}$	0.999$_{\pm 0.001}$	0.3	0.0	0.0$_{\pm 0.0}$	13.12$_{\pm 3.83}$	0.999$_{\pm 0.001}$	0.3
	CD†	0.0	0.0$_{\pm 0.0}$	7.58$_{\pm 1.54}$	0.638$_{\pm 0.033}$	1.2$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	7.58$_{\pm 1.54}$	0.638$_{\pm 0.033}$	0.5$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	7.58$_{\pm 1.54}$	0.638$_{\pm 0.033}$	1.2$_{\pm 0.0}$
	MMD-CBS†	0.0	0.0$_{\pm 0.0}$	—	0.797$_{\pm 0.027}$	9.5$_{\pm 18.8}$	0.0	0.0$_{\pm 0.0}$	—	0.797$_{\pm 0.027}$	9.5$_{\pm 18.8}$	0.0	0.0$_{\pm 0.0}$	—	0.797$_{\pm 0.027}$	9.5$_{\pm 18.8}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	104.7$_{\pm 8.5}$	0.0	—	—	0.000$_{\pm 0.000}$	113.0$_{\pm 6.5}$	0.0	—	—	0.000$_{\pm 0.000}$	109.8$_{\pm 6.6}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	7.89$_{\pm 1.38}$	0.340$_{\pm 0.044}$	8.6$_{\pm 0.8}$	0.0	0.0$_{\pm 0.0}$	8.44$_{\pm 1.23}$	0.336$_{\pm 0.044}$	3.1$_{\pm 0.3}$	0.0	0.0$_{\pm 0.0}$	8.87$_{\pm 1.18}$	0.334$_{\pm 0.043}$	4.9$_{\pm 0.4}$
	PCD-SHD	0.0	0.0$_{\pm 0.0}$	1.98$_{\pm 0.70}$	0.713$_{\pm 0.052}$	3.9$_{\pm 0.2}$	0.0	0.0$_{\pm 0.0}$	1.93$_{\pm 0.69}$	0.718$_{\pm 0.051}$	3.7$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	1.97$_{\pm 0.65}$	0.721$_{\pm 0.049}$	3.8$_{\pm 0.1}$
	Final Proj.	60.0	38.0$_{\pm 42.5}$	0.72$_{\pm 1.00}$	0.960$_{\pm 0.021}$	6.6$_{\pm 0.7}$	60.0	40.6$_{\pm 42.5}$	0.36$_{\pm 0.53}$	0.966$_{\pm 0.018}$	4.6$_{\pm 1.5}$	60.0	43.6$_{\pm 43.7}$	0.28$_{\pm 0.38}$	**0.971**$_{\pm 0.015}$	6.5$_{\pm 0.7}$
	DiRecT	**100.0**	99.2$_{\pm 2.1}$	0.00$_{\pm 0.01}$	0.905$_{\pm 0.047}$	41.5$_{\pm 0.9}$	95.0	82.7$_{\pm 35.4}$	0.00$_{\pm 0.01}$	0.906$_{\pm 0.046}$	41.4$_{\pm 0.9}$	85.0	79.8$_{\pm 39.7}$	0.04$_{\pm 0.15}$	0.915$_{\pm 0.042}$	42.4$_{\pm 1.4}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.983**$_{\pm 0.084}$	5.2$_{\pm 13.2}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.984**$_{\pm 0.078}$	4.7$_{\pm 1.0}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.968$_{\pm 0.065}$	0.4$_{\pm 0.2}$
15	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	22.25$_{\pm 4.65}$	0.999$_{\pm 0.001}$	0.3	0.0	0.0$_{\pm 0.0}$	22.25$_{\pm 4.65}$	0.999$_{\pm 0.001}$	0.3	0.0	0.0$_{\pm 0.0}$	22.25$_{\pm 4.65}$	0.999$_{\pm 0.001}$	0.4
	CD†	0.0	0.0$_{\pm 0.0}$	13.94$_{\pm 2.28}$	0.625$_{\pm 0.038}$	1.2$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	13.94$_{\pm 2.28}$	0.625$_{\pm 0.038}$	1.2$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	13.94$_{\pm 2.28}$	0.625$_{\pm 0.038}$	1.2$_{\pm 0.0}$
	MMD-CBS†	0.0	0.0$_{\pm 0.0}$	—	—	6.6$_{\pm 3.5}$	0.0	0.0$_{\pm 0.0}$	—	—	6.6$_{\pm 3.5}$	0.0	0.0$_{\pm 0.0}$	—	—	6.6$_{\pm 3.5}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	150.5$_{\pm 11.2}$	0.0	—	—	0.000$_{\pm 0.000}$	165.2$_{\pm 13.5}$	0.0	—	—	0.000$_{\pm 0.000}$	160.4$_{\pm 31.5}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	14.53$_{\pm 2.15}$	0.324$_{\pm 0.037}$	9.2$_{\pm 1.2}$	0.0	0.0$_{\pm 0.0}$	15.91$_{\pm 2.00}$	0.320$_{\pm 0.032}$	6.8$_{\pm 0.6}$	0.0	0.0$_{\pm 0.0}$	17.16$_{\pm 1.99}$	0.321$_{\pm 0.026}$	5.3$_{\pm 0.4}$
	PCD-SHD	0.0	0.0$_{\pm 0.0}$	3.93$_{\pm 1.17}$	0.674$_{\pm 0.039}$	4.3$_{\pm 0.2}$	0.0	0.0$_{\pm 0.0}$	3.89$_{\pm 1.14}$	0.679$_{\pm 0.038}$	4.1$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	3.84$_{\pm 1.12}$	0.682$_{\pm 0.037}$	4.2$_{\pm 0.2}$
	Final Proj.	75.0	44.5$_{\pm 40.5}$	1.35$_{\pm 1.67}$	0.946$_{\pm 0.027}$	5.2$_{\pm 0.5}$	85.0	54.4$_{\pm 41.8}$	0.79$_{\pm 1.19}$	0.954$_{\pm 0.023}$	9.6$_{\pm 0.6}$	90.0	60.1$_{\pm 41.6}$	0.50$_{\pm 0.81}$	**0.960**$_{\pm 0.020}$	6.0$_{\pm 2.1}$
	DiRecT	**100.0**	97.7$_{\pm 4.9}$	0.03$_{\pm 0.05}$	0.887$_{\pm 0.041}$	64.3$_{\pm 2.3}$	**100.0**	92.0$_{\pm 20.0}$	0.02$_{\pm 0.05}$	0.890$_{\pm 0.039}$	71.4$_{\pm 11.9}$	95.0	91.3$_{\pm 22.3}$	0.05$_{\pm 0.12}$	0.895$_{\pm 0.038}$	72.6$_{\pm 13.9}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**1.000**$_{\pm 0.001}$	5.7$_{\pm 29.2}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**1.000**$_{\pm 0.000}$	5.3$_{\pm 11.8}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.959$_{\pm 0.082}$	0.9$_{\pm 0.3}$
18	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	28.87$_{\pm 4.75}$	0.999$_{\pm 0.001}$	0.4	0.0	0.0$_{\pm 0.0}$	28.87$_{\pm 4.75}$	0.999$_{\pm 0.001}$	0.5	0.0	0.0$_{\pm 0.0}$	28.87$_{\pm 4.75}$	0.999$_{\pm 0.001}$	0.4
	CD†	0.0	0.0$_{\pm 0.0}$	21.88$_{\pm 2.51}$	0.642$_{\pm 0.035}$	0.6$_{\pm 0.2}$	0.0	0.0$_{\pm 0.0}$	21.88$_{\pm 2.51}$	0.642$_{\pm 0.035}$	1.3$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	21.88$_{\pm 2.51}$	0.642$_{\pm 0.035}$	1.3$_{\pm 0.0}$
	MMD-CBS†	0.0	0.0$_{\pm 0.0}$	—	0.829$_{\pm 0.000}$	9.2$_{\pm 15.5}$	0.0	0.0$_{\pm 0.0}$	—	0.829$_{\pm 0.000}$	9.2$_{\pm 15.5}$	0.0	0.0$_{\pm 0.0}$	—	0.829$_{\pm 0.000}$	9.2$_{\pm 15.5}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	149.1$_{\pm 10.6}$	0.0	—	—	0.000$_{\pm 0.000}$	143.3$_{\pm 7.8}$	0.0	—	—	0.000$_{\pm 0.000}$	142.9$_{\pm 7.8}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	22.51$_{\pm 2.98}$	0.333$_{\pm 0.034}$	9.3$_{\pm 0.9}$	0.0	0.0$_{\pm 0.0}$	24.71$_{\pm 2.90}$	0.337$_{\pm 0.031}$	3.6$_{\pm 0.3}$	0.0	0.0$_{\pm 0.0}$	27.06$_{\pm 2.84}$	0.342$_{\pm 0.032}$	5.5$_{\pm 0.4}$
	PCD-SHD	0.0	0.0$_{\pm 0.0}$	6.51$_{\pm 1.58}$	0.683$_{\pm 0.043}$	3.9$_{\pm 0.9}$	0.0	0.0$_{\pm 0.0}$	6.28$_{\pm 1.53}$	0.688$_{\pm 0.043}$	4.2$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	6.19$_{\pm 1.47}$	0.691$_{\pm 0.043}$	4.2$_{\pm 0.1}$
	Final Proj.	65.0	19.4$_{\pm 30.9}$	2.66$_{\pm 2.15}$	0.941$_{\pm 0.029}$	12.1$_{\pm 0.7}$	65.0	22.1$_{\pm 32.5}$	1.84$_{\pm 1.67}$	0.949$_{\pm 0.025}$	12.0$_{\pm 0.6}$	65.0	24.5$_{\pm 32.3}$	1.48$_{\pm 1.35}$	**0.956**$_{\pm 0.021}$	7.3$_{\pm 2.0}$
	DiRecT	**100.0**	92.6$_{\pm 11.6}$	0.06$_{\pm 0.11}$	0.881$_{\pm 0.045}$	93.6$_{\pm 18.4}$	90.0	78.4$_{\pm 36.7}$	0.21$_{\pm 0.39}$	0.888$_{\pm 0.044}$	92.6$_{\pm 16.9}$	90.0	77.4$_{\pm 37.0}$	0.31$_{\pm 0.60}$	0.893$_{\pm 0.042}$	84.1$_{\pm 7.1}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**1.000**$_{\pm 0.000}$	6.4$_{\pm 22.9}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**1.000**$_{\pm 0.000}$	5.3$_{\pm 4.2}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.953$_{\pm 0.079}$	1.0$_{\pm 0.5}$
21	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	42.00$_{\pm 5.66}$	0.999$_{\pm 0.001}$	0.4	0.0	0.0$_{\pm 0.0}$	42.00$_{\pm 5.66}$	0.999$_{\pm 0.001}$	0.4	0.0	0.0$_{\pm 0.0}$	42.00$_{\pm 5.66}$	0.999$_{\pm 0.001}$	0.8
	CD†	0.0	0.0$_{\pm 0.0}$	31.60$_{\pm 2.41}$	0.636$_{\pm 0.033}$	1.3$_{\pm 0.0}$	0.0	0.0$_{\pm 0.0}$	31.60$_{\pm 2.41}$	0.636$_{\pm 0.033}$	1.3$_{\pm 0.0}$	0.0	0.0$_{\pm 0.0}$	31.60$_{\pm 2.41}$	0.636$_{\pm 0.033}$	1.3$_{\pm 0.1}$
	MMD-CBS†	0.0	0.0$_{\pm 0.0}$	—	—	4.4$_{\pm 2.8}$	0.0	0.0$_{\pm 0.0}$	—	—	4.4$_{\pm 2.8}$	0.0	0.0$_{\pm 0.0}$	—	—	4.4$_{\pm 2.8}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	165.5$_{\pm 10.3}$	0.0	—	—	0.000$_{\pm 0.000}$	162.1$_{\pm 8.8}$	0.0	—	—	0.000$_{\pm 0.000}$	162.7$_{\pm 8.7}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	32.93$_{\pm 3.29}$	0.337$_{\pm 0.031}$	11.2$_{\pm 1.0}$	0.0	0.0$_{\pm 0.0}$	36.75$_{\pm 3.00}$	0.335$_{\pm 0.026}$	8.2$_{\pm 0.6}$	0.0	0.0$_{\pm 0.0}$	40.37$_{\pm 3.01}$	0.338$_{\pm 0.025}$	6.4$_{\pm 0.3}$
	PCD-SHD	0.0	0.0$_{\pm 0.0}$	8.70$_{\pm 1.44}$	0.662$_{\pm 0.030}$	5.0$_{\pm 0.3}$	0.0	0.0$_{\pm 0.0}$	8.63$_{\pm 1.36}$	0.668$_{\pm 0.028}$	4.6$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	8.37$_{\pm 1.38}$	0.672$_{\pm 0.026}$	4.6$_{\pm 0.1}$
	Final Proj.	25.0	7.7$_{\pm 20.9}$	4.65$_{\pm 4.26}$	0.946$_{\pm 0.026}$	15.1$_{\pm 0.6}$	40.0	8.7$_{\pm 21.3}$	3.21$_{\pm 2.92}$	0.954$_{\pm 0.023}$	11.8$_{\pm 3.7}$	35.0	9.5$_{\pm 21.0}$	2.42$_{\pm 2.03}$	**0.960**$_{\pm 0.020}$	15.1$_{\pm 0.7}$
	DiRecT	**100.0**	93.9$_{\pm 8.7}$	**0.06**$_{\pm 0.10}$	0.875$_{\pm 0.039}$	109.4$_{\pm 9.7}$	95.0	60.2$_{\pm 38.2}$	0.11$_{\pm 0.13}$	0.880$_{\pm 0.038}$	101.1$_{\pm 21.0}$	85.0	54.4$_{\pm 43.5}$	0.64$_{\pm 0.70}$	0.894$_{\pm 0.040}$	78.0$_{\pm 0.8}$
	GlobMP (Ours)	95.0	**95.0**$_{\pm 21.8}$	2.35$_{\pm 10.24}$	**1.000**$_{\pm 0.003}$	9.7$_{\pm 47.9}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**1.000**$_{\pm 0.002}$	6.3$_{\pm 3.8}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.945$_{\pm 0.098}$	1.5$_{\pm 2.5}$

Table 5: Multi robot planning performance on `Dense` for $N_a \in \{3, 6, \ldots, 21\}$ under the Low, Medium, and High velocity caps. *SR* and *CS* measure trajectory feasibility, *Coll* reports inter robot collisions, *DA* measures adherence to the environment specific motion pattern, and *Time* is the planning time per configuration. † marks methods that apply no velocity limit, so their three blocks report the same run; in addition, MMD-CBS returns a single plan per configuration rather than a batch of 64 candidates. A dash under *Coll* marks a count that is not commensurable with the others: MDOC and MMD-CBS count colliding time steps, where every other row counts agent pairs with any collision. MDOC's *CS* is omitted because its plans that do not cover every agent were counted as feasible. MMD-CBS plans under a 60 s budget per configuration and GlobMP (Ours) under 300 s per team. For GlobMP (Ours), *Time* is the median over the configurations and its subscript the standard deviation; for the baselines it is the mean, so the two are not directly comparable.

		Low ($v_{\max} = 0.565$)					Medium ($v_{\max} = 0.617$)					High ($v_{\max} = 0.675$)				
N_a	Method	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓
3	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$0.56_{\pm 0.63}$	$0.999_{\pm 0.001}$	0.4	0.0	$0.0_{\pm 0.0}$	$0.56_{\pm 0.63}$	$0.999_{\pm 0.001}$	0.4	0.0	$0.0_{\pm 0.0}$	$0.56_{\pm 0.63}$	$0.999_{\pm 0.001}$	0.5
	CD†	5.0	$0.0_{\pm 0.0}$	$0.14_{\pm 0.20}$	$0.674_{\pm 0.096}$	$1.1_{\pm 0.1}$	5.0	$0.0_{\pm 0.0}$	$0.14_{\pm 0.20}$	$0.674_{\pm 0.096}$	$1.1_{\pm 0.0}$	5.0	$0.0_{\pm 0.0}$	$0.14_{\pm 0.20}$	$0.674_{\pm 0.096}$	$1.1_{\pm 0.0}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$0.8_{\pm 0.5}$	0.0	$0.0_{\pm 0.0}$	—	—	$0.8_{\pm 0.5}$	0.0	$0.0_{\pm 0.0}$	—	—	$0.8_{\pm 0.5}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$34.0_{\pm 5.3}$	0.0	—	—	$0.000_{\pm 0.000}$	$45.5_{\pm 8.4}$	0.0	—	—	$0.000_{\pm 0.000}$	$50.9_{\pm 17.6}$
	PCD-LB	10.0	$5.2_{\pm 19.5}$	$0.01_{\pm 0.02}$	$0.386_{\pm 0.134}$	$5.1_{\pm 1.5}$	10.0	$4.8_{\pm 20.1}$	$0.00_{\pm 0.01}$	$0.381_{\pm 0.133}$	$2.1_{\pm 0.6}$	10.0	$4.7_{\pm 19.7}$	$0.00_{\pm 0.01}$	$0.376_{\pm 0.132}$	$4.0_{\pm 0.8}$
	PCD-SHD	5.0	$0.1_{\pm 0.3}$	$0.11_{\pm 0.20}$	$0.798_{\pm 0.100}$	$3.1_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.11_{\pm 0.20}$	$0.796_{\pm 0.099}$	$3.1_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.12_{\pm 0.20}$	$0.794_{\pm 0.099}$	$3.0_{\pm 0.1}$
	Final Proj.	45.0	$19.2_{\pm 35.9}$	$0.11_{\pm 0.27}$	$\mathbf{0.983}_{\pm 0.021}$	$5.1_{\pm 0.7}$	45.0	$19.2_{\pm 35.9}$	$0.10_{\pm 0.27}$	$\mathbf{0.984}_{\pm 0.019}$	$5.1_{\pm 0.8}$	45.0	$19.2_{\pm 35.9}$	$0.09_{\pm 0.24}$	$0.986_{\pm 0.017}$	$5.1_{\pm 0.8}$
	DiRecT	50.0	$22.2_{\pm 36.6}$	$0.04_{\pm 0.11}$	$0.928_{\pm 0.073}$	$36.2_{\pm 9.4}$	50.0	$22.0_{\pm 36.8}$	$0.04_{\pm 0.10}$	$0.928_{\pm 0.073}$	$37.1_{\pm 11.0}$	50.0	$21.9_{\pm 36.7}$	$0.02_{\pm 0.09}$	$0.928_{\pm 0.073}$	$37.9_{\pm 10.4}$
	GlobMP (Ours)	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$0.980_{\pm 0.087}$	$4.0_{\pm 8.0}$	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$0.864_{\pm 0.153}$	$0.2_{\pm 5.7}$	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$\mathbf{0.989}_{\pm 0.024}$	$0.2_{\pm 0.1}$
6	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$1.42_{\pm 0.99}$	$0.999_{\pm 0.001}$	0.4	0.0	$0.0_{\pm 0.0}$	$1.42_{\pm 0.99}$	$0.999_{\pm 0.001}$	0.4	0.0	$0.0_{\pm 0.0}$	$1.42_{\pm 0.99}$	$0.999_{\pm 0.001}$	0.4
	CD†	0.0	$0.0_{\pm 0.0}$	$0.80_{\pm 0.48}$	$0.700_{\pm 0.074}$	$1.1_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.80_{\pm 0.48}$	$0.700_{\pm 0.074}$	$0.5_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.80_{\pm 0.48}$	$0.700_{\pm 0.074}$	$1.1_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$1.1_{\pm 0.8}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.1_{\pm 0.8}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.1_{\pm 0.8}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$66.0_{\pm 9.4}$	0.0	—	—	$0.000_{\pm 0.000}$	$76.7_{\pm 6.1}$	0.0	—	—	$0.000_{\pm 0.000}$	$63.7_{\pm 15.4}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$0.53_{\pm 0.39}$	$0.378_{\pm 0.084}$	$5.3_{\pm 1.1}$	0.0	$0.0_{\pm 0.0}$	$0.57_{\pm 0.41}$	$0.373_{\pm 0.082}$	$4.6_{\pm 0.8}$	0.0	$0.0_{\pm 0.0}$	$0.60_{\pm 0.41}$	$0.372_{\pm 0.086}$	$4.0_{\pm 0.6}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$1.04_{\pm 0.65}$	$0.788_{\pm 0.062}$	$3.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$1.07_{\pm 0.64}$	$0.786_{\pm 0.061}$	$3.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$1.06_{\pm 0.63}$	$0.785_{\pm 0.061}$	$3.2_{\pm 0.1}$
	Final Proj.	30.0	$8.3_{\pm 20.9}$	$0.16_{\pm 0.24}$	$0.980_{\pm 0.013}$	$4.2_{\pm 1.4}$	30.0	$9.0_{\pm 22.4}$	$0.10_{\pm 0.14}$	$\mathbf{0.981}_{\pm 0.012}$	$5.6_{\pm 0.8}$	30.0	$9.1_{\pm 22.6}$	$0.08_{\pm 0.11}$	$0.983_{\pm 0.011}$	$5.6_{\pm 0.5}$
	DiRecT	35.0	$13.0_{\pm 26.8}$	$0.04_{\pm 0.08}$	$0.922_{\pm 0.046}$	$44.0_{\pm 12.8}$	35.0	$13.2_{\pm 26.9}$	$0.03_{\pm 0.07}$	$0.923_{\pm 0.046}$	$47.7_{\pm 14.0}$	35.0	$13.4_{\pm 27.2}$	$0.02_{\pm 0.07}$	$0.923_{\pm 0.045}$	$47.4_{\pm 14.6}$
	GlobMP (Ours)	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$\mathbf{1.000}_{\pm 0.000}$	$4.2_{\pm 5.9}$	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$0.915_{\pm 0.143}$	$4.1_{\pm 2.0}$	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$\mathbf{0.985}_{\pm 0.051}$	$0.2_{\pm 0.1}$
9	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$4.18_{\pm 1.43}$	$0.999_{\pm 0.001}$	0.4	0.0	$0.0_{\pm 0.0}$	$4.18_{\pm 1.43}$	$0.999_{\pm 0.001}$	0.4	0.0	$0.0_{\pm 0.0}$	$4.18_{\pm 1.43}$	$0.999_{\pm 0.001}$	0.4
	CD†	0.0	$0.0_{\pm 0.0}$	$3.39_{\pm 0.77}$	$0.712_{\pm 0.056}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$3.39_{\pm 0.77}$	$0.712_{\pm 0.056}$	$1.2_{\pm 0.0}$	0.0	$0.0_{\pm 0.0}$	$3.39_{\pm 0.77}$	$0.712_{\pm 0.056}$	$1.2_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$1.0_{\pm 0.5}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.0_{\pm 0.5}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.0_{\pm 0.5}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$82.2_{\pm 12.5}$	0.0	—	—	$0.000_{\pm 0.000}$	$73.9_{\pm 5.1}$	0.0	—	—	$0.000_{\pm 0.000}$	$77.0_{\pm 6.6}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$2.95_{\pm 1.17}$	$0.403_{\pm 0.093}$	$5.2_{\pm 1.7}$	0.0	$0.0_{\pm 0.0}$	$3.12_{\pm 1.15}$	$0.400_{\pm 0.089}$	$5.5_{\pm 1.0}$	0.0	$0.0_{\pm 0.0}$	$3.30_{\pm 1.13}$	$0.399_{\pm 0.084}$	$4.8_{\pm 0.7}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$2.08_{\pm 0.66}$	$0.799_{\pm 0.047}$	$3.6_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$2.12_{\pm 0.62}$	$0.798_{\pm 0.046}$	$1.7_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$2.17_{\pm 0.62}$	$0.796_{\pm 0.045}$	$3.5_{\pm 0.1}$
	Final Proj.	15.0	$8.8_{\pm 25.7}$	$0.46_{\pm 0.63}$	$0.984_{\pm 0.010}$	$8.2_{\pm 0.7}$	15.0	$8.8_{\pm 25.7}$	$0.36_{\pm 0.52}$	$\mathbf{0.985}_{\pm 0.009}$	$5.6_{\pm 2.1}$	15.0	$8.8_{\pm 25.7}$	$0.34_{\pm 0.50}$	$\mathbf{0.987}_{\pm 0.009}$	$8.4_{\pm 0.7}$
	DiRecT	25.0	$11.1_{\pm 26.4}$	$0.15_{\pm 0.18}$	$0.939_{\pm 0.034}$	$74.7_{\pm 22.0}$	25.0	$11.1_{\pm 26.7}$	$0.12_{\pm 0.14}$	$0.940_{\pm 0.033}$	$58.3_{\pm 13.7}$	20.0	$11.2_{\pm 26.9}$	$0.09_{\pm 0.13}$	$0.940_{\pm 0.033}$	$53.8_{\pm 0.9}$
	GlobMP (Ours)	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$\mathbf{1.000}_{\pm 0.000}$	$4.3_{\pm 15.9}$	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$0.938_{\pm 0.129}$	$4.3_{\pm 12.5}$	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$0.984_{\pm 0.037}$	$0.3_{\pm 0.1}$
12	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$7.86_{\pm 2.57}$	$0.999_{\pm 0.001}$	0.4	0.0	$0.0_{\pm 0.0}$	$7.86_{\pm 2.57}$	$0.999_{\pm 0.001}$	0.4	0.0	$0.0_{\pm 0.0}$	$7.86_{\pm 2.57}$	$0.999_{\pm 0.001}$	0.4
	CD†	0.0	$0.0_{\pm 0.0}$	$6.72_{\pm 1.77}$	$0.745_{\pm 0.049}$	$0.6_{\pm 0.0}$	0.0	$0.0_{\pm 0.0}$	$6.72_{\pm 1.77}$	$0.745_{\pm 0.049}$	$1.2_{\pm 0.0}$	0.0	$0.0_{\pm 0.0}$	$6.72_{\pm 1.77}$	$0.745_{\pm 0.049}$	$1.2_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$0.9_{\pm 0.4}$	0.0	$0.0_{\pm 0.0}$	—	—	$0.9_{\pm 0.4}$	0.0	$0.0_{\pm 0.0}$	—	—	$0.9_{\pm 0.4}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$111.9_{\pm 6.7}$	0.0	—	—	$0.000_{\pm 0.000}$	$108.8_{\pm 6.3}$	0.0	—	—	$0.000_{\pm 0.000}$	$114.3_{\pm 6.4}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$6.12_{\pm 2.04}$	$0.405_{\pm 0.071}$	$7.2_{\pm 1.0}$	0.0	$0.0_{\pm 0.0}$	$6.57_{\pm 2.01}$	$0.404_{\pm 0.073}$	$5.9_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	$7.05_{\pm 2.03}$	$0.405_{\pm 0.074}$	$5.1_{\pm 0.6}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$4.12_{\pm 1.60}$	$0.815_{\pm 0.039}$	$3.8_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$4.12_{\pm 1.62}$	$0.814_{\pm 0.037}$	$3.7_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$4.26_{\pm 1.62}$	$0.814_{\pm 0.037}$	$3.7_{\pm 0.1}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	$1.10_{\pm 1.28}$	$0.985_{\pm 0.009}$	$10.1_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	$1.00_{\pm 1.19}$	$0.986_{\pm 0.008}$	$6.3_{\pm 1.8}$	0.0	$0.0_{\pm 0.0}$	$0.92_{\pm 1.18}$	$\mathbf{0.987}_{\pm 0.007}$	$10.1_{\pm 0.7}$
	DiRecT	0.0	$0.0_{\pm 0.0}$	$0.49_{\pm 0.41}$	$0.940_{\pm 0.029}$	$68.9_{\pm 1.6}$	0.0	$0.0_{\pm 0.0}$	$0.40_{\pm 0.39}$	$0.941_{\pm 0.029}$	$69.3_{\pm 5.5}$	0.0	$0.0_{\pm 0.0}$	$0.33_{\pm 0.39}$	$0.942_{\pm 0.029}$	$74.8_{\pm 12.7}$
	GlobMP (Ours)	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$\mathbf{1.000}_{\pm 0.000}$	$4.3_{\pm 18.6}$	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$\mathbf{0.992}_{\pm 0.054}$	$4.3_{\pm 11.5}$	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$0.972_{\pm 0.073}$	$0.3_{\pm 0.1}$
15	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$12.70_{\pm 3.56}$	$0.999_{\pm 0.001}$	0.4	0.0	$0.0_{\pm 0.0}$	$12.70_{\pm 3.56}$	$0.999_{\pm 0.001}$	0.4	0.0	$0.0_{\pm 0.0}$	$12.70_{\pm 3.56}$	$0.999_{\pm 0.001}$	0.4
	CD†	0.0	$0.0_{\pm 0.0}$	$13.31_{\pm 3.25}$	$0.712_{\pm 0.044}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$13.31_{\pm 3.25}$	$0.712_{\pm 0.044}$	$1.2_{\pm 0.0}$	0.0	$0.0_{\pm 0.0}$	$13.31_{\pm 3.25}$	$0.712_{\pm 0.044}$	$1.2_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$0.9_{\pm 0.4}$	0.0	$0.0_{\pm 0.0}$	—	—	$0.9_{\pm 0.4}$	0.0	$0.0_{\pm 0.0}$	—	—	$0.9_{\pm 0.4}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$155.2_{\pm 7.7}$	0.0	—	—	$0.000_{\pm 0.000}$	$171.4_{\pm 25.9}$	0.0	—	—	$0.000_{\pm 0.000}$	$133.6_{\pm 10.9}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$13.05_{\pm 2.30}$	$0.368_{\pm 0.041}$	$8.1_{\pm 1.1}$	0.0	$0.0_{\pm 0.0}$	$14.03_{\pm 2.31}$	$0.367_{\pm 0.039}$	$6.4_{\pm 1.2}$	0.0	$0.0_{\pm 0.0}$	$15.16_{\pm 2.36}$	$0.366_{\pm 0.039}$	$5.6_{\pm 0.7}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$7.59_{\pm 2.13}$	$0.799_{\pm 0.032}$	$4.1_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$7.65_{\pm 2.17}$	$0.798_{\pm 0.032}$	$4.0_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$7.81_{\pm 2.24}$	$0.797_{\pm 0.033}$	$4.0_{\pm 0.1}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	$1.55_{\pm 1.38}$	$0.979_{\pm 0.007}$	$12.8_{\pm 3.3}$	0.0	$0.0_{\pm 0.0}$	$1.23_{\pm 1.21}$	$0.980_{\pm 0.007}$	$14.7_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	$1.06_{\pm 1.02}$	$\mathbf{0.982}_{\pm 0.006}$	$14.6_{\pm 0.7}$
	DiRecT	0.0	$0.0_{\pm 0.0}$	$0.75_{\pm 0.68}$	$0.922_{\pm 0.023}$	$113.1_{\pm 16.3}$	0.0	$0.0_{\pm 0.0}$	$0.53_{\pm 0.48}$	$0.923_{\pm 0.023}$	$114.6_{\pm 19.5}$	0.0	$0.0_{\pm 0.0}$	$0.36_{\pm 0.36}$	$0.923_{\pm 0.023}$	$102.4_{\pm 7.6}$
	GlobMP (Ours)	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$\mathbf{1.000}_{\pm 0.000}$	$4.6_{\pm 33.3}$	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$\mathbf{0.982}_{\pm 0.069}$	$4.5_{\pm 15.1}$	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$0.974_{\pm 0.071}$	$0.4_{\pm 0.2}$
18	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$16.75_{\pm 4.10}$	$0.999_{\pm 0.000}$	0.4	0.0	$0.0_{\pm 0.0}$	$16.75_{\pm 4.10}$	$0.999_{\pm 0.000}$	0.4	0.0	$0.0_{\pm 0.0}$	$16.75_{\pm 4.10}$	$0.999_{\pm 0.000}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$19.75_{\pm 3.14}$	$0.702_{\pm 0.038}$	$1.3_{\pm 0.0}$	0.0	$0.0_{\pm 0.0}$	$19.75_{\pm 3.14}$	$0.702_{\pm 0.038}$	$0.6_{\pm 0.0}$	0.0	$0.0_{\pm 0.0}$	$19.75_{\pm 3.14}$	$0.702_{\pm 0.038}$	$1.3_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$0.8_{\pm 0.4}$	0.0	$0.0_{\pm 0.0}$	—	—	$0.8_{\pm 0.4}$	0.0	$0.0_{\pm 0.0}$	—	—	$0.8_{\pm 0.4}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$147.2_{\pm 8.4}$	0.0	—	—	$0.000_{\pm 0.000}$	$143.6_{\pm 7.6}$	0.0	—	—	$0.000_{\pm 0.000}$	$143.0_{\pm 8.6}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$20.86_{\pm 4.27}$	$0.353_{\pm 0.037}$	$8.0_{\pm 1.0}$	0.0	$0.0_{\pm 0.0}$	$22.51_{\pm 4.22}$	$0.356_{\pm 0.035}$	$6.7_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	$24.22_{\pm 4.20}$	$0.359_{\pm 0.034}$	$5.7_{\pm 0.5}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$10.38_{\pm 2.46}$	$0.792_{\pm 0.031}$	$4.3_{\pm 0.2}$	0.0	$0.0_{\pm 0.0}$	$10.52_{\pm 2.40}$	$0.790_{\pm 0.031}$	$2.0_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$10.74_{\pm 2.40}$	$0.789_{\pm 0.030}$	$4.1_{\pm 0.1}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	$2.43_{\pm 1.79}$	$0.984_{\pm 0.007}$	$14.1_{\pm 3.8}$	0.0	$0.0_{\pm 0.0}$	$2.01_{\pm 1.52}$	$\mathbf{0.985}_{\pm 0.006}$	$17.3_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	$1.75_{\pm 1.40}$	$\mathbf{0.986}_{\pm 0.006}$	$9.2_{\pm 0.7}$
	DiRecT	5.0	$0.1_{\pm 0.3}$	$0.79_{\pm 0.60}$	$0.936_{\pm 0.022}$	$124.6_{\pm 10.6}$	5.0	$0.1_{\pm 0.3}$	$0.65_{\pm 0.52}$	$0.937_{\pm 0.022}$	$107.4_{\pm 20.5}$	0.0	$0.0_{\pm 0.0}$	$0.54_{\pm 0.52}$	$0.937_{\pm 0.022}$	$89.8_{\pm 1.3}$
	GlobMP (Ours)	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$\mathbf{1.000}_{\pm 0.000}$	$5.0_{\pm 20.7}$	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$0.972_{\pm 0.090}$	$4.8_{\pm 10.4}$	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$0.972_{\pm 0.069}$	$0.5_{\pm 0.2}$
21	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$24.25_{\pm 6.19}$	$0.999_{\pm 0.000}$	0.4	0.0	$0.0_{\pm 0.0}$	$24.25_{\pm 6.19}$	$0.999_{\pm 0.000}$	0.3	0.0	$0.0_{\pm 0.0}$	$24.25_{\pm 6.19}$	$0.999_{\pm 0.000}$	0.4
	CD†	0.0	$0.0_{\pm 0.0}$	$29.53_{\pm 5.13}$	$0.706_{\pm 0.022}$	$1.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$29.53_{\pm 5.13}$	$0.706_{\pm 0.022}$	$1.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$29.53_{\pm 5.13}$	$0.706_{\pm 0.022}$	$1.3_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$0.9_{\pm 0.3}$	0.0	$0.0_{\pm 0.0}$	—	—	$0.9_{\pm 0.3}$	0.0	$0.0_{\pm 0.0}$	—	—	$0.9_{\pm 0.3}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$165.3_{\pm 8.7}$	0.0	—	—	$0.000_{\pm 0.000}$	$164.1_{\pm 8.1}$	0.0	—	—	$0.000_{\pm 0.000}$	$160.9_{\pm 8.6}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$30.67_{\pm 3.84}$	$0.356_{\pm 0.029}$	$9.6_{\pm 1.4}$	0.0	$0.0_{\pm 0.0}$	$33.64_{\pm 3.88}$	$0.357_{\pm 0.028}$	$7.9_{\pm 1.1}$	0.0	$0.0_{\pm 0.0}$	$36.36_{\pm 3.94}$	$0.359_{\pm 0.027}$	$3.5_{\pm 0.8}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$14.48_{\pm 2.36}$	$0.796_{\pm 0.024}$	$4.9_{\pm 0.4}$	0.0	$0.0_{\pm 0.0}$	$14.69_{\pm 2.44}$	$0.795_{\pm 0.024}$	$4.7_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$14.90_{\pm 2.43}$	$0.794_{\pm 0.023}$	$4.6_{\pm 0.1}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	$4.31_{\pm 3.10}$	$0.982_{\pm 0.007}$	$23.1_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	$3.68_{\pm 2.74}$	$\mathbf{0.983}_{\pm 0.007}$	$23.1_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	$3.20_{\pm 2.45}$	$\mathbf{0.984}_{\pm 0.006}$	$16.5_{\pm 5.1}$
	DiRecT	0.0	$0.0_{\pm 0.0}$	$1.36_{\pm 0.97}$	$0.931_{\pm 0.019}$	$120.1_{\pm 1.0}$	0.0	$0.0_{\pm 0.0}$	$1.06_{\pm 0.78}$	$0.932_{\pm 0.019}$	$120.0_{\pm 1.1}$	0.0	$0.0_{\pm 0.0}$	$0.62_{\pm 0.57}$	$0.933_{\pm 0.019}$	$120.2_{\pm 0.7}$
	GlobMP (Ours)	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$\mathbf{1.000}_{\pm 0.000}$	$5.1_{\pm 43.5}$	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$0.978_{\pm 0.081}$	$5.0_{\pm 26.0}$	**100.0**	$\mathbf{100.0}_{\pm 0.0}$	$\mathbf{0.00}_{\pm 0.00}$	$0.970_{\pm 0.071}$	$0.9_{\pm 0.3}$

Table 6: Multi robot planning performance on `Conveyor` for $N_a \in \{3, 6, \ldots, 21\}$ under the Low, Medium, and High velocity caps. *SR* and *CS* measure trajectory feasibility, *Coll* reports inter robot collisions, *DA* measures adherence to the environment specific motion pattern, and *Time* is the planning time per configuration. *DA* on this map scores an end-to-end traversal of one conveyor lane, which the uniformly drawn start and goal pairs almost never require; it is therefore below 0.07 for every method and does not separate them. † marks methods that apply no velocity limit, so their three blocks report the same run; in addition, MMD-CBS returns a single plan per configuration rather than a batch of 64 candidates. A dash under *Coll* marks a count that is not commensurable with the others: MDOC and MMD-CBS count colliding time steps, where every other row counts agent pairs with any collision. MDOC's *CS* is omitted because its plans that do not cover every agent were counted as feasible. MMD-CBS plans under a 60 s budget per configuration and GlobMP (Ours) under 300 s per team. For GlobMP (Ours), *Time* is the median over the configurations and its subscript the standard deviation; for the baselines it is the mean, so the two are not directly comparable.

		Low ($v_{\max} = 1.21$)					Medium ($v_{\max} = 1.46$)					High ($v_{\max} = 1.76$)				
N_a	Method	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓
3	Diff. Policy†	5.0	$4.9_{\pm 21.5}$	$0.37_{\pm 0.53}$	$0.016_{\pm 0.070}$	0.6	5.0	$4.9_{\pm 21.5}$	$0.37_{\pm 0.53}$	$0.016_{\pm 0.070}$	0.6	5.0	$4.9_{\pm 21.5}$	$0.37_{\pm 0.53}$	$0.016_{\pm 0.070}$	0.6
	CD†	45.0	$17.4_{\pm 35.0}$	$0.02_{\pm 0.04}$	$0.001_{\pm 0.003}$	$0.6_{\pm 0.1}$	45.0	$18.3_{\pm 36.2}$	$0.02_{\pm 0.04}$	$0.001_{\pm 0.003}$	$1.2_{\pm 0.1}$	45.0	$19.2_{\pm 37.3}$	$0.02_{\pm 0.04}$	$0.001_{\pm 0.003}$	$1.1_{\pm 0.1}$
	MMD-CBS†	20.0	$20.0_{\pm 40.0}$	—	$0.000_{\pm 0.000}$	$1.3_{\pm 0.7}$	20.0	$20.0_{\pm 40.0}$	—	$0.000_{\pm 0.000}$	$1.3_{\pm 0.7}$	20.0	$20.0_{\pm 40.0}$	—	$0.000_{\pm 0.000}$	$1.3_{\pm 0.7}$
	MDOC†	5.0	—	—	$0.000_{\pm 0.000}$	$40.1_{\pm 7.0}$	5.0	—	—	$0.000_{\pm 0.000}$	$36.4_{\pm 2.6}$	5.0	—	—	$0.000_{\pm 0.000}$	$36.3_{\pm 2.8}$
	PCD-LB	70.0	$36.2_{\pm 42.5}$	$0.01_{\pm 0.03}$	$0.004_{\pm 0.016}$	$3.0_{\pm 0.1}$	65.0	$37.1_{\pm 43.6}$	$0.01_{\pm 0.02}$	$0.004_{\pm 0.016}$	$3.1_{\pm 0.1}$	65.0	$36.8_{\pm 43.5}$	$0.01_{\pm 0.02}$	$0.004_{\pm 0.017}$	$3.1_{\pm 0.1}$
	PCD-SHD	10.0	$9.5_{\pm 28.4}$	$0.01_{\pm 0.03}$	$0.003_{\pm 0.011}$	$2.1_{\pm 0.7}$	10.0	$9.5_{\pm 28.4}$	$0.01_{\pm 0.02}$	$0.002_{\pm 0.010}$	$3.0_{\pm 0.1}$	10.0	$9.6_{\pm 28.9}$	$0.01_{\pm 0.02}$	$0.002_{\pm 0.009}$	$3.0_{\pm 0.1}$
	Final Proj.	10.0	$10.0_{\pm 30.0}$	$0.00_{\pm 0.01}$	$0.000_{\pm 0.000}$	$3.6_{\pm 0.5}$	10.0	$10.0_{\pm 30.0}$	$0.00_{\pm 0.00}$	$0.000_{\pm 0.000}$	$3.7_{\pm 0.7}$	10.0	$10.0_{\pm 30.0}$	**0.00**$_{\pm 0.00}$	$0.000_{\pm 0.000}$	$3.7_{\pm 0.6}$
	DiRecT	10.0	$9.9_{\pm 29.8}$	**0.00**$_{\pm 0.00}$	$0.000_{\pm 0.000}$	$24.3_{\pm 5.8}$	10.0	$10.0_{\pm 30.0}$	**0.00**$_{\pm 0.00}$	$0.000_{\pm 0.000}$	$24.5_{\pm 5.7}$	10.0	$10.0_{\pm 30.0}$	**0.00**$_{\pm 0.00}$	$0.000_{\pm 0.000}$	$26.4_{\pm 6.7}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.017**$_{\pm 0.128}$	$0.4_{\pm 0.2}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.033**$_{\pm 0.180}$	$0.5_{\pm 0.4}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.067**$_{\pm 0.249}$	$0.7_{\pm 0.6}$
6	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$1.22_{\pm 1.04}$	$0.000_{\pm 0.000}$	0.6	0.0	$0.0_{\pm 0.0}$	$1.22_{\pm 1.04}$	$0.000_{\pm 0.000}$	0.6	0.0	$0.0_{\pm 0.0}$	$1.22_{\pm 1.04}$	$0.000_{\pm 0.000}$	0.6
	CD†	5.0	$0.1_{\pm 0.3}$	$0.25_{\pm 0.29}$	$0.008_{\pm 0.032}$	$1.2_{\pm 0.1}$	5.0	$0.1_{\pm 0.3}$	$0.25_{\pm 0.29}$	$0.008_{\pm 0.032}$	$1.2_{\pm 0.1}$	5.0	$0.2_{\pm 0.7}$	$0.25_{\pm 0.29}$	$0.008_{\pm 0.032}$	$1.1_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$1.5_{\pm 1.1}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.5_{\pm 1.1}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.5_{\pm 1.1}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$62.5_{\pm 3.3}$	0.0	—	—	$0.000_{\pm 0.000}$	$82.6_{\pm 23.1}$	0.0	—	—	$0.000_{\pm 0.000}$	$85.2_{\pm 20.8}$
	PCD-LB	25.0	$6.1_{\pm 14.7}$	$0.85_{\pm 0.74}$	$0.006_{\pm 0.013}$	$2.8_{\pm 0.7}$	40.0	$7.8_{\pm 16.5}$	$0.72_{\pm 0.60}$	$0.007_{\pm 0.013}$	$3.1_{\pm 0.4}$	45.0	$13.5_{\pm 20.5}$	$0.44_{\pm 0.34}$	$0.007_{\pm 0.014}$	$3.3_{\pm 0.1}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$0.10_{\pm 0.18}$	$0.001_{\pm 0.006}$	$3.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.06_{\pm 0.09}$	$0.001_{\pm 0.006}$	$3.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.03_{\pm 0.05}$	$0.001_{\pm 0.005}$	$3.2_{\pm 0.1}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	$0.05_{\pm 0.14}$	$0.000_{\pm 0.000}$	$1.9_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	$0.03_{\pm 0.10}$	$0.000_{\pm 0.000}$	$3.4_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	$0.01_{\pm 0.03}$	$0.000_{\pm 0.000}$	$3.4_{\pm 0.6}$
	DiRecT	0.0	$0.0_{\pm 0.0}$	$0.00_{\pm 0.01}$	$0.001_{\pm 0.005}$	$23.4_{\pm 6.1}$	0.0	$0.0_{\pm 0.0}$	$0.01_{\pm 0.02}$	$0.001_{\pm 0.002}$	$24.0_{\pm 6.6}$	0.0	$0.0_{\pm 0.0}$	$0.01_{\pm 0.02}$	$0.000_{\pm 0.001}$	$25.0_{\pm 7.0}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.008**$_{\pm 0.091}$	$0.8_{\pm 0.8}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.008**$_{\pm 0.091}$	$1.2_{\pm 2.2}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.017**$_{\pm 0.128}$	$1.8_{\pm 10.5}$
9	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$3.16_{\pm 1.58}$	$0.006_{\pm 0.024}$	0.6	0.0	$0.0_{\pm 0.0}$	$3.16_{\pm 1.58}$	$0.006_{\pm 0.024}$	0.3	0.0	$0.0_{\pm 0.0}$	$3.16_{\pm 1.58}$	$0.006_{\pm 0.024}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$1.47_{\pm 0.91}$	$0.000_{\pm 0.002}$	$0.6_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$1.47_{\pm 0.91}$	$0.000_{\pm 0.002}$	$1.1_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$1.47_{\pm 0.91}$	$0.000_{\pm 0.002}$	$1.1_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$1.8_{\pm 1.1}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.8_{\pm 1.1}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.8_{\pm 1.1}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$113.3_{\pm 17.4}$	0.0	—	—	$0.000_{\pm 0.000}$	$97.2_{\pm 18.9}$	0.0	—	—	$0.000_{\pm 0.000}$	$74.7_{\pm 4.0}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$4.20_{\pm 1.42}$	$0.004_{\pm 0.008}$	$3.5_{\pm 0.1}$	10.0	$0.4_{\pm 1.4}$	$3.09_{\pm 1.02}$	$0.004_{\pm 0.010}$	$3.5_{\pm 0.1}$	25.0	$2.4_{\pm 5.4}$	$1.54_{\pm 0.57}$	$0.005_{\pm 0.011}$	$3.5_{\pm 0.1}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$0.13_{\pm 0.16}$	$0.003_{\pm 0.014}$	$3.6_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.08_{\pm 0.10}$	$0.003_{\pm 0.014}$	$3.6_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.05_{\pm 0.06}$	$0.003_{\pm 0.014}$	$3.6_{\pm 0.1}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	$0.05_{\pm 0.10}$	$0.005_{\pm 0.024}$	$4.3_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	$0.02_{\pm 0.04}$	$0.005_{\pm 0.024}$	$4.3_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	**0.02**$_{\pm 0.04}$	$0.005_{\pm 0.024}$	$4.3_{\pm 0.6}$
	DiRecT	0.0	$0.0_{\pm 0.0}$	$0.04_{\pm 0.06}$	$0.007_{\pm 0.025}$	$34.5_{\pm 9.7}$	0.0	$0.0_{\pm 0.0}$	$0.01_{\pm 0.02}$	$0.006_{\pm 0.024}$	$35.6_{\pm 10.5}$	0.0	$0.0_{\pm 0.0}$	$0.02_{\pm 0.03}$	$0.006_{\pm 0.024}$	$25.9_{\pm 1.3}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.028**$_{\pm 0.164}$	$2.0_{\pm 8.3}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.028**$_{\pm 0.164}$	$2.6_{\pm 2.7}$	**95.0**	**95.0**$_{\pm 21.8}$	$0.10_{\pm 0.44}$	**0.028**$_{\pm 0.164}$	$5.5_{\pm 64.2}$
12	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$6.33_{\pm 2.50}$	$0.012_{\pm 0.029}$	0.6	0.0	$0.0_{\pm 0.0}$	$6.33_{\pm 2.50}$	$0.012_{\pm 0.029}$	0.6	0.0	$0.0_{\pm 0.0}$	$6.33_{\pm 2.50}$	$0.012_{\pm 0.029}$	0.6
	CD†	0.0	$0.0_{\pm 0.0}$	$3.96_{\pm 1.28}$	$0.018_{\pm 0.026}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$3.96_{\pm 1.28}$	$0.018_{\pm 0.026}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$3.96_{\pm 1.28}$	$0.018_{\pm 0.026}$	$0.6_{\pm 0.0}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$1.9_{\pm 1.6}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.9_{\pm 1.6}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.9_{\pm 1.6}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$107.9_{\pm 7.1}$	0.0	—	—	$0.000_{\pm 0.000}$	$120.6_{\pm 5.3}$	0.0	—	—	$0.000_{\pm 0.000}$	$127.3_{\pm 7.3}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$9.25_{\pm 1.68}$	$0.005_{\pm 0.007}$	$3.7_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$7.14_{\pm 1.37}$	$0.005_{\pm 0.007}$	$1.9_{\pm 0.2}$	0.0	$0.0_{\pm 0.0}$	$3.76_{\pm 0.86}$	$0.005_{\pm 0.006}$	$3.7_{\pm 0.5}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$0.62_{\pm 0.51}$	$0.013_{\pm 0.022}$	$3.9_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.37_{\pm 0.32}$	$0.012_{\pm 0.022}$	$3.9_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.19_{\pm 0.18}$	$0.012_{\pm 0.022}$	$1.8_{\pm 0.2}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	$0.20_{\pm 0.29}$	$0.010_{\pm 0.025}$	$5.4_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	**0.05**$_{\pm 0.08}$	$0.010_{\pm 0.025}$	$5.4_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	**0.06**$_{\pm 0.11}$	$0.010_{\pm 0.025}$	$5.4_{\pm 0.7}$
	DiRecT	0.0	$0.0_{\pm 0.0}$	**0.06**$_{\pm 0.08}$	$0.012_{\pm 0.026}$	$33.6_{\pm 1.4}$	0.0	$0.0_{\pm 0.0}$	$0.07_{\pm 0.09}$	$0.013_{\pm 0.028}$	$34.0_{\pm 1.9}$	0.0	$0.0_{\pm 0.0}$	$0.09_{\pm 0.16}$	$0.014_{\pm 0.029}$	$34.2_{\pm 1.9}$
	GlobMP (Ours)	**90.0**	**90.0**$_{\pm 30.0}$	$0.85_{\pm 2.55}$	**0.042**$_{\pm 0.200}$	$3.3_{\pm 91.3}$	**95.0**	**95.0**$_{\pm 21.8}$	$0.45_{\pm 1.96}$	**0.037**$_{\pm 0.190}$	$5.7_{\pm 72.9}$	**90.0**	**90.0**$_{\pm 30.0}$	$0.60_{\pm 1.80}$	**0.033**$_{\pm 0.180}$	$13.3_{\pm 79.3}$
15	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$9.77_{\pm 2.61}$	$0.003_{\pm 0.013}$	0.3	0.0	$0.0_{\pm 0.0}$	$9.77_{\pm 2.61}$	$0.003_{\pm 0.013}$	0.3	0.0	$0.0_{\pm 0.0}$	$9.77_{\pm 2.61}$	$0.003_{\pm 0.013}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$7.35_{\pm 1.96}$	$0.003_{\pm 0.009}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$7.35_{\pm 1.96}$	$0.003_{\pm 0.009}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$7.35_{\pm 1.96}$	$0.003_{\pm 0.009}$	$1.2_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$1.9_{\pm 1.4}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.9_{\pm 1.4}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.9_{\pm 1.4}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$155.4_{\pm 11.8}$	0.0	—	—	$0.000_{\pm 0.000}$	$172.5_{\pm 14.6}$	0.0	—	—	$0.000_{\pm 0.000}$	$174.3_{\pm 27.4}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$20.74_{\pm 3.01}$	$0.005_{\pm 0.006}$	$4.1_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$17.02_{\pm 2.64}$	$0.004_{\pm 0.006}$	$4.1_{\pm 0.2}$	0.0	$0.0_{\pm 0.0}$	$11.84_{\pm 1.88}$	$0.005_{\pm 0.006}$	$4.1_{\pm 0.1}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$0.75_{\pm 0.40}$	$0.003_{\pm 0.010}$	$4.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.48_{\pm 0.28}$	$0.003_{\pm 0.010}$	$4.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.29_{\pm 0.19}$	$0.003_{\pm 0.010}$	$4.3_{\pm 0.1}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	$0.49_{\pm 0.42}$	$0.003_{\pm 0.015}$	$3.7_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	$0.23_{\pm 0.25}$	$0.003_{\pm 0.015}$	$6.9_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	**0.18**$_{\pm 0.24}$	$0.003_{\pm 0.015}$	$6.0_{\pm 1.6}$
	DiRecT	0.0	$0.0_{\pm 0.0}$	**0.14**$_{\pm 0.15}$	$0.003_{\pm 0.013}$	$44.0_{\pm 1.3}$	0.0	$0.0_{\pm 0.0}$	**0.16**$_{\pm 0.12}$	$0.003_{\pm 0.013}$	$44.9_{\pm 2.2}$	0.0	$0.0_{\pm 0.0}$	**0.18**$_{\pm 0.15}$	$0.003_{\pm 0.013}$	$47.2_{\pm 5.5}$
	GlobMP (Ours)	**90.0**	**90.0**$_{\pm 30.0}$	$1.00_{\pm 3.15}$	**0.027**$_{\pm 0.161}$	$5.5_{\pm 68.3}$	**85.0**	**85.0**$_{\pm 35.7}$	$1.45_{\pm 3.58}$	**0.023**$_{\pm 0.151}$	$11.2_{\pm 74.1}$	**90.0**	**90.0**$_{\pm 30.0}$	$0.90_{\pm 2.70}$	**0.023**$_{\pm 0.151}$	$22.8_{\pm 91.2}$
18	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$15.01_{\pm 3.59}$	$0.000_{\pm 0.000}$	0.3	0.0	$0.0_{\pm 0.0}$	$15.01_{\pm 3.59}$	$0.000_{\pm 0.000}$	0.3	0.0	$0.0_{\pm 0.0}$	$15.01_{\pm 3.59}$	$0.000_{\pm 0.000}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$12.92_{\pm 1.90}$	$0.003_{\pm 0.006}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$12.92_{\pm 1.90}$	$0.003_{\pm 0.006}$	$1.2_{\pm 0.2}$	0.0	$0.0_{\pm 0.0}$	$12.92_{\pm 1.90}$	$0.003_{\pm 0.006}$	$1.2_{\pm 0.0}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$1.4_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.4_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.4_{\pm 0.9}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$162.1_{\pm 8.6}$	0.0	—	—	$0.000_{\pm 0.000}$	$156.7_{\pm 7.6}$	0.0	—	—	$0.000_{\pm 0.000}$	$157.5_{\pm 7.0}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$35.56_{\pm 3.24}$	$0.002_{\pm 0.003}$	$4.2_{\pm 0.2}$	0.0	$0.0_{\pm 0.0}$	$28.42_{\pm 2.79}$	$0.002_{\pm 0.003}$	$4.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$19.50_{\pm 1.96}$	$0.003_{\pm 0.004}$	$2.8_{\pm 0.9}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$1.18_{\pm 0.69}$	$0.003_{\pm 0.004}$	$4.4_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.80_{\pm 0.52}$	$0.003_{\pm 0.004}$	$4.4_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.55_{\pm 0.37}$	$0.002_{\pm 0.004}$	$4.4_{\pm 0.1}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	$0.84_{\pm 0.83}$	$0.000_{\pm 0.000}$	$8.7_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	$0.33_{\pm 0.33}$	$0.000_{\pm 0.000}$	$7.3_{\pm 2.1}$	0.0	$0.0_{\pm 0.0}$	**0.23**$_{\pm 0.23}$	$0.000_{\pm 0.000}$	$8.7_{\pm 0.7}$
	DiRecT	0.0	$0.0_{\pm 0.0}$	**0.20**$_{\pm 0.15}$	$0.003_{\pm 0.009}$	$65.8_{\pm 10.7}$	0.0	$0.0_{\pm 0.0}$	**0.22**$_{\pm 0.18}$	$0.003_{\pm 0.008}$	$64.5_{\pm 10.8}$	0.0	$0.0_{\pm 0.0}$	$0.26_{\pm 0.22}$	$0.003_{\pm 0.008}$	$64.4_{\pm 9.0}$
	GlobMP (Ours)	**90.0**	**90.0**$_{\pm 30.0}$	$1.85_{\pm 5.55}$	**0.047**$_{\pm 0.212}$	$12.4_{\pm 89.4}$	**85.0**	**85.0**$_{\pm 35.7}$	$2.75_{\pm 6.77}$	**0.044**$_{\pm 0.206}$	$14.2_{\pm 102.5}$	**75.0**	**75.0**$_{\pm 43.3}$	$3.60_{\pm 6.61}$	**0.039**$_{\pm 0.193}$	$36.0_{\pm 118.0}$
21	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$20.41_{\pm 2.03}$	$0.002_{\pm 0.010}$	0.3	0.0	$0.0_{\pm 0.0}$	$20.41_{\pm 2.03}$	$0.002_{\pm 0.010}$	0.3	0.0	$0.0_{\pm 0.0}$	$20.41_{\pm 2.03}$	$0.002_{\pm 0.010}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$20.28_{\pm 2.30}$	$0.011_{\pm 0.016}$	$1.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$20.28_{\pm 2.30}$	$0.011_{\pm 0.016}$	$1.2_{\pm 0.0}$	0.0	$0.0_{\pm 0.0}$	$20.28_{\pm 2.30}$	$0.011_{\pm 0.016}$	$1.3_{\pm 0.0}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$1.7_{\pm 1.4}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.7_{\pm 1.4}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.7_{\pm 1.4}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$182.3_{\pm 8.9}$	0.0	—	—	$0.000_{\pm 0.000}$	$181.9_{\pm 9.0}$	0.0	—	—	$0.000_{\pm 0.000}$	$163.8_{\pm 8.1}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$53.35_{\pm 2.98}$	$0.002_{\pm 0.003}$	$4.7_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$40.89_{\pm 3.07}$	$0.002_{\pm 0.003}$	$4.8_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$26.92_{\pm 2.55}$	$0.002_{\pm 0.003}$	$4.9_{\pm 0.1}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$1.60_{\pm 0.65}$	$0.006_{\pm 0.011}$	$5.0_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$1.09_{\pm 0.49}$	$0.006_{\pm 0.011}$	$3.0_{\pm 1.0}$	0.0	$0.0_{\pm 0.0}$	$0.68_{\pm 0.32}$	$0.006_{\pm 0.011}$	$5.0_{\pm 0.1}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	$0.98_{\pm 0.65}$	$0.003_{\pm 0.010}$	$11.5_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	$0.43_{\pm 0.32}$	$0.003_{\pm 0.010}$	$10.8_{\pm 1.3}$	0.0	$0.0_{\pm 0.0}$	$0.28_{\pm 0.19}$	$0.003_{\pm 0.010}$	$11.4_{\pm 0.7}$
	DiRecT	0.0	$0.0_{\pm 0.0}$	**0.20**$_{\pm 0.14}$	$0.006_{\pm 0.013}$	$89.0_{\pm 12.7}$	0.0	$0.0_{\pm 0.0}$	**0.21**$_{\pm 0.11}$	$0.007_{\pm 0.015}$	$84.7_{\pm 5.9}$	0.0	$0.0_{\pm 0.0}$	**0.22**$_{\pm 0.13}$	$0.007_{\pm 0.015}$	$89.1_{\pm 11.6}$
	GlobMP (Ours)	**80.0**	**80.0**$_{\pm 40.0}$	$5.00_{\pm 10.27}$	**0.038**$_{\pm 0.191}$	$16.4_{\pm 112.4}$	**70.0**	**70.0**$_{\pm 45.8}$	$6.30_{\pm 9.77}$	**0.026**$_{\pm 0.160}$	$41.4_{\pm 119.1}$	**65.0**	**65.0**$_{\pm 47.7}$	$5.95_{\pm 9.13}$	**0.026**$_{\pm 0.160}$	$108.4_{\pm 116.6}$

Table 7: Multi robot planning performance on `Highways` for $N_a \in \{3, 6, \ldots, 21\}$ under the Low, Medium, and High velocity caps. *SR* and *CS* measure trajectory feasibility, *Coll* reports inter robot collisions, *DA* measures adherence to the environment specific motion pattern, and *Time* is the planning time per configuration. † marks methods that apply no velocity limit, so their three blocks report the same run; in addition, MMD-CBS returns a single plan per configuration rather than a batch of 64 candidates. A dash under *Coll* marks a count that is not commensurable with the others: MDOC and MMD-CBS count colliding time steps, where every other row counts agent pairs with any collision. MDOC's *CS* is omitted because its plans that do not cover every agent were counted as feasible. MMD-CBS plans under a 60 s budget per configuration and GlobMP (Ours) under 300 s per team. For GlobMP (Ours), *Time* is the median over the configurations and its subscript the standard deviation; for the baselines it is the mean, so the two are not directly comparable.

		Low ($v_{\max} = 0.647$)					Medium ($v_{\max} = 0.783$)					High ($v_{\max} = 0.878$)				
N_a	Method	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓
3	Diff. Policy†	10.0	10.0$_{\pm 30.0}$	0.15$_{\pm 0.35}$	0.567$_{\pm 0.298}$	0.3	10.0	10.0$_{\pm 30.0}$	0.15$_{\pm 0.35}$	0.567$_{\pm 0.298}$	0.3	10.0	10.0$_{\pm 30.0}$	0.15$_{\pm 0.35}$	0.567$_{\pm 0.298}$	0.3
	CD†	100.0	0.0$_{\pm 0.0}$	0.25$_{\pm 0.32}$	0.564$_{\pm 0.289}$	1.1$_{\pm 0.1}$	100.0	0.0$_{\pm 0.0}$	0.25$_{\pm 0.32}$	0.564$_{\pm 0.289}$	1.2$_{\pm 0.1}$	100.0	0.0$_{\pm 0.0}$	0.25$_{\pm 0.32}$	0.564$_{\pm 0.289}$	1.2$_{\pm 0.1}$
	MMD-CBS†	90.0	45.0$_{\pm 49.7}$	—	0.567$_{\pm 0.318}$	3.8$_{\pm 3.2}$	90.0	55.0$_{\pm 49.7}$	—	0.567$_{\pm 0.318}$	3.8$_{\pm 3.2}$	90.0	55.0$_{\pm 49.7}$	—	0.567$_{\pm 0.318}$	3.8$_{\pm 3.2}$
	MDOC†	15.0	—	—	0.083$_{\pm 0.233}$	41.3$_{\pm 5.6}$	15.0	—	—	0.083$_{\pm 0.233}$	37.2$_{\pm 3.4}$	15.0	—	—	0.083$_{\pm 0.233}$	36.8$_{\pm 3.3}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	0.02$_{\pm 0.02}$	0.565$_{\pm 0.293}$	3.5$_{\pm 0.4}$	0.0	0.0$_{\pm 0.0}$	0.05$_{\pm 0.11}$	0.564$_{\pm 0.292}$	3.0$_{\pm 0.2}$	0.0	0.0$_{\pm 0.0}$	0.05$_{\pm 0.13}$	0.564$_{\pm 0.292}$	3.0$_{\pm 0.1}$
	PCD-SHD	**100.0**	89.8$_{\pm 15.8}$	0.09$_{\pm 0.17}$	0.561$_{\pm 0.281}$	2.8$_{\pm 0.1}$	**100.0**	90.9$_{\pm 13.8}$	0.08$_{\pm 0.14}$	0.562$_{\pm 0.279}$	2.8$_{\pm 0.1}$	**100.0**	90.4$_{\pm 13.7}$	0.08$_{\pm 0.14}$	0.562$_{\pm 0.279}$	2.8$_{\pm 0.1}$
	Final Proj.	95.0	85.9$_{\pm 33.0}$	0.18$_{\pm 0.47}$	0.567$_{\pm 0.298}$	3.8$_{\pm 0.6}$	**100.0**	86.1$_{\pm 32.4}$	0.18$_{\pm 0.46}$	0.567$_{\pm 0.298}$	3.9$_{\pm 0.7}$	**100.0**	84.6$_{\pm 32.6}$	0.20$_{\pm 0.46}$	0.567$_{\pm 0.298}$	3.9$_{\pm 0.6}$
	DiRecT	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.568$_{\pm 0.294}$	26.4$_{\pm 6.4}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.568**$_{\pm 0.294}$	27.6$_{\pm 7.1}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.568**$_{\pm 0.294}$	29.3$_{\pm 7.7}$
	GlobMP (Ours)	95.0	95.0$_{\pm 21.8}$	**0.00**$_{\pm 0.00}$	**0.583**$_{\pm 0.493}$	0.2$_{\pm 15.3}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.567$_{\pm 0.496}$	0.3$_{\pm 0.2}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.550$_{\pm 0.497}$	0.2$_{\pm 0.2}$
6	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	1.39$_{\pm 0.78}$	0.582$_{\pm 0.187}$	0.3	0.0	0.0$_{\pm 0.0}$	1.39$_{\pm 0.78}$	0.582$_{\pm 0.187}$	0.3	0.0	0.0$_{\pm 0.0}$	1.39$_{\pm 0.78}$	0.582$_{\pm 0.187}$	0.3
	CD†	30.0	0.0$_{\pm 0.0}$	2.61$_{\pm 0.80}$	0.555$_{\pm 0.192}$	1.2$_{\pm 0.1}$	30.0	0.0$_{\pm 0.0}$	2.61$_{\pm 0.80}$	0.555$_{\pm 0.192}$	1.2$_{\pm 0.1}$	30.0	0.0$_{\pm 0.0}$	2.61$_{\pm 0.80}$	0.555$_{\pm 0.192}$	0.5$_{\pm 0.0}$
	MMD-CBS†	75.0	0.0$_{\pm 0.0}$	—	0.558$_{\pm 0.219}$	32.0$_{\pm 16.7}$	75.0	0.0$_{\pm 0.0}$	—	0.558$_{\pm 0.219}$	32.0$_{\pm 16.7}$	75.0	0.0$_{\pm 0.0}$	—	0.558$_{\pm 0.219}$	32.0$_{\pm 16.7}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	63.5$_{\pm 3.2}$	0.0	—	—	0.000$_{\pm 0.000}$	121.4$_{\pm 150.4}$	0.0	—	—	0.000$_{\pm 0.000}$	72.0$_{\pm 6.6}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	0.26$_{\pm 0.26}$	0.539$_{\pm 0.205}$	3.6$_{\pm 0.5}$	0.0	0.0$_{\pm 0.0}$	0.19$_{\pm 0.16}$	0.540$_{\pm 0.204}$	1.6$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	0.15$_{\pm 0.15}$	0.540$_{\pm 0.203}$	3.1$_{\pm 0.1}$
	PCD-SHD	**100.0**	52.6$_{\pm 20.0}$	0.59$_{\pm 0.34}$	0.561$_{\pm 0.180}$	3.0$_{\pm 0.1}$	**100.0**	52.8$_{\pm 21.6}$	0.59$_{\pm 0.35}$	0.562$_{\pm 0.182}$	2.6$_{\pm 0.7}$	**100.0**	53.3$_{\pm 21.7}$	0.57$_{\pm 0.36}$	0.563$_{\pm 0.183}$	2.9$_{\pm 0.5}$
	Final Proj.	**100.0**	69.9$_{\pm 39.6}$	0.53$_{\pm 0.90}$	**0.582**$_{\pm 0.187}$	2.5$_{\pm 1.2}$	**100.0**	75.9$_{\pm 33.4}$	0.33$_{\pm 0.56}$	**0.582**$_{\pm 0.187}$	4.1$_{\pm 0.6}$	**100.0**	79.1$_{\pm 31.5}$	0.29$_{\pm 0.51}$	**0.582**$_{\pm 0.187}$	4.1$_{\pm 0.7}$
	DiRecT	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.570$_{\pm 0.187}$	32.0$_{\pm 8.6}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.570$_{\pm 0.187}$	32.8$_{\pm 9.3}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.570$_{\pm 0.187}$	34.9$_{\pm 9.7}$
	GlobMP (Ours)	80.0	80.0$_{\pm 40.0}$	0.10$_{\pm 0.44}$	0.575$_{\pm 0.494}$	0.7$_{\pm 68.6}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.567$_{\pm 0.496}$	0.6$_{\pm 10.0}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.567$_{\pm 0.496}$	0.7$_{\pm 0.7}$
9	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	4.58$_{\pm 1.91}$	0.450$_{\pm 0.172}$	0.3	0.0	0.0$_{\pm 0.0}$	4.58$_{\pm 1.91}$	0.450$_{\pm 0.172}$	0.3	0.0	0.0$_{\pm 0.0}$	4.58$_{\pm 1.91}$	0.450$_{\pm 0.172}$	0.3
	CD†	0.0	0.0$_{\pm 0.0}$	7.04$_{\pm 1.79}$	0.463$_{\pm 0.170}$	1.2$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	7.04$_{\pm 1.79}$	0.463$_{\pm 0.170}$	1.2$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	7.04$_{\pm 1.79}$	0.463$_{\pm 0.170}$	1.2$_{\pm 0.1}$
	MMD-CBS†	5.0	0.0$_{\pm 0.0}$	—	0.478$_{\pm 0.186}$	59.0$_{\pm 9.8}$	5.0	0.0$_{\pm 0.0}$	—	0.478$_{\pm 0.186}$	59.0$_{\pm 9.8}$	5.0	0.0$_{\pm 0.0}$	—	0.478$_{\pm 0.186}$	59.0$_{\pm 9.8}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	121.0$_{\pm 15.7}$	0.0	—	—	0.000$_{\pm 0.000}$	91.0$_{\pm 13.2}$	0.0	—	—	0.000$_{\pm 0.000}$	82.7$_{\pm 3.8}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	1.29$_{\pm 0.58}$	**0.469**$_{\pm 0.162}$	4.4$_{\pm 0.4}$	0.0	0.0$_{\pm 0.0}$	1.26$_{\pm 0.55}$	**0.469**$_{\pm 0.161}$	3.4$_{\pm 0.5}$	0.0	0.0$_{\pm 0.0}$	1.28$_{\pm 0.56}$	0.469$_{\pm 0.160}$	3.5$_{\pm 0.1}$
	PCD-SHD	80.0	9.9$_{\pm 14.2}$	2.60$_{\pm 0.95}$	0.460$_{\pm 0.166}$	3.3$_{\pm 0.1}$	90.0	10.8$_{\pm 15.0}$	2.38$_{\pm 0.89}$	0.461$_{\pm 0.166}$	3.4$_{\pm 0.1}$	90.0	10.9$_{\pm 13.2}$	2.33$_{\pm 0.93}$	0.462$_{\pm 0.166}$	3.4$_{\pm 0.1}$
	Final Proj.	70.0	41.5$_{\pm 40.8}$	1.60$_{\pm 1.72}$	0.452$_{\pm 0.174}$	5.5$_{\pm 0.9}$	80.0	44.0$_{\pm 39.7}$	1.23$_{\pm 1.35}$	0.452$_{\pm 0.174}$	5.6$_{\pm 0.7}$	80.0	41.5$_{\pm 37.4}$	1.12$_{\pm 1.13}$	0.451$_{\pm 0.174}$	5.6$_{\pm 0.7}$
	DiRecT	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.454$_{\pm 0.176}$	49.0$_{\pm 13.7}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.454$_{\pm 0.175}$	36.4$_{\pm 7.2}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.454$_{\pm 0.176}$	35.2$_{\pm 1.3}$
	GlobMP (Ours)	65.0	65.0$_{\pm 47.7}$	1.00$_{\pm 1.76}$	0.450$_{\pm 0.497}$	2.5$_{\pm 127.9}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.467$_{\pm 0.499}$	1.1$_{\pm 5.4}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.483**$_{\pm 0.500}$	1.2$_{\pm 13.1}$
12	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	8.75$_{\pm 3.11}$	0.464$_{\pm 0.154}$	0.3	0.0	0.0$_{\pm 0.0}$	8.75$_{\pm 3.11}$	0.464$_{\pm 0.154}$	0.3	0.0	0.0$_{\pm 0.0}$	8.75$_{\pm 3.11}$	0.464$_{\pm 0.154}$	0.3
	CD†	0.0	0.0$_{\pm 0.0}$	15.44$_{\pm 3.23}$	0.466$_{\pm 0.145}$	1.2$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	15.44$_{\pm 3.23}$	0.466$_{\pm 0.145}$	1.2$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	15.44$_{\pm 3.23}$	0.466$_{\pm 0.145}$	0.6$_{\pm 0.1}$
	MMD-CBS†	0.0	0.0$_{\pm 0.0}$	—	0.450$_{\pm 0.150}$	61.2$_{\pm 0.9}$	0.0	0.0$_{\pm 0.0}$	—	0.450$_{\pm 0.150}$	61.2$_{\pm 0.9}$	0.0	0.0$_{\pm 0.0}$	—	0.450$_{\pm 0.150}$	61.2$_{\pm 0.9}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	100.8$_{\pm 7.3}$	0.0	—	—	0.000$_{\pm 0.000}$	124.5$_{\pm 6.1}$	0.0	—	—	0.000$_{\pm 0.000}$	122.5$_{\pm 5.6}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	5.02$_{\pm 0.76}$	0.456$_{\pm 0.140}$	4.4$_{\pm 0.4}$	0.0	0.0$_{\pm 0.0}$	6.04$_{\pm 0.83}$	0.457$_{\pm 0.140}$	3.7$_{\pm 0.2}$	0.0	0.0$_{\pm 0.0}$	6.68$_{\pm 0.77}$	0.459$_{\pm 0.140}$	3.7$_{\pm 0.1}$
	PCD-SHD	20.0	0.6$_{\pm 1.3}$	5.75$_{\pm 1.94}$	**0.468**$_{\pm 0.142}$	3.6$_{\pm 0.1}$	20.0	0.5$_{\pm 1.1}$	5.46$_{\pm 1.97}$	0.469$_{\pm 0.142}$	3.6$_{\pm 0.1}$	20.0	0.9$_{\pm 2.0}$	5.23$_{\pm 1.77}$	**0.469**$_{\pm 0.142}$	3.7$_{\pm 0.1}$
	Final Proj.	50.0	29.1$_{\pm 39.5}$	3.49$_{\pm 3.31}$	0.463$_{\pm 0.154}$	6.3$_{\pm 1.4}$	55.0	29.2$_{\pm 40.0}$	2.76$_{\pm 2.80}$	0.463$_{\pm 0.154}$	6.8$_{\pm 0.7}$	65.0	30.2$_{\pm 39.5}$	2.23$_{\pm 2.24}$	0.463$_{\pm 0.154}$	6.8$_{\pm 0.7}$
	DiRecT	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.467$_{\pm 0.148}$	44.1$_{\pm 2.2}$	**100.0**	99.9$_{\pm 0.3}$	0.00$_{\pm 0.01}$	0.466$_{\pm 0.148}$	43.4$_{\pm 1.8}$	**100.0**	99.9$_{\pm 0.3}$	0.00$_{\pm 0.00}$	0.466$_{\pm 0.149}$	43.7$_{\pm 1.7}$
	GlobMP (Ours)	60.0	60.0$_{\pm 49.0}$	3.45$_{\pm 4.99}$	0.467$_{\pm 0.499}$	25.2$_{\pm 102.3}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.492**$_{\pm 0.500}$	2.1$_{\pm 24.5}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.458$_{\pm 0.498}$	2.2$_{\pm 12.3}$
15	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	14.06$_{\pm 4.00}$	0.484$_{\pm 0.109}$	0.3	0.0	0.0$_{\pm 0.0}$	14.06$_{\pm 4.00}$	0.484$_{\pm 0.109}$	0.3	0.0	0.0$_{\pm 0.0}$	14.06$_{\pm 4.00}$	0.484$_{\pm 0.109}$	0.4
	CD†	0.0	0.0$_{\pm 0.0}$	26.51$_{\pm 3.99}$	0.481$_{\pm 0.107}$	1.2$_{\pm 0.0}$	0.0	0.0$_{\pm 0.0}$	26.51$_{\pm 3.99}$	0.481$_{\pm 0.107}$	1.3$_{\pm 0.0}$	0.0	0.0$_{\pm 0.0}$	26.51$_{\pm 3.99}$	0.481$_{\pm 0.107}$	1.2$_{\pm 0.1}$
	MMD-CBS†	0.0	0.0$_{\pm 0.0}$	—	0.483$_{\pm 0.111}$	63.0$_{\pm 2.1}$	0.0	0.0$_{\pm 0.0}$	—	0.483$_{\pm 0.111}$	63.0$_{\pm 2.1}$	0.0	0.0$_{\pm 0.0}$	—	0.483$_{\pm 0.111}$	63.0$_{\pm 2.1}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	141.9$_{\pm 12.5}$	0.0	—	—	0.000$_{\pm 0.000}$	168.6$_{\pm 16.0}$	0.0	—	—	0.000$_{\pm 0.000}$	177.0$_{\pm 31.2}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	11.99$_{\pm 1.72}$	0.485$_{\pm 0.110}$	5.1$_{\pm 0.6}$	0.0	0.0$_{\pm 0.0}$	14.72$_{\pm 1.73}$	**0.485**$_{\pm 0.109}$	4.2$_{\pm 0.3}$	0.0	0.0$_{\pm 0.0}$	16.38$_{\pm 1.74}$	**0.486**$_{\pm 0.109}$	2.8$_{\pm 1.0}$
	PCD-SHD	0.0	0.0$_{\pm 0.0}$	10.01$_{\pm 2.01}$	0.480$_{\pm 0.106}$	3.3$_{\pm 0.9}$	0.0	0.0$_{\pm 0.0}$	9.49$_{\pm 1.89}$	0.481$_{\pm 0.106}$	4.0$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	9.22$_{\pm 1.82}$	0.481$_{\pm 0.105}$	4.1$_{\pm 0.1}$
	Final Proj.	30.0	11.0$_{\pm 26.4}$	5.93$_{\pm 4.23}$	0.485$_{\pm 0.109}$	6.7$_{\pm 2.3}$	35.0	11.1$_{\pm 23.1}$	4.71$_{\pm 3.34}$	0.485$_{\pm 0.109}$	9.1$_{\pm 0.8}$	35.0	10.9$_{\pm 23.2}$	4.14$_{\pm 2.88}$	0.485$_{\pm 0.109}$	9.1$_{\pm 0.7}$
	DiRecT	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.482$_{\pm 0.109}$	60.0$_{\pm 2.7}$	**100.0**	99.9$_{\pm 0.3}$	0.00$_{\pm 0.01}$	0.482$_{\pm 0.109}$	69.0$_{\pm 10.6}$	**100.0**	99.7$_{\pm 1.4}$	0.01$_{\pm 0.03}$	0.482$_{\pm 0.109}$	70.2$_{\pm 10.7}$
	GlobMP (Ours)	30.0	30.0$_{\pm 45.8}$	9.00$_{\pm 7.40}$	**0.487**$_{\pm 0.500}$	300.1$_{\pm 136.1}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.467$_{\pm 0.499}$	2.1$_{\pm 74.3}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.467$_{\pm 0.499}$	2.6$_{\pm 61.0}$
18	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	19.84$_{\pm 5.64}$	0.504$_{\pm 0.119}$	0.3	0.0	0.0$_{\pm 0.0}$	19.84$_{\pm 5.64}$	0.504$_{\pm 0.119}$	0.3	0.0	0.0$_{\pm 0.0}$	19.84$_{\pm 5.64}$	0.504$_{\pm 0.119}$	0.3
	CD†	0.0	0.0$_{\pm 0.0}$	39.00$_{\pm 3.90}$	0.508$_{\pm 0.116}$	1.3$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	39.00$_{\pm 3.90}$	0.508$_{\pm 0.116}$	1.3$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	39.00$_{\pm 3.90}$	0.508$_{\pm 0.116}$	1.3$_{\pm 0.1}$
	MMD-CBS†	0.0	0.0$_{\pm 0.0}$	—	0.503$_{\pm 0.121}$	64.8$_{\pm 2.5}$	0.0	0.0$_{\pm 0.0}$	—	0.503$_{\pm 0.121}$	64.8$_{\pm 2.5}$	0.0	0.0$_{\pm 0.0}$	—	0.503$_{\pm 0.121}$	64.8$_{\pm 2.5}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	163.3$_{\pm 8.1}$	0.0	—	—	0.000$_{\pm 0.000}$	159.7$_{\pm 7.1}$	0.0	—	—	0.000$_{\pm 0.000}$	160.6$_{\pm 7.0}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	20.39$_{\pm 2.78}$	**0.511**$_{\pm 0.110}$	5.0$_{\pm 0.5}$	0.0	0.0$_{\pm 0.0}$	25.36$_{\pm 2.89}$	0.511$_{\pm 0.109}$	4.2$_{\pm 0.2}$	0.0	0.0$_{\pm 0.0}$	28.30$_{\pm 2.87}$	0.511$_{\pm 0.108}$	4.1$_{\pm 0.1}$
	PCD-SHD	0.0	0.0$_{\pm 0.0}$	15.17$_{\pm 2.97}$	0.508$_{\pm 0.114}$	4.1$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	14.31$_{\pm 2.75}$	0.509$_{\pm 0.114}$	4.1$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	14.20$_{\pm 2.61}$	0.509$_{\pm 0.113}$	4.2$_{\pm 0.1}$
	Final Proj.	15.0	3.1$_{\pm 12.2}$	10.49$_{\pm 5.78}$	0.504$_{\pm 0.118}$	7.5$_{\pm 2.3}$	25.0	4.7$_{\pm 17.3}$	8.44$_{\pm 5.26}$	0.504$_{\pm 0.118}$	11.5$_{\pm 0.7}$	35.0	4.8$_{\pm 14.9}$	7.71$_{\pm 4.86}$	0.504$_{\pm 0.118}$	10.0$_{\pm 2.6}$
	DiRecT	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.507$_{\pm 0.118}$	87.5$_{\pm 9.0}$	**100.0**	**99.8**$_{\pm 0.7}$	**0.01**$_{\pm 0.02}$	0.507$_{\pm 0.118}$	88.4$_{\pm 12.7}$	**100.0**	**99.9**$_{\pm 0.3}$	**0.00**$_{\pm 0.01}$	0.507$_{\pm 0.118}$	77.5$_{\pm 8.1}$
	GlobMP (Ours)	0.0	0.0$_{\pm 0.0}$	19.10$_{\pm 5.20}$	0.503$_{\pm 0.500}$	300.1$_{\pm 0.3}$	90.0	90.0$_{\pm 30.0}$	2.85$_{\pm 8.79}$	**0.525**$_{\pm 0.499}$	9.2$_{\pm 101.2}$	90.0	90.0$_{\pm 30.0}$	3.55$_{\pm 10.66}$	**0.519**$_{\pm 0.500}$	4.3$_{\pm 101.1}$
21	Diff. Policy†	0.0	0.0$_{\pm 0.0}$	23.25$_{\pm 7.73}$	0.500$_{\pm 0.121}$	0.3	0.0	0.0$_{\pm 0.0}$	23.25$_{\pm 7.73}$	0.500$_{\pm 0.121}$	0.3	0.0	0.0$_{\pm 0.0}$	23.25$_{\pm 7.73}$	0.500$_{\pm 0.121}$	0.3
	CD†	0.0	0.0$_{\pm 0.0}$	52.34$_{\pm 7.72}$	0.500$_{\pm 0.118}$	0.7$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	52.34$_{\pm 7.72}$	0.500$_{\pm 0.118}$	1.3$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	52.34$_{\pm 7.72}$	0.500$_{\pm 0.118}$	1.3$_{\pm 0.0}$
	MMD-CBS†	0.0	0.0$_{\pm 0.0}$	—	0.502$_{\pm 0.122}$	66.3$_{\pm 4.0}$	0.0	0.0$_{\pm 0.0}$	—	0.502$_{\pm 0.122}$	66.3$_{\pm 4.0}$	0.0	0.0$_{\pm 0.0}$	—	0.502$_{\pm 0.122}$	66.3$_{\pm 4.0}$
	MDOC†	0.0	—	—	0.000$_{\pm 0.000}$	165.8$_{\pm 8.8}$	0.0	—	—	0.000$_{\pm 0.000}$	183.6$_{\pm 8.1}$	0.0	—	—	0.000$_{\pm 0.000}$	180.1$_{\pm 8.2}$
	PCD-LB	0.0	0.0$_{\pm 0.0}$	33.40$_{\pm 2.61}$	0.502$_{\pm 0.113}$	5.5$_{\pm 0.8}$	0.0	0.0$_{\pm 0.0}$	40.60$_{\pm 3.30}$	0.502$_{\pm 0.113}$	4.7$_{\pm 0.3}$	0.0	0.0$_{\pm 0.0}$	44.22$_{\pm 3.72}$	0.502$_{\pm 0.113}$	4.7$_{\pm 0.1}$
	PCD-SHD	0.0	0.0$_{\pm 0.0}$	19.55$_{\pm 3.93}$	0.500$_{\pm 0.115}$	4.5$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	18.93$_{\pm 3.44}$	0.500$_{\pm 0.115}$	4.6$_{\pm 0.1}$	0.0	0.0$_{\pm 0.0}$	18.51$_{\pm 3.52}$	0.500$_{\pm 0.115}$	4.7$_{\pm 0.1}$
	Final Proj.	25.0	3.2$_{\pm 8.2}$	7.22$_{\pm 6.39}$	0.501$_{\pm 0.122}$	14.9$_{\pm 0.7}$	40.0	5.4$_{\pm 11.2}$	5.81$_{\pm 5.73}$	0.501$_{\pm 0.122}$	14.9$_{\pm 0.8}$	45.0	6.6$_{\pm 14.1}$	5.29$_{\pm 5.39}$	0.501$_{\pm 0.122}$	10.2$_{\pm 3.4}$
	DiRecT	**100.0**	**99.9**$_{\pm 0.3}$	**0.00**$_{\pm 0.00}$	0.501$_{\pm 0.123}$	99.2$_{\pm 1.5}$	**100.0**	**99.8**$_{\pm 0.5}$	**0.00**$_{\pm 0.01}$	0.501$_{\pm 0.123}$	97.0$_{\pm 8.7}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	0.501$_{\pm 0.123}$	77.5$_{\pm 0.7}$
	GlobMP (Ours)	15.0	15.0$_{\pm 35.7}$	20.90$_{\pm 11.92}$	**0.502**$_{\pm 0.500}$	300.1$_{\pm 68.7}$	55.0	55.0$_{\pm 49.7}$	11.35$_{\pm 14.57}$	**0.519**$_{\pm 0.500}$	35.2$_{\pm 143.5}$	65.0	65.0$_{\pm 47.7}$	10.75$_{\pm 15.56}$	**0.507**$_{\pm 0.500}$	19.1$_{\pm 136.7}$

Table 8: Multi robot planning performance on `Drop-Region` for $N_a \in \{3, 6, \ldots, 21\}$ under the Low, Medium, and High velocity caps. *SR* and *CS* measure trajectory feasibility, *Coll* reports inter robot collisions, *DA* measures adherence to the environment specific motion pattern, and *Time* is the planning time per configuration. † marks methods that apply no velocity limit, so their three blocks report the same run; in addition, MMD-CBS returns a single plan per configuration rather than a batch of 64 candidates. A dash under *Coll* marks a count that is not commensurable with the others: MDOC and MMD-CBS count colliding time steps, where every other row counts agent pairs with any collision. MDOC's *CS* is omitted because its plans that do not cover every agent were counted as feasible. MMD-CBS plans under a 60 s budget per configuration and GlobMP (Ours) under 300 s per team. For GlobMP (Ours), *Time* is the median over the configurations and its subscript the standard deviation; for the baselines it is the mean, so the two are not directly comparable.

		Low ($v_{\max} = 0.928$)					Medium ($v_{\max} = 1.13$)					High ($v_{\max} = 1.34$)				
N_a	Method	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓
3	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$0.13_{\pm 0.34}$	$0.139_{\pm 0.175}$	0.3	0.0	$0.0_{\pm 0.0}$	$0.13_{\pm 0.34}$	$0.139_{\pm 0.175}$	0.3	0.0	$0.0_{\pm 0.0}$	$0.13_{\pm 0.34}$	$0.139_{\pm 0.175}$	0.3
	CD†	45.0	$5.2_{\pm 21.7}$	$0.11_{\pm 0.32}$	$0.000_{\pm 0.001}$	$0.5_{\pm 0.0}$	45.0	$11.9_{\pm 24.6}$	$0.11_{\pm 0.32}$	$0.000_{\pm 0.001}$	$1.0_{\pm 0.2}$	45.0	$22.2_{\pm 34.5}$	$0.11_{\pm 0.32}$	$0.000_{\pm 0.001}$	$1.2_{\pm 0.0}$
	MMD-CBS†	30.0	$25.0_{\pm 43.3}$	—	$0.000_{\pm 0.000}$	$2.2_{\pm 1.5}$	30.0	$30.0_{\pm 45.8}$	—	$0.000_{\pm 0.000}$	$2.2_{\pm 1.5}$	30.0	$30.0_{\pm 45.8}$	—	$0.000_{\pm 0.000}$	$2.2_{\pm 1.5}$
	MDOC†	25.0	—	—	$0.000_{\pm 0.000}$	$42.7_{\pm 6.5}$	25.0	—	—	$0.000_{\pm 0.000}$	$33.2_{\pm 2.7}$	25.0	—	—	$0.000_{\pm 0.000}$	$38.6_{\pm 3.8}$
	PCD-LB	75.0	$55.9_{\pm 44.1}$	$0.00_{\pm 0.01}$	$0.000_{\pm 0.000}$	$3.1_{\pm 0.1}$	75.0	$56.8_{\pm 44.0}$	$0.01_{\pm 0.02}$	$0.000_{\pm 0.000}$	$2.7_{\pm 0.6}$	80.0	$57.0_{\pm 43.5}$	$0.01_{\pm 0.03}$	$0.000_{\pm 0.000}$	$3.0_{\pm 0.1}$
	PCD-SHD	30.0	$11.3_{\pm 28.3}$	$0.09_{\pm 0.23}$	$0.047_{\pm 0.094}$	$1.2_{\pm 0.1}$	20.0	$11.2_{\pm 28.4}$	$0.08_{\pm 0.21}$	$0.049_{\pm 0.094}$	$1.2_{\pm 0.1}$	20.0	$11.3_{\pm 28.5}$	$0.06_{\pm 0.15}$	$0.049_{\pm 0.092}$	$1.2_{\pm 0.1}$
	Final Proj.	95.0	$86.2_{\pm 26.3}$	$0.11_{\pm 0.28}$	$\mathbf{0.194}_{\pm 0.211}$	$3.4_{\pm 0.5}$	95.0	$86.2_{\pm 26.4}$	$0.11_{\pm 0.28}$	$\mathbf{0.204}_{\pm 0.215}$	$3.6_{\pm 0.6}$	**100.0**	$96.1_{\pm 5.1}$	$0.01_{\pm 0.03}$	$\mathbf{0.234}_{\pm 0.215}$	$3.6_{\pm 0.6}$
	DiRecT	**100.0**	$97.5_{\pm 8.1}$	$0.03_{\pm 0.09}$	$0.114_{\pm 0.149}$	$24.6_{\pm 6.3}$	**100.0**	$96.3_{\pm 12.5}$	$0.04_{\pm 0.13}$	$0.115_{\pm 0.150}$	$26.2_{\pm 6.6}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.115_{\pm 0.150}$	$26.2_{\pm 6.9}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.083_{\pm 0.276}$	$0.3_{\pm 0.1}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.083_{\pm 0.276}$	$0.5_{\pm 0.2}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.083_{\pm 0.276}$	$0.4_{\pm 0.2}$
6	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$1.16_{\pm 0.91}$	$0.148_{\pm 0.107}$	0.3	0.0	$0.0_{\pm 0.0}$	$1.16_{\pm 0.91}$	$0.148_{\pm 0.107}$	0.3	0.0	$0.0_{\pm 0.0}$	$1.16_{\pm 0.91}$	$0.148_{\pm 0.107}$	0.3
	CD†	15.0	$0.0_{\pm 0.0}$	$0.91_{\pm 0.78}$	$0.001_{\pm 0.002}$	$1.2_{\pm 0.0}$	15.0	$0.0_{\pm 0.0}$	$0.91_{\pm 0.78}$	$0.001_{\pm 0.002}$	$1.2_{\pm 0.1}$	15.0	$0.4_{\pm 1.0}$	$0.91_{\pm 0.78}$	$0.001_{\pm 0.002}$	$1.1_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$2.2_{\pm 1.3}$	0.0	$0.0_{\pm 0.0}$	—	—	$2.2_{\pm 1.3}$	0.0	$0.0_{\pm 0.0}$	—	—	$2.2_{\pm 1.3}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$62.9_{\pm 3.7}$	0.0	—	—	$0.000_{\pm 0.000}$	$83.6_{\pm 24.8}$	0.0	—	—	$0.000_{\pm 0.000}$	$83.0_{\pm 17.9}$
	PCD-LB	5.0	$0.1_{\pm 0.3}$	$1.32_{\pm 0.80}$	$0.000_{\pm 0.000}$	$3.2_{\pm 0.2}$	10.0	$0.4_{\pm 1.4}$	$1.36_{\pm 0.81}$	$0.000_{\pm 0.000}$	$3.1_{\pm 0.1}$	20.0	$0.9_{\pm 2.3}$	$1.31_{\pm 0.79}$	$0.000_{\pm 0.000}$	$3.1_{\pm 0.1}$
	PCD-SHD	5.0	$0.1_{\pm 0.3}$	$0.24_{\pm 0.35}$	$0.027_{\pm 0.047}$	$1.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.21_{\pm 0.30}$	$0.028_{\pm 0.049}$	$1.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.17_{\pm 0.23}$	$0.027_{\pm 0.049}$	$1.4_{\pm 0.1}$
	Final Proj.	**100.0**	$47.8_{\pm 35.8}$	$0.56_{\pm 0.55}$	$\mathbf{0.161}_{\pm 0.127}$	$3.4_{\pm 0.6}$	90.0	$45.7_{\pm 37.1}$	$0.57_{\pm 0.50}$	$\mathbf{0.168}_{\pm 0.127}$	$3.4_{\pm 0.6}$	**100.0**	$62.5_{\pm 32.4}$	$0.33_{\pm 0.40}$	$\mathbf{0.212}_{\pm 0.137}$	$3.4_{\pm 0.7}$
	DiRecT	**100.0**	$98.2_{\pm 6.1}$	$0.02_{\pm 0.06}$	$0.073_{\pm 0.106}$	$24.9_{\pm 6.8}$	**100.0**	$97.8_{\pm 6.2}$	$0.02_{\pm 0.06}$	$0.075_{\pm 0.106}$	$25.7_{\pm 6.8}$	**100.0**	$99.8_{\pm 0.7}$	$0.00_{\pm 0.01}$	$0.076_{\pm 0.107}$	$26.7_{\pm 7.4}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.117_{\pm 0.321}$	$0.6_{\pm 0.3}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.125_{\pm 0.331}$	$0.9_{\pm 1.1}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.133_{\pm 0.340}$	$1.1_{\pm 3.6}$
9	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$2.92_{\pm 1.22}$	$0.124_{\pm 0.096}$	0.3	0.0	$0.0_{\pm 0.0}$	$2.92_{\pm 1.22}$	$0.124_{\pm 0.096}$	0.3	0.0	$0.0_{\pm 0.0}$	$2.92_{\pm 1.22}$	$0.124_{\pm 0.096}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$3.40_{\pm 1.07}$	$0.001_{\pm 0.002}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$3.40_{\pm 1.07}$	$0.001_{\pm 0.002}$	$1.2_{\pm 0.0}$	0.0	$0.0_{\pm 0.0}$	$3.40_{\pm 1.07}$	$0.001_{\pm 0.002}$	$1.2_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$2.3_{\pm 1.9}$	0.0	$0.0_{\pm 0.0}$	—	—	$2.3_{\pm 1.9}$	0.0	$0.0_{\pm 0.0}$	—	—	$2.3_{\pm 1.9}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$108.4_{\pm 21.4}$	0.0	—	—	$0.000_{\pm 0.000}$	$95.4_{\pm 14.3}$	0.0	—	—	$0.000_{\pm 0.000}$	$83.7_{\pm 3.8}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$4.77_{\pm 1.42}$	$0.000_{\pm 0.000}$	$3.5_{\pm 0.2}$	0.0	$0.0_{\pm 0.0}$	$5.00_{\pm 1.45}$	$0.000_{\pm 0.000}$	$3.4_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$4.58_{\pm 1.31}$	$0.000_{\pm 0.000}$	$3.5_{\pm 0.1}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$1.06_{\pm 0.75}$	$0.031_{\pm 0.043}$	$1.5_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.98_{\pm 0.67}$	$0.034_{\pm 0.044}$	$1.5_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.78_{\pm 0.56}$	$0.035_{\pm 0.043}$	$1.5_{\pm 0.1}$
	Final Proj.	50.0	$17.3_{\pm 31.9}$	$1.92_{\pm 1.82}$	$\mathbf{0.180}_{\pm 0.109}$	$4.3_{\pm 0.7}$	40.0	$13.0_{\pm 26.5}$	$2.00_{\pm 1.72}$	$\mathbf{0.209}_{\pm 0.118}$	$3.9_{\pm 1.1}$	80.0	$32.2_{\pm 32.6}$	$1.22_{\pm 1.33}$	$\mathbf{0.244}_{\pm 0.123}$	$4.4_{\pm 0.7}$
	DiRecT	90.0	$74.3_{\pm 35.6}$	$0.28_{\pm 0.40}$	$0.080_{\pm 0.084}$	$37.2_{\pm 9.7}$	95.0	$69.5_{\pm 37.5}$	$0.34_{\pm 0.44}$	$0.081_{\pm 0.085}$	$30.0_{\pm 6.7}$	**100.0**	**99.6**$_{\pm 1.4}$	**0.00**$_{\pm 0.01}$	$0.082_{\pm 0.086}$	$31.1_{\pm 7.7}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.061_{\pm 0.240}$	$1.3_{\pm 0.4}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.067_{\pm 0.249}$	$2.0_{\pm 2.7}$	95.0	$95.0_{\pm 21.8}$	$0.35_{\pm 1.53}$	$0.067_{\pm 0.249}$	$2.1_{\pm 66.9}$
12	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$6.19_{\pm 2.67}$	$0.142_{\pm 0.077}$	0.3	0.0	$0.0_{\pm 0.0}$	$6.19_{\pm 2.67}$	$0.142_{\pm 0.077}$	0.3	0.0	$0.0_{\pm 0.0}$	$6.19_{\pm 2.67}$	$0.142_{\pm 0.077}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$7.84_{\pm 1.65}$	$0.002_{\pm 0.005}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$7.84_{\pm 1.65}$	$0.002_{\pm 0.005}$	$0.6_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$7.84_{\pm 1.65}$	$0.002_{\pm 0.005}$	$1.2_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$1.7_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.7_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.7_{\pm 0.9}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$110.6_{\pm 10.0}$	0.0	—	—	$0.000_{\pm 0.000}$	$121.3_{\pm 5.9}$	0.0	—	—	$0.000_{\pm 0.000}$	$107.6_{\pm 5.6}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$11.55_{\pm 2.61}$	$0.000_{\pm 0.002}$	$1.7_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$11.87_{\pm 2.46}$	$0.000_{\pm 0.001}$	$3.7_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$11.07_{\pm 2.25}$	$0.000_{\pm 0.002}$	$3.8_{\pm 0.1}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$2.48_{\pm 1.14}$	$0.021_{\pm 0.022}$	$1.6_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$2.31_{\pm 1.05}$	$0.022_{\pm 0.023}$	$1.6_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$1.88_{\pm 0.92}$	$0.024_{\pm 0.023}$	$1.6_{\pm 0.1}$
	Final Proj.	10.0	$0.8_{\pm 3.1}$	$4.75_{\pm 2.81}$	$\mathbf{0.175}_{\pm 0.078}$	$5.4_{\pm 0.7}$	5.0	$0.7_{\pm 3.1}$	$4.67_{\pm 2.64}$	$\mathbf{0.206}_{\pm 0.089}$	$5.4_{\pm 0.7}$	30.0	$10.5_{\pm 24.4}$	$2.49_{\pm 1.41}$	$\mathbf{0.255}_{\pm 0.096}$	$5.4_{\pm 0.7}$
	DiRecT	55.0	$32.3_{\pm 37.5}$	$1.09_{\pm 0.78}$	$0.054_{\pm 0.048}$	$41.0_{\pm 10.3}$	55.0	$30.4_{\pm 37.4}$	$1.21_{\pm 0.85}$	$0.056_{\pm 0.050}$	$45.0_{\pm 12.0}$	**100.0**	**98.4**$_{\pm 2.7}$	**0.02**$_{\pm 0.03}$	$0.057_{\pm 0.051}$	$47.4_{\pm 12.7}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.067_{\pm 0.249}$	$2.0_{\pm 8.3}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.071_{\pm 0.257}$	$3.2_{\pm 1.6}$	95.0	$95.0_{\pm 21.8}$	$0.50_{\pm 2.18}$	$0.071_{\pm 0.257}$	$4.0_{\pm 78.9}$
15	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$8.53_{\pm 2.25}$	$0.183_{\pm 0.062}$	0.3	0.0	$0.0_{\pm 0.0}$	$8.53_{\pm 2.25}$	$0.183_{\pm 0.062}$	0.3	0.0	$0.0_{\pm 0.0}$	$8.53_{\pm 2.25}$	$0.183_{\pm 0.062}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$13.91_{\pm 2.49}$	$0.001_{\pm 0.002}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$13.91_{\pm 2.49}$	$0.001_{\pm 0.002}$	$1.2_{\pm 0.0}$	0.0	$0.0_{\pm 0.0}$	$13.91_{\pm 2.49}$	$0.001_{\pm 0.002}$	$1.2_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$1.2_{\pm 0.8}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.2_{\pm 0.8}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.2_{\pm 0.8}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$158.3_{\pm 11.1}$	0.0	—	—	$0.000_{\pm 0.000}$	$175.3_{\pm 13.5}$	0.0	—	—	$0.000_{\pm 0.000}$	$167.0_{\pm 72.6}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$22.65_{\pm 2.75}$	$0.000_{\pm 0.000}$	$4.1_{\pm 0.2}$	0.0	$0.0_{\pm 0.0}$	$23.62_{\pm 2.63}$	$0.000_{\pm 0.000}$	$4.1_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$22.71_{\pm 2.48}$	$0.000_{\pm 0.000}$	$4.1_{\pm 0.1}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$2.78_{\pm 1.30}$	$0.026_{\pm 0.028}$	$1.7_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$2.60_{\pm 1.24}$	$0.025_{\pm 0.027}$	$1.8_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$2.11_{\pm 1.00}$	$0.025_{\pm 0.027}$	$1.8_{\pm 0.1}$
	Final Proj.	15.0	$3.2_{\pm 12.9}$	$5.40_{\pm 2.82}$	$\mathbf{0.211}_{\pm 0.071}$	$7.0_{\pm 0.7}$	15.0	$2.0_{\pm 7.8}$	$5.43_{\pm 2.64}$	$\mathbf{0.254}_{\pm 0.074}$	$4.7_{\pm 1.8}$	50.0	$7.0_{\pm 18.1}$	$2.96_{\pm 1.97}$	$\mathbf{0.285}_{\pm 0.076}$	$7.0_{\pm 0.7}$
	DiRecT	70.0	$41.2_{\pm 40.4}$	$1.16_{\pm 1.06}$	$0.083_{\pm 0.037}$	$57.0_{\pm 14.4}$	65.0	$38.3_{\pm 40.2}$	$1.28_{\pm 1.12}$	$0.085_{\pm 0.036}$	$51.8_{\pm 7.9}$	**100.0**	$97.5_{\pm 7.6}$	$0.03_{\pm 0.08}$	$0.087_{\pm 0.036}$	$52.8_{\pm 7.9}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.047_{\pm 0.211}$	$2.3_{\pm 2.3}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.063_{\pm 0.244}$	$4.3_{\pm 3.1}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.067_{\pm 0.249}$	$11.8_{\pm 18.1}$
18	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$14.42_{\pm 4.49}$	$0.124_{\pm 0.051}$	0.3	0.0	$0.0_{\pm 0.0}$	$14.42_{\pm 4.49}$	$0.124_{\pm 0.051}$	0.3	0.0	$0.0_{\pm 0.0}$	$14.42_{\pm 4.49}$	$0.124_{\pm 0.051}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$23.43_{\pm 2.96}$	$0.002_{\pm 0.004}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$23.43_{\pm 2.96}$	$0.002_{\pm 0.004}$	$1.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$23.43_{\pm 2.96}$	$0.002_{\pm 0.004}$	$1.2_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$2.1_{\pm 1.2}$	0.0	$0.0_{\pm 0.0}$	—	—	$2.1_{\pm 1.2}$	0.0	$0.0_{\pm 0.0}$	—	—	$2.1_{\pm 1.2}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$166.5_{\pm 9.9}$	0.0	—	—	$0.000_{\pm 0.000}$	$159.0_{\pm 7.7}$	0.0	—	—	$0.000_{\pm 0.000}$	$142.2_{\pm 7.0}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$37.15_{\pm 5.84}$	$0.000_{\pm 0.000}$	$4.1_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$38.50_{\pm 5.83}$	$0.000_{\pm 0.000}$	$4.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$36.52_{\pm 5.40}$	$0.000_{\pm 0.000}$	$4.0_{\pm 0.6}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$4.46_{\pm 2.06}$	$0.020_{\pm 0.023}$	$1.8_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$4.29_{\pm 1.94}$	$0.022_{\pm 0.026}$	$1.8_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$3.56_{\pm 1.70}$	$0.023_{\pm 0.026}$	$1.8_{\pm 0.1}$
	Final Proj.	10.0	$1.5_{\pm 4.8}$	$8.88_{\pm 5.02}$	$\mathbf{0.140}_{\pm 0.049}$	$8.6_{\pm 0.8}$	10.0	$0.9_{\pm 2.6}$	$8.67_{\pm 4.89}$	$\mathbf{0.166}_{\pm 0.052}$	$7.4_{\pm 1.7}$	10.0	$0.7_{\pm 2.2}$	$5.66_{\pm 2.88}$	$\mathbf{0.199}_{\pm 0.066}$	$8.6_{\pm 0.7}$
	DiRecT	60.0	$31.9_{\pm 37.5}$	$2.05_{\pm 2.15}$	$0.059_{\pm 0.043}$	$68.0_{\pm 9.8}$	55.0	$30.5_{\pm 37.9}$	$2.19_{\pm 2.27}$	$0.060_{\pm 0.042}$	$64.9_{\pm 9.0}$	**100.0**	**91.6**$_{\pm 15.7}$	**0.11**$_{\pm 0.22}$	$0.062_{\pm 0.042}$	$64.4_{\pm 8.0}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.056_{\pm 0.229}$	$3.6_{\pm 9.4}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.056_{\pm 0.229}$	$9.5_{\pm 7.6}$	90.0	$90.0_{\pm 30.0}$	$2.15_{\pm 6.45}$	$0.067_{\pm 0.249}$	$24.2_{\pm 86.0}$
21	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$18.84_{\pm 3.94}$	$0.161_{\pm 0.078}$	0.4	0.0	$0.0_{\pm 0.0}$	$18.84_{\pm 3.94}$	$0.161_{\pm 0.078}$	0.3	0.0	$0.0_{\pm 0.0}$	$18.84_{\pm 3.94}$	$0.161_{\pm 0.078}$	0.4
	CD†	0.0	$0.0_{\pm 0.0}$	$33.21_{\pm 2.01}$	$0.002_{\pm 0.003}$	$0.8_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$33.21_{\pm 2.01}$	$0.002_{\pm 0.003}$	$1.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$33.21_{\pm 2.01}$	$0.002_{\pm 0.003}$	$1.3_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$2.6_{\pm 1.7}$	0.0	$0.0_{\pm 0.0}$	—	—	$2.6_{\pm 1.7}$	0.0	$0.0_{\pm 0.0}$	—	—	$2.6_{\pm 1.7}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$185.4_{\pm 9.2}$	0.0	—	—	$0.000_{\pm 0.000}$	$181.0_{\pm 8.7}$	0.0	—	—	$0.000_{\pm 0.000}$	$178.6_{\pm 8.8}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$53.13_{\pm 5.24}$	$0.000_{\pm 0.000}$	$4.6_{\pm 0.2}$	0.0	$0.0_{\pm 0.0}$	$54.89_{\pm 5.28}$	$0.000_{\pm 0.000}$	$4.6_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$51.21_{\pm 4.97}$	$0.000_{\pm 0.000}$	$4.6_{\pm 0.1}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$6.27_{\pm 1.83}$	$0.027_{\pm 0.023}$	$2.0_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$5.99_{\pm 1.81}$	$0.029_{\pm 0.024}$	$2.0_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$4.79_{\pm 1.53}$	$0.030_{\pm 0.024}$	$2.0_{\pm 0.1}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	$14.01_{\pm 6.39}$	$\mathbf{0.160}_{\pm 0.079}$	$8.4_{\pm 2.8}$	0.0	$0.0_{\pm 0.0}$	$13.54_{\pm 6.03}$	$\mathbf{0.194}_{\pm 0.084}$	$11.9_{\pm 0.8}$	10.0	$0.2_{\pm 0.7}$	$8.83_{\pm 4.51}$	$\mathbf{0.232}_{\pm 0.086}$	$11.8_{\pm 0.7}$
	DiRecT	20.0	$12.2_{\pm 26.9}$	**3.57**$_{\pm 2.00}$	$0.062_{\pm 0.041}$	$89.9_{\pm 13.2}$	20.0	$9.7_{\pm 23.9}$	$3.73_{\pm 2.03}$	$0.066_{\pm 0.043}$	$78.6_{\pm 1.2}$	**100.0**	$88.0_{\pm 10.0}$	**0.14**$_{\pm 0.12}$	$0.070_{\pm 0.046}$	$79.9_{\pm 1.2}$
	GlobMP (Ours)	**65.0**	$65.0_{\pm 47.7}$	$5.45_{\pm 7.65}$	$0.095_{\pm 0.294}$	$15.2_{\pm 136.9}$	**95.0**	**95.0**$_{\pm 21.8}$	**1.05**$_{\pm 4.58}$	$0.086_{\pm 0.280}$	$15.1_{\pm 64.4}$	85.0	$85.0_{\pm 35.7}$	$3.95_{\pm 9.98}$	$0.093_{\pm 0.290}$	$27.1_{\pm 97.5}$

Table 9: Multi robot planning performance on `Corridor-Bay` for $N_a \in \{3, 6, \ldots, 21\}$ under the Low, Medium, and High velocity caps. *SR* and *CS* measure trajectory feasibility, *Coll* reports inter robot collisions, *DA* measures adherence to the environment specific motion pattern, and *Time* is the planning time per configuration. † marks methods that apply no velocity limit, so their three blocks report the same run; in addition, MMD-CBS returns a single plan per configuration rather than a batch of 64 candidates. A dash under *Coll* marks a count that is not commensurable with the others: MDOC and MMD-CBS count colliding time steps, where every other row counts agent pairs with any collision. MDOC's *CS* is omitted because its plans that do not cover every agent were counted as feasible. MMD-CBS plans under a 60 s budget per configuration and GlobMP (Ours) under 300 s per team. For GlobMP (Ours), *Time* is the median over the configurations and its subscript the standard deviation; for the baselines it is the mean, so the two are not directly comparable.

		Low ($v_{\max} = 1.21$)					Medium ($v_{\max} = 1.46$)					High ($v_{\max} = 1.76$)				
N_a	Method	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓
3	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$0.31_{\pm 0.53}$	$0.997_{\pm 0.002}$	0.3	0.0	$0.0_{\pm 0.0}$	$0.31_{\pm 0.53}$	$0.997_{\pm 0.002}$	0.3	0.0	$0.0_{\pm 0.0}$	$0.31_{\pm 0.53}$	$0.997_{\pm 0.002}$	0.3
	CD†	85.0	$12.5_{\pm 20.0}$	$0.12_{\pm 0.26}$	$0.683_{\pm 0.079}$	$1.2_{\pm 0.0}$	85.0	$16.3_{\pm 21.3}$	$0.12_{\pm 0.26}$	$0.683_{\pm 0.079}$	$1.1_{\pm 0.0}$	85.0	$18.0_{\pm 21.5}$	$0.12_{\pm 0.26}$	$0.683_{\pm 0.079}$	$1.2_{\pm 0.0}$
	MMD-CBS†	45.0	$45.0_{\pm 49.7}$	—	$0.890_{\pm 0.068}$	$5.7_{\pm 6.5}$	45.0	$45.0_{\pm 49.7}$	—	$0.890_{\pm 0.068}$	$5.7_{\pm 6.5}$	45.0	$45.0_{\pm 49.7}$	—	$0.890_{\pm 0.068}$	$5.7_{\pm 6.5}$
	MDOC†	10.0	—	—	$0.072_{\pm 0.215}$	$36.1_{\pm 6.6}$	10.0	—	—	$0.072_{\pm 0.215}$	$36.4_{\pm 2.9}$	10.0	—	—	$0.072_{\pm 0.215}$	$37.0_{\pm 3.6}$
	PCD-LB	10.0	$0.5_{\pm 1.7}$	$0.02_{\pm 0.06}$	—	$3.0_{\pm 0.0}$	10.0	$0.5_{\pm 2.1}$	$0.01_{\pm 0.05}$	—	$3.1_{\pm 0.1}$	20.0	$1.5_{\pm 3.8}$	$0.01_{\pm 0.03}$	—	$3.1_{\pm 0.1}$
	PCD-SHD	45.0	$25.9_{\pm 36.9}$	$0.06_{\pm 0.09}$	—	$1.9_{\pm 0.8}$	50.0	$26.9_{\pm 38.0}$	$0.07_{\pm 0.12}$	—	$3.1_{\pm 0.1}$	40.0	$26.9_{\pm 38.5}$	$0.06_{\pm 0.09}$	—	$3.1_{\pm 0.1}$
	Final Proj.	80.0	$53.4_{\pm 42.9}$	$0.00_{\pm 0.00}$	$0.912_{\pm 0.039}$	$2.3_{\pm 0.9}$	85.0	$51.9_{\pm 43.4}$	$0.02_{\pm 0.08}$	$0.911_{\pm 0.040}$	$3.7_{\pm 0.6}$	85.0	$51.9_{\pm 43.4}$	**0.00**$_{\pm 0.00}$	$0.914_{\pm 0.039}$	$3.7_{\pm 0.6}$
	DiRecT	**95.0**	$87.2_{\pm 24.1}$	**0.00**$_{\pm 0.00}$	**0.920**$_{\pm 0.056}$	$18.0_{\pm 0.6}$	**95.0**	$82.0_{\pm 25.8}$	**0.00**$_{\pm 0.00}$	**0.920**$_{\pm 0.056}$	$18.2_{\pm 0.7}$	**95.0**	$80.0_{\pm 27.4}$	**0.00**$_{\pm 0.00}$	**0.921**$_{\pm 0.054}$	$18.2_{\pm 0.7}$
	GlobMP (Ours)	**95.0**	**95.0**$_{\pm 21.8}$	$0.05_{\pm 0.22}$	$0.914_{\pm 0.070}$	$0.7_{\pm 65.2}$	**95.0**	**95.0**$_{\pm 21.8}$	$0.05_{\pm 0.22}$	$0.918_{\pm 0.069}$	$0.9_{\pm 65.2}$	**95.0**	**95.0**$_{\pm 21.8}$	$0.05_{\pm 0.22}$	$0.920_{\pm 0.068}$	$0.7_{\pm 65.2}$
6	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$1.27_{\pm 1.05}$	$0.996_{\pm 0.002}$	0.3	0.0	$0.0_{\pm 0.0}$	$1.27_{\pm 1.05}$	$0.996_{\pm 0.002}$	0.3	0.0	$0.0_{\pm 0.0}$	$1.27_{\pm 1.05}$	$0.996_{\pm 0.002}$	0.3
	CD†	40.0	$3.8_{\pm 12.3}$	$0.70_{\pm 0.78}$	$0.669_{\pm 0.032}$	$1.2_{\pm 0.1}$	40.0	$4.8_{\pm 12.6}$	$0.70_{\pm 0.78}$	$0.669_{\pm 0.032}$	$0.5_{\pm 0.0}$	40.0	$5.8_{\pm 13.6}$	$0.70_{\pm 0.78}$	$0.669_{\pm 0.032}$	$1.2_{\pm 0.1}$
	MMD-CBS†	50.0	$35.0_{\pm 47.7}$	—	$0.880_{\pm 0.034}$	$13.5_{\pm 13.0}$	50.0	$50.0_{\pm 50.0}$	—	$0.880_{\pm 0.034}$	$13.5_{\pm 13.0}$	50.0	$50.0_{\pm 50.0}$	—	$0.880_{\pm 0.034}$	$13.5_{\pm 13.0}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$55.8_{\pm 3.5}$	0.0	—	—	$0.000_{\pm 0.000}$	$74.4_{\pm 18.6}$	0.0	—	—	$0.000_{\pm 0.000}$	$92.2_{\pm 23.6}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$0.84_{\pm 0.63}$	—	$1.4_{\pm 0.2}$	0.0	$0.0_{\pm 0.0}$	$0.66_{\pm 0.47}$	—	$3.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.39_{\pm 0.30}$	—	$3.4_{\pm 0.1}$
	PCD-SHD	40.0	$18.4_{\pm 34.0}$	$0.44_{\pm 0.47}$	—	$3.2_{\pm 0.1}$	40.0	$19.1_{\pm 33.2}$	$0.39_{\pm 0.39}$	—	$3.2_{\pm 0.1}$	50.0	$19.2_{\pm 33.2}$	$0.32_{\pm 0.28}$	—	$3.2_{\pm 0.1}$
	Final Proj.	60.0	$24.5_{\pm 37.2}$	$0.02_{\pm 0.07}$	$0.897_{\pm 0.042}$	$3.8_{\pm 0.6}$	60.0	$23.2_{\pm 36.9}$	$0.01_{\pm 0.03}$	$0.899_{\pm 0.039}$	$2.0_{\pm 0.6}$	60.0	$23.1_{\pm 37.0}$	$0.01_{\pm 0.03}$	$0.903_{\pm 0.037}$	$3.8_{\pm 0.7}$
	DiRecT	**100.0**	$65.8_{\pm 36.9}$	$0.00_{\pm 0.01}$	**0.915**$_{\pm 0.037}$	$18.3_{\pm 0.7}$	**100.0**	$56.8_{\pm 38.4}$	$0.00_{\pm 0.00}$	$0.915_{\pm 0.036}$	$18.3_{\pm 0.7}$	**95.0**	$51.8_{\pm 37.4}$	**0.00**$_{\pm 0.00}$	**0.917**$_{\pm 0.035}$	$18.3_{\pm 0.7}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.915_{\pm 0.071}$	$0.4_{\pm 2.6}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	**0.917**$_{\pm 0.069}$	$0.6_{\pm 1.5}$	85.0	**85.0**$_{\pm 35.7}$	$0.20_{\pm 0.51}$	$0.917_{\pm 0.071}$	$1.7_{\pm 106.5}$
9	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$3.48_{\pm 1.58}$	$0.997_{\pm 0.001}$	0.3	0.0	$0.0_{\pm 0.0}$	$3.48_{\pm 1.58}$	$0.997_{\pm 0.001}$	0.6	0.0	$0.0_{\pm 0.0}$	$3.48_{\pm 1.58}$	$0.997_{\pm 0.001}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$2.86_{\pm 1.16}$	$0.698_{\pm 0.034}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$2.86_{\pm 1.16}$	$0.698_{\pm 0.034}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$2.86_{\pm 1.16}$	$0.698_{\pm 0.034}$	$1.2_{\pm 0.1}$
	MMD-CBS†	5.0	$0.0_{\pm 0.0}$	—	$0.901_{\pm 0.039}$	$22.1_{\pm 32.2}$	5.0	$0.0_{\pm 0.0}$	—	$0.901_{\pm 0.039}$	$22.1_{\pm 32.2}$	5.0	$5.0_{\pm 21.8}$	—	$0.901_{\pm 0.039}$	$22.1_{\pm 32.2}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$117.4_{\pm 19.9}$	0.0	—	—	$0.000_{\pm 0.000}$	$98.4_{\pm 14.9}$	0.0	—	—	$0.000_{\pm 0.000}$	$82.5_{\pm 4.3}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$2.66_{\pm 1.28}$	—	$3.6_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$2.05_{\pm 0.97}$	—	$3.6_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$1.13_{\pm 0.50}$	—	$3.7_{\pm 0.1}$
	PCD-SHD	5.0	$0.5_{\pm 2.4}$	$1.54_{\pm 0.58}$	—	$3.6_{\pm 0.1}$	10.0	$0.9_{\pm 3.4}$	$1.19_{\pm 0.42}$	—	$3.6_{\pm 0.1}$	10.0	$1.0_{\pm 3.8}$	$1.04_{\pm 0.34}$	—	$3.7_{\pm 0.1}$
	Final Proj.	75.0	$22.0_{\pm 28.3}$	$0.03_{\pm 0.03}$	$0.886_{\pm 0.024}$	$4.9_{\pm 0.6}$	70.0	$19.5_{\pm 27.8}$	$0.03_{\pm 0.09}$	$0.889_{\pm 0.025}$	$4.9_{\pm 0.7}$	70.0	$19.5_{\pm 27.8}$	$0.00_{\pm 0.01}$	$0.894_{\pm 0.024}$	$2.6_{\pm 0.6}$
	DiRecT	**95.0**	$76.7_{\pm 36.3}$	**0.00**$_{\pm 0.00}$	**0.919**$_{\pm 0.024}$	$25.0_{\pm 0.8}$	**90.0**	$67.1_{\pm 34.6}$	**0.00**$_{\pm 0.00}$	**0.920**$_{\pm 0.024}$	$25.0_{\pm 0.7}$	**90.0**	$60.2_{\pm 34.5}$	**0.00**$_{\pm 0.00}$	**0.922**$_{\pm 0.023}$	$25.0_{\pm 0.9}$
	GlobMP (Ours)	**95.0**	**95.0**$_{\pm 21.8}$	$0.35_{\pm 1.53}$	$0.911_{\pm 0.064}$	$1.8_{\pm 69.7}$	75.0	**75.0**$_{\pm 43.3}$	$1.10_{\pm 2.07}$	$0.909_{\pm 0.066}$	$2.4_{\pm 128.5}$	75.0	**75.0**$_{\pm 43.3}$	$0.85_{\pm 1.53}$	$0.912_{\pm 0.064}$	$12.5_{\pm 126.4}$
12	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$5.98_{\pm 1.89}$	$0.997_{\pm 0.001}$	0.6	0.0	$0.0_{\pm 0.0}$	$5.98_{\pm 1.89}$	$0.997_{\pm 0.001}$	0.3	0.0	$0.0_{\pm 0.0}$	$5.98_{\pm 1.89}$	$0.997_{\pm 0.001}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$5.62_{\pm 1.27}$	$0.694_{\pm 0.024}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$5.62_{\pm 1.27}$	$0.694_{\pm 0.024}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$5.62_{\pm 1.27}$	$0.694_{\pm 0.024}$	$0.6_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	$0.921_{\pm 0.000}$	$5.1_{\pm 12.7}$	0.0	$0.0_{\pm 0.0}$	—	$0.921_{\pm 0.000}$	$5.1_{\pm 12.7}$	0.0	$0.0_{\pm 0.0}$	—	$0.921_{\pm 0.000}$	$5.1_{\pm 12.7}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$108.7_{\pm 7.5}$	0.0	—	—	$0.000_{\pm 0.000}$	$124.3_{\pm 6.0}$	0.0	—	—	$0.000_{\pm 0.000}$	$128.6_{\pm 6.3}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$7.83_{\pm 1.80}$	—	$3.8_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$5.87_{\pm 1.42}$	—	$4.0_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$3.64_{\pm 0.99}$	—	$1.9_{\pm 0.2}$
	PCD-SHD	5.0	$0.1_{\pm 0.3}$	$2.98_{\pm 1.29}$	—	$3.9_{\pm 0.1}$	5.0	$0.1_{\pm 0.3}$	$2.48_{\pm 1.12}$	—	$2.0_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$2.15_{\pm 0.97}$	—	$4.0_{\pm 0.1}$
	Final Proj.	45.0	$16.4_{\pm 29.4}$	$0.09_{\pm 0.10}$	$0.878_{\pm 0.036}$	$6.1_{\pm 0.7}$	40.0	$15.5_{\pm 30.9}$	$0.04_{\pm 0.08}$	$0.882_{\pm 0.035}$	$6.1_{\pm 0.6}$	40.0	$15.4_{\pm 31.0}$	$0.00_{\pm 0.00}$	$0.887_{\pm 0.033}$	$6.1_{\pm 0.7}$
	DiRecT	**95.0**	$63.2_{\pm 37.9}$	**0.00**$_{\pm 0.01}$	**0.921**$_{\pm 0.027}$	$31.1_{\pm 0.7}$	**95.0**	$54.1_{\pm 38.4}$	**0.00**$_{\pm 0.00}$	**0.923**$_{\pm 0.026}$	$31.1_{\pm 0.7}$	**95.0**	$49.1_{\pm 39.8}$	**0.00**$_{\pm 0.00}$	**0.925**$_{\pm 0.024}$	$31.1_{\pm 0.7}$
	GlobMP (Ours)	85.0	**85.0**$_{\pm 35.7}$	$1.15_{\pm 2.74}$	$0.903_{\pm 0.067}$	$4.5_{\pm 105.6}$	75.0	**75.0**$_{\pm 43.3}$	$1.75_{\pm 3.05}$	$0.905_{\pm 0.068}$	$8.4_{\pm 127.3}$	70.0	**70.0**$_{\pm 45.8}$	$1.75_{\pm 2.77}$	$0.908_{\pm 0.066}$	$17.1_{\pm 130.8}$
15	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$10.78_{\pm 2.90}$	$0.996_{\pm 0.001}$	0.3	0.0	$0.0_{\pm 0.0}$	$10.78_{\pm 2.90}$	$0.996_{\pm 0.001}$	0.3	0.0	$0.0_{\pm 0.0}$	$10.78_{\pm 2.90}$	$0.996_{\pm 0.001}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$10.72_{\pm 2.10}$	$0.686_{\pm 0.021}$	$1.2_{\pm 0.0}$	0.0	$0.0_{\pm 0.0}$	$10.72_{\pm 2.10}$	$0.686_{\pm 0.021}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$10.72_{\pm 2.10}$	$0.686_{\pm 0.021}$	$1.2_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	$0.844_{\pm 0.000}$	$8.1_{\pm 17.6}$	0.0	$0.0_{\pm 0.0}$	—	$0.844_{\pm 0.000}$	$8.1_{\pm 17.6}$	0.0	$0.0_{\pm 0.0}$	—	$0.844_{\pm 0.000}$	$8.1_{\pm 17.6}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$164.4_{\pm 13.3}$	0.0	—	—	$0.000_{\pm 0.000}$	$175.2_{\pm 19.9}$	0.0	—	—	$0.000_{\pm 0.000}$	$171.8_{\pm 26.1}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$18.53_{\pm 1.79}$	—	$4.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$15.04_{\pm 1.50}$	—	$4.4_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$10.75_{\pm 1.19}$	—	$4.4_{\pm 0.1}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$5.32_{\pm 1.80}$	—	$4.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$4.44_{\pm 1.50}$	—	$4.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$3.73_{\pm 1.21}$	—	$4.4_{\pm 0.1}$
	Final Proj.	40.0	$5.2_{\pm 13.1}$	$0.22_{\pm 0.24}$	$0.851_{\pm 0.030}$	$4.3_{\pm 1.0}$	30.0	$6.7_{\pm 17.7}$	$0.08_{\pm 0.13}$	$0.858_{\pm 0.030}$	$7.8_{\pm 0.7}$	30.0	$7.0_{\pm 18.3}$	$0.02_{\pm 0.04}$	$0.863_{\pm 0.028}$	$7.8_{\pm 0.7}$
	DiRecT	**100.0**	$60.2_{\pm 30.4}$	**0.00**$_{\pm 0.01}$	**0.904**$_{\pm 0.018}$	$40.4_{\pm 0.7}$	**95.0**	$39.7_{\pm 33.0}$	**0.00**$_{\pm 0.00}$	**0.906**$_{\pm 0.017}$	$40.4_{\pm 0.7}$	**90.0**	$33.6_{\pm 32.8}$	**0.00**$_{\pm 0.00}$	**0.908**$_{\pm 0.017}$	$40.5_{\pm 0.9}$
	GlobMP (Ours)	75.0	**75.0**$_{\pm 43.3}$	$2.85_{\pm 4.99}$	$0.892_{\pm 0.074}$	$3.9_{\pm 127.2}$	65.0	**65.0**$_{\pm 47.7}$	$3.70_{\pm 5.45}$	$0.895_{\pm 0.072}$	$36.9_{\pm 133.7}$	65.0	**65.0**$_{\pm 47.7}$	$3.15_{\pm 4.63}$	$0.899_{\pm 0.072}$	$93.9_{\pm 129.5}$
18	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$15.87_{\pm 4.43}$	$0.996_{\pm 0.001}$	0.3	0.0	$0.0_{\pm 0.0}$	$15.87_{\pm 4.43}$	$0.996_{\pm 0.001}$	0.3	0.0	$0.0_{\pm 0.0}$	$15.87_{\pm 4.43}$	$0.996_{\pm 0.001}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$16.96_{\pm 3.44}$	$0.684_{\pm 0.022}$	$1.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$16.96_{\pm 3.44}$	$0.684_{\pm 0.022}$	$1.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$16.96_{\pm 3.44}$	$0.684_{\pm 0.022}$	$1.3_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	$0.898_{\pm 0.016}$	$8.5_{\pm 18.7}$	0.0	$0.0_{\pm 0.0}$	—	$0.898_{\pm 0.016}$	$8.5_{\pm 18.7}$	0.0	$0.0_{\pm 0.0}$	—	$0.898_{\pm 0.016}$	$8.5_{\pm 18.7}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$147.2_{\pm 7.4}$	0.0	—	—	$0.000_{\pm 0.000}$	$157.6_{\pm 7.0}$	0.0	—	—	$0.000_{\pm 0.000}$	$159.9_{\pm 7.2}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$30.77_{\pm 4.62}$	—	$4.4_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$24.93_{\pm 3.85}$	—	$4.5_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$18.60_{\pm 2.71}$	—	$4.6_{\pm 0.1}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$8.11_{\pm 2.72}$	—	$4.4_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$6.79_{\pm 2.30}$	—	$4.4_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$5.77_{\pm 1.97}$	—	$4.5_{\pm 0.1}$
	Final Proj.	20.0	$6.6_{\pm 18.2}$	$0.38_{\pm 0.36}$	$0.837_{\pm 0.040}$	$10.2_{\pm 0.7}$	20.0	$6.6_{\pm 15.8}$	$0.09_{\pm 0.09}$	$0.846_{\pm 0.038}$	$8.8_{\pm 2.0}$	20.0	$5.5_{\pm 13.1}$	$0.02_{\pm 0.03}$	$0.853_{\pm 0.038}$	$10.2_{\pm 0.8}$
	DiRecT	**90.0**	$43.4_{\pm 34.4}$	**0.03**$_{\pm 0.04}$	**0.902**$_{\pm 0.026}$	$53.3_{\pm 0.9}$	**90.0**	$30.7_{\pm 30.5}$	**0.00**$_{\pm 0.00}$	**0.905**$_{\pm 0.025}$	$53.3_{\pm 0.9}$	**85.0**	$23.4_{\pm 28.2}$	**0.00**$_{\pm 0.00}$	**0.908**$_{\pm 0.025}$	$53.1_{\pm 0.7}$
	GlobMP (Ours)	65.0	**65.0**$_{\pm 47.7}$	$6.05_{\pm 8.70}$	$0.883_{\pm 0.071}$	$28.6_{\pm 135.8}$	60.0	**60.0**$_{\pm 49.0}$	$7.85_{\pm 10.09}$	$0.892_{\pm 0.070}$	$11.2_{\pm 143.1}$	40.0	**40.0**$_{\pm 49.0}$	$9.95_{\pm 8.68}$	$0.894_{\pm 0.068}$	$300.0_{\pm 137.3}$
21	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$20.72_{\pm 5.04}$	$0.997_{\pm 0.001}$	0.3	0.0	$0.0_{\pm 0.0}$	$20.72_{\pm 5.04}$	$0.997_{\pm 0.001}$	0.3	0.0	$0.0_{\pm 0.0}$	$20.72_{\pm 5.04}$	$0.997_{\pm 0.001}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$24.06_{\pm 3.00}$	$0.692_{\pm 0.016}$	$1.3_{\pm 0.0}$	0.0	$0.0_{\pm 0.0}$	$24.06_{\pm 3.00}$	$0.692_{\pm 0.016}$	$0.8_{\pm 0.2}$	0.0	$0.0_{\pm 0.0}$	$24.06_{\pm 3.00}$	$0.692_{\pm 0.016}$	$1.3_{\pm 0.0}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	$0.954_{\pm 0.015}$	$19.8_{\pm 41.4}$	0.0	$0.0_{\pm 0.0}$	—	$0.954_{\pm 0.015}$	$19.8_{\pm 41.4}$	0.0	$0.0_{\pm 0.0}$	—	$0.954_{\pm 0.015}$	$19.8_{\pm 41.4}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$183.7_{\pm 8.7}$	0.0	—	—	$0.000_{\pm 0.000}$	$180.8_{\pm 8.7}$	0.0	—	—	$0.000_{\pm 0.000}$	$179.5_{\pm 8.6}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$42.63_{\pm 4.11}$	—	$5.0_{\pm 0.2}$	0.0	$0.0_{\pm 0.0}$	$34.30_{\pm 3.31}$	—	$5.1_{\pm 0.2}$	0.0	$0.0_{\pm 0.0}$	$26.27_{\pm 2.54}$	—	$4.4_{\pm 1.1}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$10.66_{\pm 3.55}$	—	$4.8_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$9.02_{\pm 2.99}$	—	$4.9_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$7.59_{\pm 2.61}$	—	$2.5_{\pm 0.2}$
	Final Proj.	25.0	$8.2_{\pm 21.8}$	$0.70_{\pm 0.81}$	$0.832_{\pm 0.033}$	$13.0_{\pm 0.8}$	30.0	$6.7_{\pm 19.6}$	$0.23_{\pm 0.30}$	$0.840_{\pm 0.032}$	$9.6_{\pm 2.9}$	35.0	$6.7_{\pm 19.5}$	$0.07_{\pm 0.12}$	$0.846_{\pm 0.032}$	$13.1_{\pm 0.8}$
	DiRecT	**100.0**	**52.2**$_{\pm 29.1}$	**0.04**$_{\pm 0.06}$	**0.906**$_{\pm 0.024}$	$68.5_{\pm 0.7}$	**100.0**	$35.5_{\pm 29.5}$	**0.00**$_{\pm 0.00}$	**0.910**$_{\pm 0.022}$	$68.5_{\pm 0.7}$	**85.0**	$26.8_{\pm 29.7}$	**0.00**$_{\pm 0.00}$	**0.913**$_{\pm 0.021}$	$68.5_{\pm 0.7}$
	GlobMP (Ours)	45.0	$45.0_{\pm 49.7}$	$12.90_{\pm 12.22}$	$0.886_{\pm 0.077}$	$300.0_{\pm 141.1}$	60.0	**60.0**$_{\pm 49.0}$	$8.45_{\pm 10.77}$	$0.900_{\pm 0.068}$	$33.5_{\pm 137.9}$	70.0	**70.0**$_{\pm 45.8}$	$5.60_{\pm 8.79}$	$0.899_{\pm 0.071}$	$83.2_{\pm 116.2}$

Table 10: Multi robot planning performance on `Cross-Room` for $N_a \in \{3, 6, \ldots, 21\}$ under the Low, Medium, and High velocity caps. *SR* and *CS* measure trajectory feasibility, *Coll* reports inter robot collisions, *DA* measures adherence to the environment specific motion pattern, and *Time* is the planning time per configuration. † marks methods that apply no velocity limit, so their three blocks report the same run; in addition, MMD-CBS returns a single plan per configuration rather than a batch of 64 candidates. A dash under *Coll* marks a count that is not commensurable with the others: MDOC and MMD-CBS count colliding time steps, where every other row counts agent pairs with any collision. MDOC's *CS* is omitted because its plans that do not cover every agent were counted as feasible. MMD-CBS plans under a 60 s budget per configuration and GlobMP (Ours) under 300 s per team. For GlobMP (Ours), *Time* is the median over the configurations and its subscript the standard deviation; for the baselines it is the mean, so the two are not directly comparable.

		Low ($v_{\max} = 0.928$)					Medium ($v_{\max} = 1.13$)					High ($v_{\max} = 1.34$)				
N_a	Method	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓	SR↑	CS↑	Coll↓	DA↑	Time↓
3	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$0.30_{\pm 0.55}$	$0.985_{\pm 0.007}$	0.6	0.0	$0.0_{\pm 0.0}$	$0.30_{\pm 0.55}$	$0.985_{\pm 0.007}$	0.5	0.0	$0.0_{\pm 0.0}$	$0.30_{\pm 0.55}$	$0.985_{\pm 0.007}$	0.5
	CD†	20.0	$5.6_{\pm 15.6}$	$0.02_{\pm 0.05}$	$0.662_{\pm 0.070}$	$1.1_{\pm 0.0}$	20.0	$9.3_{\pm 25.5}$	$0.02_{\pm 0.05}$	$0.662_{\pm 0.070}$	$1.1_{\pm 0.1}$	20.0	$10.1_{\pm 27.3}$	$0.02_{\pm 0.05}$	$0.662_{\pm 0.070}$	$1.1_{\pm 0.0}$
	MMD-CBS†	10.0	$10.0_{\pm 30.0}$	—	$0.887_{\pm 0.049}$	$0.7_{\pm 0.4}$	10.0	$10.0_{\pm 30.0}$	—	$0.887_{\pm 0.049}$	$0.7_{\pm 0.4}$	10.0	$10.0_{\pm 30.0}$	—	$0.887_{\pm 0.049}$	$0.7_{\pm 0.4}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$37.9_{\pm 2.4}$	0.0	—	—	$0.000_{\pm 0.000}$	$36.2_{\pm 2.5}$	0.0	—	—	$0.000_{\pm 0.000}$	$35.4_{\pm 2.6}$
	PCD-LB	55.0	$16.6_{\pm 30.6}$	$0.05_{\pm 0.17}$	—	$2.7_{\pm 0.2}$	45.0	$15.6_{\pm 30.8}$	$0.05_{\pm 0.15}$	—	$1.5_{\pm 0.1}$	45.0	$16.3_{\pm 30.6}$	$0.04_{\pm 0.14}$	—	$1.5_{\pm 0.1}$
	PCD-SHD	10.0	$4.0_{\pm 16.3}$	$0.00_{\pm 0.01}$	—	$2.0_{\pm 0.6}$	10.0	$3.8_{\pm 15.3}$	$0.00_{\pm 0.01}$	—	$1.6_{\pm 0.4}$	10.0	$4.0_{\pm 16.3}$	$0.00_{\pm 0.01}$	—	$1.5_{\pm 0.1}$
	Final Proj.	5.0	$2.7_{\pm 11.9}$	$0.01_{\pm 0.02}$	**0.910**$_{\pm 0.035}$	$4.4_{\pm 0.9}$	5.0	$2.7_{\pm 11.9}$	$0.01_{\pm 0.01}$	**0.907**$_{\pm 0.035}$	$4.4_{\pm 0.9}$	5.0	$2.7_{\pm 11.9}$	$0.00_{\pm 0.01}$	**0.909**$_{\pm 0.035}$	$2.4_{\pm 1.0}$
	DiRecT	10.0	$5.0_{\pm 21.1}$	$0.03_{\pm 0.08}$	$0.899_{\pm 0.038}$	$33.1_{\pm 6.8}$	10.0	$5.0_{\pm 21.1}$	$0.00_{\pm 0.01}$	$0.902_{\pm 0.038}$	$30.4_{\pm 8.1}$	10.0	$4.8_{\pm 20.8}$	$0.00_{\pm 0.00}$	$0.904_{\pm 0.038}$	$29.1_{\pm 8.4}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.804_{\pm 0.142}$	$0.4_{\pm 0.3}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.802_{\pm 0.133}$	$0.4_{\pm 0.3}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.809_{\pm 0.143}$	$0.6_{\pm 0.7}$
6	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$1.59_{\pm 0.98}$	$0.985_{\pm 0.004}$	0.6	0.0	$0.0_{\pm 0.0}$	$1.59_{\pm 0.98}$	$0.985_{\pm 0.004}$	0.6	0.0	$0.0_{\pm 0.0}$	$1.59_{\pm 0.98}$	$0.985_{\pm 0.004}$	0.6
	CD†	10.0	$1.2_{\pm 5.4}$	$0.40_{\pm 0.41}$	$0.671_{\pm 0.041}$	$1.1_{\pm 0.1}$	10.0	$3.2_{\pm 14.0}$	$0.40_{\pm 0.41}$	$0.671_{\pm 0.041}$	$0.5_{\pm 0.1}$	10.0	$3.7_{\pm 16.0}$	$0.40_{\pm 0.41}$	$0.671_{\pm 0.041}$	$1.1_{\pm 0.0}$
	MMD-CBS†	5.0	$0.0_{\pm 0.0}$	—	$0.849_{\pm 0.000}$	$1.0_{\pm 1.1}$	5.0	$0.0_{\pm 0.0}$	—	$0.849_{\pm 0.000}$	$1.0_{\pm 1.1}$	5.0	$0.0_{\pm 0.0}$	—	$0.849_{\pm 0.000}$	$1.0_{\pm 1.1}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$66.3_{\pm 3.8}$	0.0	—	—	$0.000_{\pm 0.000}$	$64.4_{\pm 6.6}$	0.0	—	—	$0.000_{\pm 0.000}$	$61.7_{\pm 6.2}$
	PCD-LB	25.0	$8.6_{\pm 22.8}$	$1.01_{\pm 0.48}$	—	$2.7_{\pm 0.7}$	20.0	$7.7_{\pm 22.1}$	$1.07_{\pm 0.46}$	—	$2.5_{\pm 0.5}$	40.0	$7.7_{\pm 21.9}$	$1.01_{\pm 0.46}$	—	$1.9_{\pm 0.1}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$0.17_{\pm 0.26}$	—	$2.1_{\pm 0.6}$	0.0	$0.0_{\pm 0.0}$	$0.14_{\pm 0.20}$	—	$2.1_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	$0.12_{\pm 0.16}$	—	$1.8_{\pm 0.1}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	$0.08_{\pm 0.12}$	$0.906_{\pm 0.029}$	$3.9_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	$0.07_{\pm 0.11}$	$0.902_{\pm 0.030}$	$3.9_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	$0.06_{\pm 0.09}$	$0.903_{\pm 0.031}$	$3.9_{\pm 0.8}$
	DiRecT	0.0	$0.0_{\pm 0.0}$	$0.07_{\pm 0.14}$	**0.908**$_{\pm 0.031}$	$24.5_{\pm 7.3}$	0.0	$0.0_{\pm 0.0}$	$0.03_{\pm 0.11}$	**0.908**$_{\pm 0.033}$	$24.4_{\pm 7.3}$	0.0	$0.0_{\pm 0.0}$	$0.01_{\pm 0.02}$	**0.909**$_{\pm 0.034}$	$27.1_{\pm 7.2}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.783_{\pm 0.133}$	$1.2_{\pm 52.1}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.786_{\pm 0.136}$	$1.5_{\pm 1.3}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.787_{\pm 0.133}$	$2.7_{\pm 2.3}$
9	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$3.23_{\pm 1.34}$	$0.985_{\pm 0.003}$	0.6	0.0	$0.0_{\pm 0.0}$	$3.23_{\pm 1.34}$	$0.985_{\pm 0.003}$	0.6	0.0	$0.0_{\pm 0.0}$	$3.23_{\pm 1.34}$	$0.985_{\pm 0.003}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$1.42_{\pm 0.71}$	$0.659_{\pm 0.030}$	$1.1_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$1.42_{\pm 0.71}$	$0.659_{\pm 0.030}$	$1.1_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$1.42_{\pm 0.71}$	$0.659_{\pm 0.030}$	$1.1_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$1.0_{\pm 0.8}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.0_{\pm 0.8}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.0_{\pm 0.8}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$94.2_{\pm 4.1}$	0.0	—	—	$0.000_{\pm 0.000}$	$92.9_{\pm 3.9}$	0.0	—	—	$0.000_{\pm 0.000}$	$93.0_{\pm 6.6}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$3.96_{\pm 1.41}$	—	$2.1_{\pm 0.4}$	0.0	$0.0_{\pm 0.0}$	$4.11_{\pm 1.38}$	—	$3.1_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	$3.68_{\pm 1.18}$	—	$2.2_{\pm 0.1}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$0.28_{\pm 0.27}$	—	$2.0_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.26_{\pm 0.22}$	—	$2.1_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.22_{\pm 0.16}$	—	$2.0_{\pm 0.1}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	$0.14_{\pm 0.17}$	$0.895_{\pm 0.032}$	$2.8_{\pm 0.8}$	0.0	$0.0_{\pm 0.0}$	$0.11_{\pm 0.13}$	$0.891_{\pm 0.034}$	$5.1_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	$0.08_{\pm 0.10}$	$0.893_{\pm 0.034}$	$5.1_{\pm 1.0}$
	DiRecT	0.0	$0.0_{\pm 0.0}$	$0.20_{\pm 0.26}$	**0.902**$_{\pm 0.027}$	$35.4_{\pm 9.8}$	0.0	$0.0_{\pm 0.0}$	$0.06_{\pm 0.08}$	**0.906**$_{\pm 0.028}$	$36.8_{\pm 9.8}$	0.0	$0.0_{\pm 0.0}$	$0.02_{\pm 0.03}$	**0.908**$_{\pm 0.027}$	$38.5_{\pm 9.6}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.782_{\pm 0.133}$	$2.3_{\pm 1.1}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.783_{\pm 0.124}$	$3.3_{\pm 3.0}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.782_{\pm 0.132}$	$7.3_{\pm 20.9}$
12	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$7.06_{\pm 2.70}$	$0.985_{\pm 0.003}$	0.3	0.0	$0.0_{\pm 0.0}$	$7.06_{\pm 2.70}$	$0.985_{\pm 0.003}$	0.6	0.0	$0.0_{\pm 0.0}$	$7.06_{\pm 2.70}$	$0.985_{\pm 0.003}$	0.6
	CD†	0.0	$0.0_{\pm 0.0}$	$4.08_{\pm 1.26}$	$0.663_{\pm 0.020}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$4.08_{\pm 1.26}$	$0.663_{\pm 0.020}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$4.08_{\pm 1.26}$	$0.663_{\pm 0.020}$	$0.6_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$1.1_{\pm 1.2}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.1_{\pm 1.2}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.1_{\pm 1.2}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$122.0_{\pm 5.3}$	0.0	—	—	$0.000_{\pm 0.000}$	$110.7_{\pm 5.9}$	0.0	—	—	$0.000_{\pm 0.000}$	$108.8_{\pm 4.8}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$9.93_{\pm 1.54}$	—	$3.4_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	$10.31_{\pm 1.59}$	—	$2.4_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$9.16_{\pm 1.46}$	—	$4.5_{\pm 0.1}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$0.60_{\pm 0.36}$	—	$4.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.53_{\pm 0.26}$	—	$2.4_{\pm 0.3}$	0.0	$0.0_{\pm 0.0}$	$0.46_{\pm 0.26}$	—	$4.1_{\pm 0.8}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	$0.41_{\pm 0.42}$	$0.881_{\pm 0.026}$	$5.4_{\pm 2.1}$	0.0	$0.0_{\pm 0.0}$	$0.33_{\pm 0.32}$	$0.876_{\pm 0.028}$	$7.5_{\pm 1.0}$	0.0	$0.0_{\pm 0.0}$	$0.33_{\pm 0.28}$	$0.877_{\pm 0.029}$	$7.5_{\pm 0.9}$
	DiRecT	0.0	$0.0_{\pm 0.0}$	$0.44_{\pm 0.48}$	**0.898**$_{\pm 0.020}$	$49.1_{\pm 12.6}$	0.0	$0.0_{\pm 0.0}$	$0.10_{\pm 0.14}$	**0.900**$_{\pm 0.020}$	$49.2_{\pm 12.6}$	0.0	$0.0_{\pm 0.0}$	$0.04_{\pm 0.05}$	**0.902**$_{\pm 0.020}$	$49.8_{\pm 12.5}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.773_{\pm 0.140}$	$3.1_{\pm 57.3}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.778_{\pm 0.135}$	$5.8_{\pm 11.5}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.778_{\pm 0.140}$	$9.8_{\pm 42.3}$
15	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$9.58_{\pm 2.09}$	$0.985_{\pm 0.003}$	0.6	0.0	$0.0_{\pm 0.0}$	$9.58_{\pm 2.09}$	$0.985_{\pm 0.003}$	0.6	0.0	$0.0_{\pm 0.0}$	$9.58_{\pm 2.09}$	$0.985_{\pm 0.003}$	0.6
	CD†	0.0	$0.0_{\pm 0.0}$	$6.69_{\pm 1.73}$	$0.662_{\pm 0.016}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$6.69_{\pm 1.73}$	$0.662_{\pm 0.016}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$6.69_{\pm 1.73}$	$0.662_{\pm 0.016}$	$1.2_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$1.0_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.0_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.0_{\pm 0.9}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$140.7_{\pm 6.7}$	0.0	—	—	$0.000_{\pm 0.000}$	$134.0_{\pm 6.8}$	0.0	—	—	$0.000_{\pm 0.000}$	$134.2_{\pm 6.1}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$20.03_{\pm 2.29}$	—	$3.0_{\pm 0.6}$	0.0	$0.0_{\pm 0.0}$	$20.97_{\pm 2.25}$	—	$2.9_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$19.26_{\pm 1.97}$	—	$3.0_{\pm 0.4}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$0.84_{\pm 0.37}$	—	$3.9_{\pm 1.1}$	0.0	$0.0_{\pm 0.0}$	$0.78_{\pm 0.35}$	—	$2.4_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.68_{\pm 0.31}$	—	$2.4_{\pm 0.1}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	$0.59_{\pm 0.46}$	$0.872_{\pm 0.019}$	$7.9_{\pm 2.4}$	0.0	$0.0_{\pm 0.0}$	$0.51_{\pm 0.37}$	$0.867_{\pm 0.020}$	$11.2_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	$0.44_{\pm 0.31}$	$0.868_{\pm 0.020}$	$8.8_{\pm 2.8}$
	DiRecT	0.0	$0.0_{\pm 0.0}$	$0.64_{\pm 0.50}$	**0.892**$_{\pm 0.014}$	$68.2_{\pm 15.4}$	0.0	$0.0_{\pm 0.0}$	$0.20_{\pm 0.24}$	**0.894**$_{\pm 0.014}$	$69.0_{\pm 15.5}$	0.0	$0.0_{\pm 0.0}$	**0.07**$_{\pm 0.08}$	**0.896**$_{\pm 0.014}$	$75.8_{\pm 12.1}$
	GlobMP (Ours)	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.774_{\pm 0.142}$	$3.9_{\pm 23.9}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.772_{\pm 0.134}$	$9.2_{\pm 44.7}$	**95.0**	**95.0**$_{\pm 21.8}$	$0.80_{\pm 3.49}$	$0.775_{\pm 0.139}$	$21.9_{\pm 62.5}$
18	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$13.49_{\pm 2.49}$	$0.985_{\pm 0.002}$	0.3	0.0	$0.0_{\pm 0.0}$	$13.49_{\pm 2.49}$	$0.985_{\pm 0.002}$	0.6	0.0	$0.0_{\pm 0.0}$	$13.49_{\pm 2.49}$	$0.985_{\pm 0.002}$	0.3
	CD†	0.0	$0.0_{\pm 0.0}$	$11.98_{\pm 2.21}$	$0.665_{\pm 0.016}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$11.98_{\pm 2.21}$	$0.665_{\pm 0.016}$	$1.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$11.98_{\pm 2.21}$	$0.665_{\pm 0.016}$	$1.2_{\pm 0.1}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$0.9_{\pm 0.6}$	0.0	$0.0_{\pm 0.0}$	—	—	$0.9_{\pm 0.6}$	0.0	$0.0_{\pm 0.0}$	—	—	$0.9_{\pm 0.6}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$161.4_{\pm 7.5}$	0.0	—	—	$0.000_{\pm 0.000}$	$199.2_{\pm 11.4}$	0.0	—	—	$0.000_{\pm 0.000}$	$173.8_{\pm 8.0}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$32.89_{\pm 2.64}$	—	$3.1_{\pm 0.2}$	0.0	$0.0_{\pm 0.0}$	$34.26_{\pm 2.78}$	—	$5.7_{\pm 0.5}$	0.0	$0.0_{\pm 0.0}$	$31.50_{\pm 2.87}$	—	$3.4_{\pm 0.1}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$1.07_{\pm 0.63}$	—	$3.2_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$0.99_{\pm 0.57}$	—	$2.9_{\pm 0.2}$	0.0	$0.0_{\pm 0.0}$	$0.88_{\pm 0.43}$	—	$3.3_{\pm 0.1}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	**0.56**$_{\pm 0.38}$	$0.869_{\pm 0.016}$	$14.9_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	$0.45_{\pm 0.33}$	$0.865_{\pm 0.017}$	$11.6_{\pm 3.7}$	0.0	$0.0_{\pm 0.0}$	$0.38_{\pm 0.26}$	$0.866_{\pm 0.018}$	$14.9_{\pm 0.9}$
	DiRecT	0.0	$0.0_{\pm 0.0}$	$1.02_{\pm 0.65}$	**0.900**$_{\pm 0.012}$	$89.6_{\pm 24.7}$	0.0	$0.0_{\pm 0.0}$	**0.27**$_{\pm 0.21}$	**0.903**$_{\pm 0.012}$	$104.4_{\pm 3.8}$	0.0	$0.0_{\pm 0.0}$	$0.09_{\pm 0.06}$	**0.905**$_{\pm 0.011}$	$58.6_{\pm 25.2}$
	GlobMP (Ours)	**95.0**	**95.0**$_{\pm 21.8}$	$1.00_{\pm 4.36}$	$0.789_{\pm 0.134}$	$8.1_{\pm 96.3}$	**95.0**	**95.0**$_{\pm 21.8}$	$1.00_{\pm 4.36}$	$0.791_{\pm 0.130}$	$11.1_{\pm 61.9}$	**100.0**	**100.0**$_{\pm 0.0}$	**0.00**$_{\pm 0.00}$	$0.779_{\pm 0.130}$	$60.5_{\pm 84.3}$
21	Diff. Policy†	0.0	$0.0_{\pm 0.0}$	$20.13_{\pm 3.78}$	$0.985_{\pm 0.002}$	0.7	0.0	$0.0_{\pm 0.0}$	$20.13_{\pm 3.78}$	$0.985_{\pm 0.002}$	0.7	0.0	$0.0_{\pm 0.0}$	$20.13_{\pm 3.78}$	$0.985_{\pm 0.002}$	0.7
	CD†	0.0	$0.0_{\pm 0.0}$	$18.02_{\pm 3.46}$	$0.659_{\pm 0.017}$	$1.2_{\pm 0.0}$	0.0	$0.0_{\pm 0.0}$	$18.02_{\pm 3.46}$	$0.659_{\pm 0.017}$	$0.6_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$18.02_{\pm 3.46}$	$0.659_{\pm 0.017}$	$1.3_{\pm 0.0}$
	MMD-CBS†	0.0	$0.0_{\pm 0.0}$	—	—	$1.1_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.1_{\pm 0.9}$	0.0	$0.0_{\pm 0.0}$	—	—	$1.1_{\pm 0.9}$
	MDOC†	0.0	—	—	$0.000_{\pm 0.000}$	$214.2_{\pm 10.8}$	0.0	—	—	$0.000_{\pm 0.000}$	$229.0_{\pm 40.8}$	0.0	—	—	$0.000_{\pm 0.000}$	$188.8_{\pm 17.4}$
	PCD-LB	0.0	$0.0_{\pm 0.0}$	$47.76_{\pm 4.94}$	—	$3.8_{\pm 0.3}$	0.0	$0.0_{\pm 0.0}$	$50.44_{\pm 4.95}$	—	$4.0_{\pm 0.4}$	0.0	$0.0_{\pm 0.0}$	$46.76_{\pm 4.28}$	—	$6.1_{\pm 1.6}$
	PCD-SHD	0.0	$0.0_{\pm 0.0}$	$2.04_{\pm 0.52}$	—	$3.9_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$1.97_{\pm 0.41}$	—	$3.3_{\pm 0.1}$	0.0	$0.0_{\pm 0.0}$	$1.75_{\pm 0.36}$	—	$3.4_{\pm 0.2}$
	Final Proj.	0.0	$0.0_{\pm 0.0}$	**1.34**$_{\pm 1.05}$	$0.842_{\pm 0.021}$	$17.5_{\pm 4.4}$	0.0	$0.0_{\pm 0.0}$	$1.08_{\pm 0.87}$	$0.837_{\pm 0.022}$	$20.4_{\pm 1.0}$	0.0	$0.0_{\pm 0.0}$	$0.90_{\pm 0.71}$	$0.837_{\pm 0.024}$	$16.0_{\pm 4.8}$
	DiRecT	0.0	$0.0_{\pm 0.0}$	$1.52_{\pm 0.87}$	**0.883**$_{\pm 0.018}$	$80.9_{\pm 35.1}$	0.0	$0.0_{\pm 0.0}$	**0.37**$_{\pm 0.35}$	**0.886**$_{\pm 0.018}$	$57.5_{\pm 0.7}$	0.0	$0.0_{\pm 0.0}$	**0.15**$_{\pm 0.12}$	**0.889**$_{\pm 0.017}$	$57.5_{\pm 0.7}$
	GlobMP (Ours)	**85.0**	**85.0**$_{\pm 35.7}$	$5.30_{\pm 13.46}$	$0.765_{\pm 0.146}$	$14.4_{\pm 116.6}$	**75.0**	**75.0**$_{\pm 43.3}$	$9.80_{\pm 17.31}$	$0.753_{\pm 0.139}$	$197.1_{\pm 116.2}$	**90.0**	**90.0**$_{\pm 30.0}$	$4.35_{\pm 13.07}$	$0.761_{\pm 0.138}$	$98.2_{\pm 103.0}$

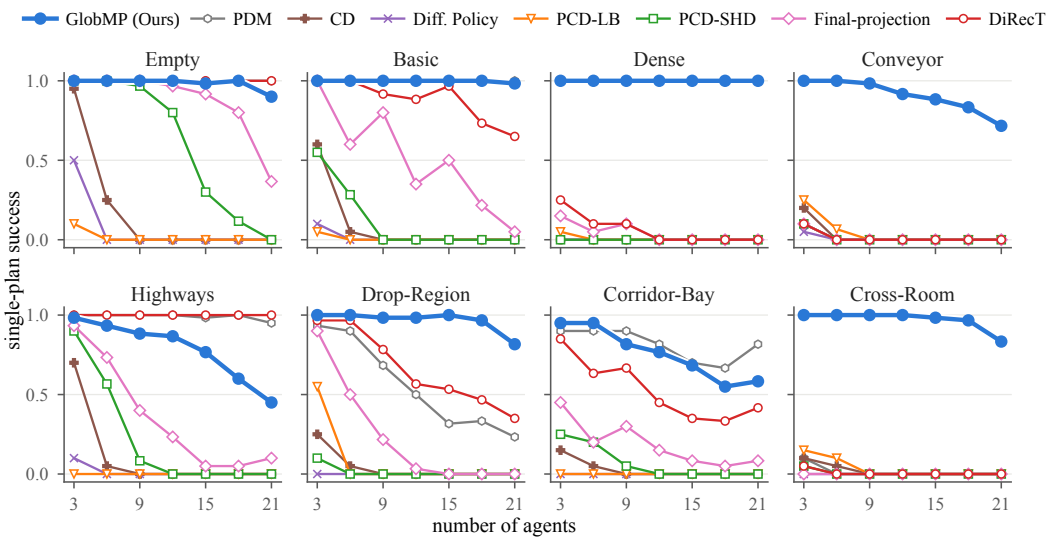


Figure 13: Success of a *single* returned plan for $N_a \in \{3,6,\ldots,21\}$ agents, one panel per map, averaged over the three velocity caps: the first candidate each baseline samples, and the one trajectory our method returns. MDOC and MMD-CBS store no per-candidate data and are omitted. Figure 11 gives every velocity cap separately, and Figure 12 the best-of-64 counterpart.

are reconstructed from Liang et al. (2025). Start–goal pairs follow the sampling procedure of Shaoul et al. (2025), which places every endpoint at least $2.1r+\varepsilon$ from an obstacle and $\max(0.15, 2.2r)+\varepsilon$ from every other endpoint with $\varepsilon = 0.01$.

Problem setup. Each agent follows a planar Bézier trajectory, with one block per agent and obstacle and one per pair of agents at separation $2r$ (§A4.5). Box obstacles use the affine blocks of §A2.1.1, one half-space per agent, segment and box, with the side selected by an independent grid seed.

The eight environments cover different forms of geometric and coordinated motion:

- `Empty` contains no obstacles and isolates inter-agent coordination.
- `Basic` and `Dense` contain ten and twenty scattered circular obstacles, respectively.
- `Conveyor` contains two narrow corridors with opposing preferred traffic directions, requiring coordinated single-file motion.
- `Highways` contains a central block that induces a warehouse-style circulation pattern.
- `Drop-Region` contains four drop-off regions and rewards motion through their centers.
- `Corridor-Bay` contains three parallel single-file channels connected by an open crossing, requiring teams to coordinate their passage.
- `Cross-Room` contains a plus-shaped divider that forms four quadrants connected only through gaps near the workspace boundary.

The demonstrations associated with each environment encode its characteristic motion pattern, which is measured by the data-adherence metric *DA*.

Robots have radius $r = 0.05$ on every map except `Empty`, where $r = 0.10$, with minimum pairwise separation $2r$. Obstacles are those of the source maps, enlarged by 2.20 (`Basic`), 1.50 (`Conveyor`), 1.45 (`Highways`), 1.60 (`Drop-Region`), 1.15 (`Corridor-Bay`) and 1.10 (`Cross-Room`); `Empty` and `Dense` keep their published sizes. Each environment is evaluated under Low, Medium, and High test-time velocity caps. In the order Low, Medium, High (Figure 14), these are $(0.483, 0.571, 0.675)$ for `Empty`, $(0.477, 0.567, 0.675)$ for `Basic`, $(0.565, 0.617, 0.675)$ for `Dense`, $(1.21, 1.46, 1.76)$ for `Conveyor`, $(0.647, 0.783, 0.878)$ for `Highways`, $(0.928, 1.13, 1.34)$ for `Drop-Region`, $(1.21, 1.46, 1.76)$ for `Corridor-Bay`, and $(0.928, 1.13, 1.34)$ for `Cross-Room`.

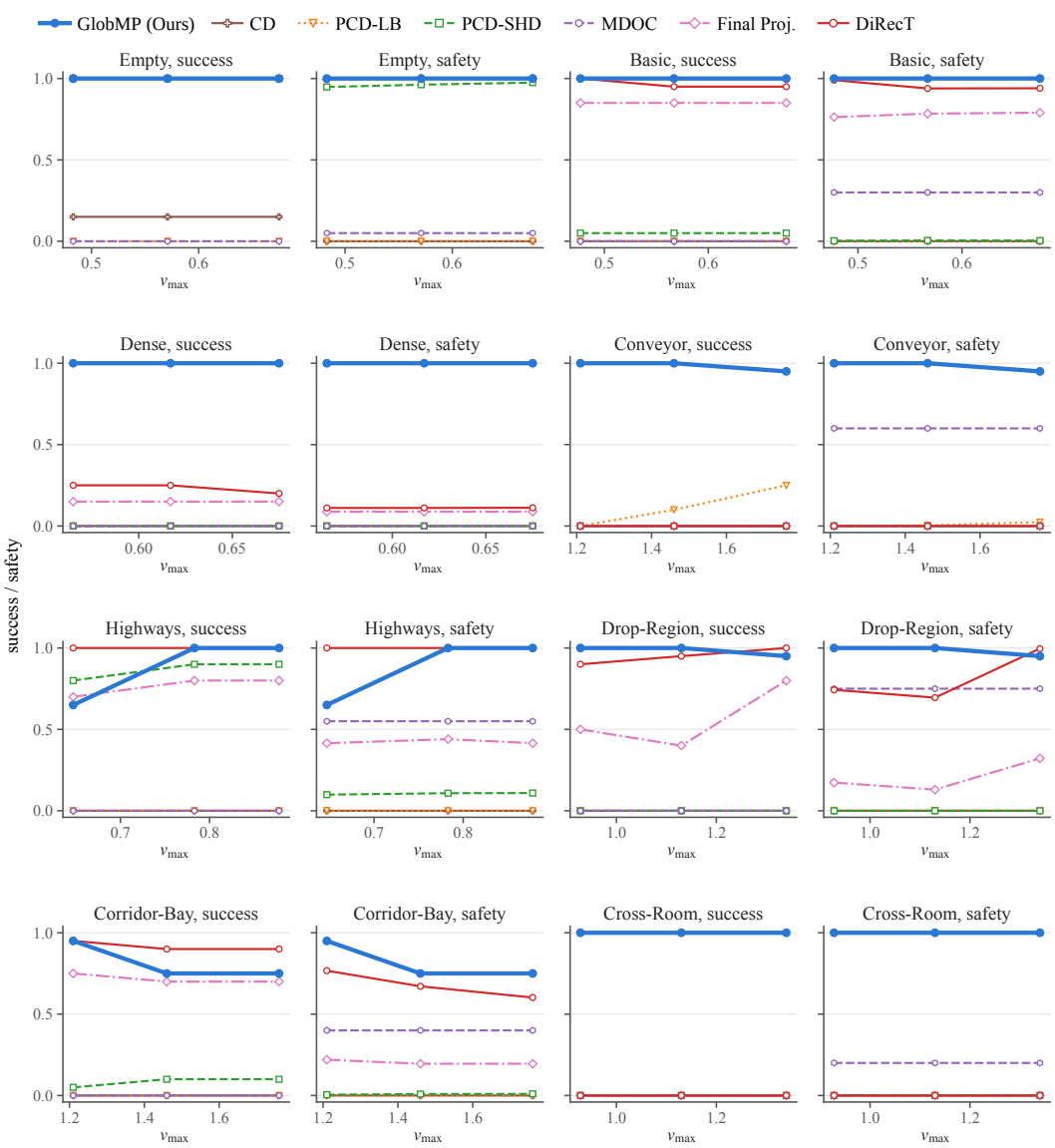


Figure 14: Success rate and constraint safety against the velocity cap $v_{\max}$ at $N_a = 9$, two panels per map. Each map has its own caps, so the horizontal ranges differ. Figure 13 sweeps the team size averaged over the caps, instead: a method that falls there but stays flat here is limited by inter-robot coupling, not by the velocity limit. MMD-CBS and Diffusion Policy are omitted, neither applying a velocity limit; CD and MDOC do not apply it either, so their curves are flat.

Baselines. We compare GlobMP against Diffusion Policy (Chi et al., 2023), CD, PCD-LB, and PCD-SHD (Luan et al., 2026), Final Proj. and DiRecT (Giaretta et al., 2026), MMD-CBS (Shaoul et al., 2025), and MDOC (He et al., 2026). Diffusion Policy provides the unguided reference; CD uses constraint guidance; PCD-LB and PCD-SHD augment guidance with projection; Final Proj. applies projection after sampling; MMD-CBS combines sampled single-agent motions through conflict-based search; DiRecT applies projection throughout denoising; and MDOC uses model-based diffusion optimal control. Each method is evaluated on the same start–goal instances.

Evaluation. We evaluate each method for $N_a \in \{3, 6, \ldots, 21\}$ on all eight maps and under three velocity caps. For each (map, team size) pair, the same 20 initial configurations are shared across methods, and each sampling baseline generates 64 candidate trajectories per configuration; MMD-CBS and MDOC return one plan, and GlobMP one trajectory. Following Giaretta et al. (2026), we

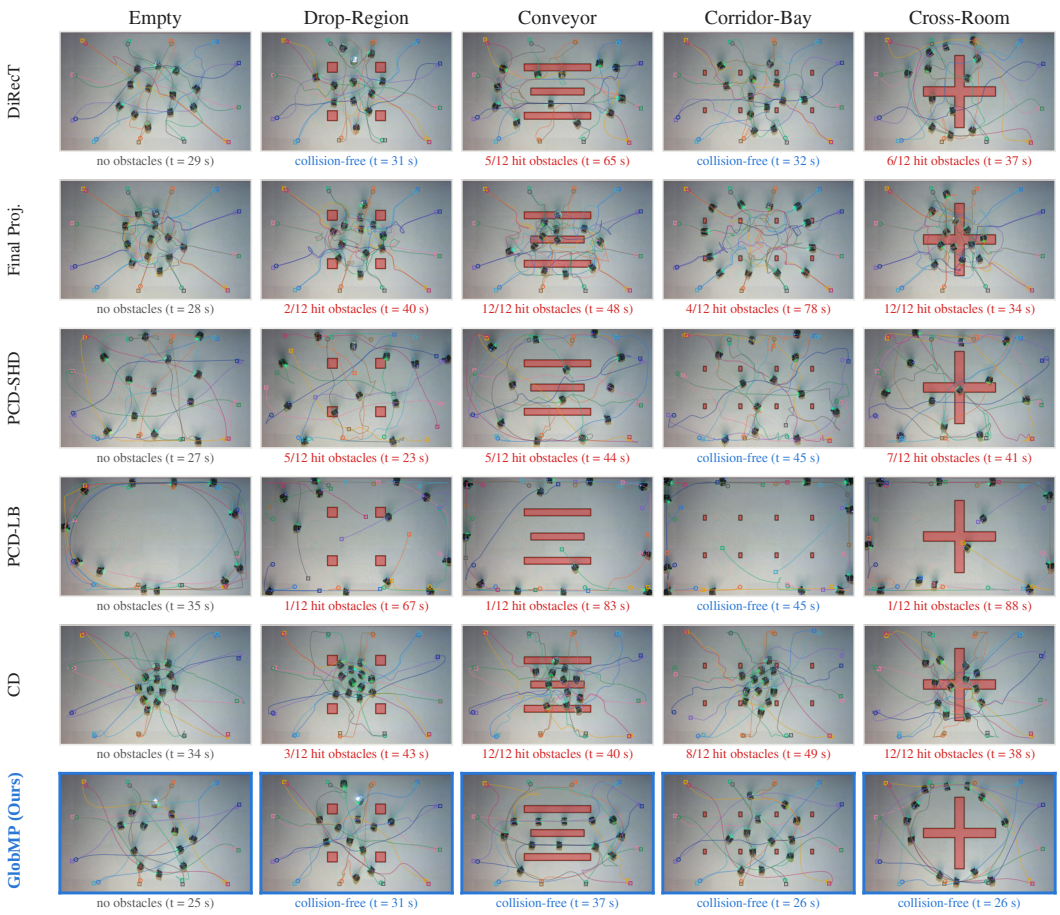


Figure 15: Every Robotarium hardware run, one row per method and one column per map: the ring instance with 12 robots, one frame of the overhead video per run (lens distortion removed) with the paths the robots drove, tracked from the video. Obstacles are virtual and drawn from the map; the text under each panel gives how many robots overlapped one and the time of the frame shown. PCD-LB's robots end far from their goals on the four obstacle maps, as do most of PCD-SHD's on `Drop-Region`.

report *SR*, the fraction of configurations with at least one collision-free trajectory; *CS*, the fraction of all trajectories satisfying the collision and kinematic constraints; *Coll*, the mean number of colliding agent pairs; *DA*, environment-specific task adherence; and *Time*, wall clock time per configuration. A dash marks a quantity that does not exist rather than a zero; the table captions state which quantity and why.

Figure 13 and Tables 3–10 summarize the results. We interpret *DA* together with feasibility, since adherence to the demonstrated motion pattern is not meaningful if the generated trajectory violates the constraints, consistent with the evaluation of Giaretta et al. (2026). At the Medium velocity cap, averaged across all maps and team sizes, *GlobMP (Ours)* attains a mean *SR* of 0.941, ahead of DiRecT at 0.601, Final Proj. at 0.419, PCD-SHD at 0.211, CD at 0.132, MMD-CBS at 0.121, PCD-LB at 0.063, MDOC at 0.025 and Diffusion Policy at 0.020.

Performance generally decreases as the number of robots increases, although the effect depends strongly on the environment. At the Medium cap, DiRecT maintains *SR*$=1.00$ on `Empty` and `Highways` at every team size and at least 0.90 on `Basic`, but at most 0.50 on `Dense`. `Conveyor` and `Cross-Room` separate the methods most sharply: the best baseline is PCD-LB, at 0.65 and 0.40 on `Conveyor` and 0.45 and 0.20 on `Cross-Room` for $N_a = 3$ and 6, and from $N_a = 9$ no baseline exceeds 0.10, while *GlobMP (Ours)* stays at 1.00 through $N_a = 15$ on `Cross-Room` and ranges

from 1.00 to 0.70 on `Conveyor`. `Drop-Region` and `Corridor-Bay` provide intermediate difficulty, with DiRecT attaining mean *SR* values of 0.70 and 0.95, respectively; DiRecT exceeds GlobMP on `Corridor-Bay` from $N_a = 9$ (at least 0.90 against 0.60 to 0.75) and on `Highways` at $N_a \geq 18$ (1.00 against 0.90 and 0.55). Changing the velocity cap has a comparatively small effect, partly because four of the nine rows (marked †) ignore it, except for DiRecT on `Drop-Region`, which reaches 1.00 at every team size only under the High cap, and for GlobMP on `Highways`, whose Low-cap SR falls from 0.80 at $N_a = 6$ to 0.60 at $N_a = 12$ and whose cells from $N_a = 15$ hit the planning budget (0.30, 0 and 0.15), suggesting that environment geometry and coupled collision avoidance dominate the difficulty of the harder instances.

The *GlobMP (Ours)* rows use the same shared instances across all eight maps, all velocity caps, and all 168 experimental cells, with a 300 s planning budget per team; MMD-CBS has 60 s per configuration. Planning times are measured on pinned CPU cores with one job per core. Cells that reach the full 300 s budget were repeated on an idle machine and showed the same behavior, indicating that these cases reflect planning difficulty rather than resource contention, namely on `Highways` at the Low cap for $N_a \geq 15$ and on `Corridor-Bay` at $N_a = 18$ (High) and $N_a = 21$ (Low).

The suite's success rate is a best-of-B quantity: an instance counts as solved when any of the $B = 64$ candidates a baseline samples is collision free, whereas our method returns one trajectory. Figure 13 therefore scores every method on the success of a single returned plan, and Figure 12 shows the best-of-64 success rate the tables report.

A4.5 Implementation details

This subsection lists how GlobMP is run in the experiments: the curve class of each experiment, the settings of Algorithms 1 and 2, the cone solve, and the checks of the implementation.

Curve class. Table 11 gives the degree, the cost order k and the boundary order ℓ of each experiment; the synthetic study uses 7 pieces (4 in the scaling study). The synthetic study uses the minimum-snap class of the baselines it is compared against ($k = \ell = 4$); the other tasks prescribe only where each robot starts and stops, so they fix position and velocity ($\ell = 1$) and minimize acceleration ($k = 2$); Maze2D uses $k = \ell = 2$. In every row $2(\ell+1) \geq k$, so Assumption B holds.

Table 11: Curve class of each experiment.

Experiment	Section	degree d	k	ℓ
Synthetic study	§4.1	9	4	4
Multi-robot suite	§A4.4	9	2	1
Robotarium	§4.2	12	2	1
Maze2D	§A4.3	9	2	2

Piecewise curves. With knots $0 = t_0 < \cdots < t_N = 1$, $\Delta_i = t_i - t_{i-1}$ and $u = (t - t_{i-1})/\Delta_i$ on piece i, the curve is $\gamma(t) = \mathbf{\Gamma}^{(i)}\mathbf{b}_d(u)$ and $\gamma^{(s)}(t) = \Delta_i^{-s}\mathbf{\Gamma}^{(i)}\mathbf{b}_d^{(s)}(u)$. The endpoint conditions of order $s \leq \ell$ and the continuity conditions of order $s \leq c$ (C^c junctions) at the $N-1$ junctions are linear in the stacked control points, so $\mathbf{\Gamma}\mathbf{A} = \mathbf{B}$ gives the same orthonormal chart as for one piece, with $d_f = N(d+1) - \operatorname{rank}(\mathbf{A})$. The cost matrix is $\operatorname{blkdiag}(\Delta_i^{1-2k}\mathbf{G}^{(k)})$, and the constraint blocks are built piece by piece with the same scaling. With $c \geq k-1$ the curves lie in $\mathscr{H}^k$, and a zero-cost curve is one polynomial of degree below k, which the endpoint conditions remove when $k \leq 2(\ell+1)$.

Restoration (Algorithm 1). The provable step $1/L_{\text{proj}}$ (Appendix A3.1.4) is too small to be practical, so the implementation calibrates the step: it starts at 10^{-3} and multiplies it by 3 while the merit (2) decreases, and it backs off when an iterate is not finite. A curve is accepted when every block satisfies $\operatorname{dist}^2(\boldsymbol{\pi}_j, \mathscr{K}_j) \leq \text{tol}$. This tolerance is in coefficient units, which carry the squared length scale of the workspace, so it is set per domain.

Cone solve. Each projection onto $\mathscr{K}_j$ uses Moreau's decomposition and fully corrective nonnegative matching pursuit (Algorithm 3), with the block rescaled to unit norm. The implementation maximizes the unnormalized correlation $\langle \mathbf{v}, \mathbf{b}_{2d}(t) \rangle$, a polynomial in t, at the real roots of its derivative and the two endpoints, then refits the nonnegative weights and drops atoms whose weight reaches zero. The number of atoms is bounded by Carathéodory's theorem, which allows at most $2d+1$ atoms.

Finishing step. In the synthetic study, the minimum-snap curvature ratio $\beta/\alpha \approx 2 \times 10^7$ makes first-order descent slow along flat directions, so each run is finished by Newton's method on the first-order conditions and the active contact equations. An interior contact at a time t_* contributes $g_j(t_*) = 0$ and $\partial_t g_j(t_*) = 0$; the unknowns are the free control points, the contact times and nonnegative multipliers. This step lowered the cost by a further factor of 2 to 3; the reported curves pass the dense recheck, but the step is numerical and is not covered by the theory. Maze2D uses the same step, after an SQP polish of the output of Algorithm 2 that is not covered by the theory either (§A4.3).

Several robots. The free control points of all robots are stacked; the robots share d, k and ℓ, and hence $\mathbf{A}$, $\mathbf{N}$ and the cost matrix, and each has its own boundary data. There is one collision block per robot and obstacle and one per pair of robots, the latter the obstacle block applied to the difference curve $\mathbf{\Gamma}_a - \mathbf{\Gamma}_b$ at the separation radius, so Algorithms 1 and 2 are unchanged.

Workspace bounds. A bound $\underline{x}_i \le \gamma_i(t) \le \bar{x}_i$ is two nonnegativity conditions with Bernstein coefficients $\mathbf{\Gamma}_{i,:} - \underline{x}_i \mathbf{u}$ and $\bar{x}_i \mathbf{u} - \mathbf{\Gamma}_{i,:}$. Since $\mathbf{u}^\top \mathbf{b}_d \equiv 1$, a degree-$d$ coefficient vector $\mathbf{c}$ is raised to degree $2d$ by $S_d\big(\frac{1}{2}(\mathbf{c}\mathbf{u}^\top + \mathbf{u}\mathbf{c}^\top)\big)$, so these blocks enter the same cone as the collision blocks and are linear in $\mathbf{\Gamma}$.

Checks of the implementation. Each of the following holds to machine precision on the benchmark instances: S_d reproduces the Bernstein coefficients of $\mathbf{b}_d^\top \mathbf{X} \mathbf{b}_d$; the chart satisfies $\mathbf{\Gamma}\mathbf{A} = \mathbf{B}$ and $\mathbf{N}^\top \mathbf{A} = 0$; $\boldsymbol{\pi}_j$ reproduces $\|\gamma(t) - \mathbf{c}_j\|^2 - r_j^2$; the cost matches $\int_0^1 \|\gamma^{(k)}\|^2 dt$; and the cone test is exact in both directions, returning zero for collision-free curves and a positive value, increasing with the penetration depth, for colliding ones.