LEARNING DYNAMICS OF STRATEGIC PUBLISHERS IN GENERATIVE AI ECOSYSTEMS

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Paper under double-blind review

ABSTRACT

Generative AI (GenAI) search systems are transforming how users access information. Given a user's question, GenAI systems generate an answer, often accompanied by attributed sources (e.g., in the form of citations). Content creators (publishers) seeking to increase exposure might behave strategically and compete with each other for users' attention. In these generative systems, the incentives take on a novel form: publishers may gain exposure through generated responses and/or attributions to these responses. We introduce a novel game-theoretic model of the emerging GenAI ecosystem in which publishers compete for attribution-based exposure. We study the learning dynamics of strategic content creators under better-response dynamics. We associate the convergence of learning dynamics to equilibrium with ecosystem stability. Using the concept of potential games, we study the stability of GenAI ecosystems under several known content selection mechanisms. We demonstrate the instability of mechanisms representing real-world modern systems and characterize a mechanism that induces a stable ecosystem. We conduct extensive simulations to analyze the stability and welfare of GenAI ecosystems under various mechanisms. The simulations support our theoretical findings and reveal an interplay among stability, publisher welfare, and user welfare. We then introduce a study illustrating how platforms can select GenAI mechanisms to achieve desired welfare trade-offs.

1 INTRODUCTION

Generative AI (GenAI) is rapidly reshaping how users search for information (Chatterji et al., 2025). Traditional search systems — e.g., Google search — present a ranked list of documents. In contrast, GenAI search systems — e.g., ChatGPT or Perplexity[1] — use documents to synthesize a response; often leveraging retrieval-augmented generation (RAG) frameworks (Lewis et al., 2020). The responses are often accompanied by attributed sources (e.g., in the form of citations embedded in the text; Kirsten et al., 2026). Consequently, in GenAI systems, content creators (publishers) may gain visibility through responses (Dekel et al., 2026) or through the attributions associated with those responses (Ding et al., 2025). More specifically, publishers may be incentivized to modify their content so that it is more likely to be cited or referenced in responses, a practice known as generative engine optimization (GEO; Aggarwal et al., 2024; Wen et al., 2026).

We propose a novel game-theoretic model for GenAI search ecosystems, focusing on generative question-answering platforms. Publishers create documents and users submit questions; GenAI platforms generate responses to questions using selected (retrieved) documents and generative models (e.g., LLMs), and assign attribution to publishers. Since publishers are interested in maximizing visibility, they may be incentivized to behave strategically. We adopt *better-response dynamics* (Milchtaich, 1996) to describe their learning dynamics. That is, we only assume that if there is a situation where a publisher may gain by deviating to any utility-improving action, then she will indeed deviate.

Although content modification through better-response dynamics was studied in prior work on ranking-based search (Nachimovsky & Tennenholtz, 2025; Madmon et al., 2025a), we are the first (to the best of our knowledge) to model, using game theory, the GenAI search ecosystem and its effect on

[1] perplexity.com

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the learning dynamics of publishers. In our model, publisher utility depends on both being attributed to generated responses and being faithful to her original content, and user utility is determined by both the relevance of responses to the question and the relevance of attributions to the response.

In ranking-based search, a central question in the study of strategic content modification is whether the resulting dynamics converge to a stable state and whether such convergence can be guaranteed (Ben-Porat & Tennenholtz, 2018; Madmon et al., 2025b). To establish tight theoretical results, we appeal to the *Nash equilibrium*[2] (Nash Jr, 1950) solution concept and convergence to equilibrium; a state that implies stability of the ecosystem. Using the concept of *potential games* (Monderer & Shapley, 1996), we revisit content selection mechanisms offered in the literature. Interestingly, we show that in a relatively narrow set of scenarios, such convergence is guaranteed. Importantly, we show that dynamics may diverge in RAG-based platforms and in platforms that are based on the winner-takes-all concept (Hillman & Riley, 1989).

While stability is interesting at its own right, we also empirically study the welfare properties of overall dynamics regardless of whether convergence occurs. Through experimental analysis, we show several intriguing results. Specifically, we introduce a novel mechanism design setting in which we show mechanisms to trade off user welfare and publisher welfare. In addition, we reveal the unexpected merits of unstable mechanisms in terms of welfare. Specifically, we show that stability does not entail higher social welfare and, in some cases, significantly lower. Finally, we empirically identify and analyze the mechanisms that maximize social welfare for various objectives. Specifically, we show that these mechanisms differ across objectives both in their family and their parameters.

Overall, our contributions are threefold: (i) we introduce a novel game-theoretic model of the emerging GenAI search ecosystem with attribution-incentivized content creators; (ii) we provide a theoretical analysis of learning dynamics in terms of stability; and (iii) we empirically analyze better-response dynamics, considering stability, welfare, and mechanism optimality.

2 Related Work

Game Theory in Search and Recommendation Ecosystems. Game theory has been extensively employed to model strategic content creation in search (Kurland & Tennenholtz, 2022) and recommendation (Ben-Porat & Tennenholtz, 2018; Boutilier et al., 2024) ecosystems. With the growing adoption of dense retrieval methods (Xu et al., 2026b), recent work has increasingly focused on models with continuous strategy spaces (Yao et al., 2023a;b; Madmon et al., 2025a;b; Yu et al., 2025). These game-theoretic formulations are commonly used to analyze stability (Reinman et al., 2026) and welfare (Yao et al., 2024) properties. At the same time, the increasing use of GenAI for information-seeking (Chatterji et al., 2025) gives rise to models that capture strategic behavior in this setting (Taitler et al., 2025; Esmaeili et al., 2026; Ohayon et al., 2026; Xu et al., 2026a). For instance, Wu et al. (2026a) introduce a model for AI-overview systems, and Keinan & Ben-Porat (2026) propose a model in which creators strategically decide both their content and whether to share it with GenAI platforms. To the best of our knowledge, we are the first to formally model the learning dynamics of strategic publishers in GenAI systems, specifically question-answering systems.

Strategic Content Modification in Ad Hoc Search and Recommendation. Strategic behavior of content creators has been widely addressed in prior work (Basat et al., 2017; Hron et al., 2023; Prasad et al., 2023; Boutilier et al., 2024; Nachimovsky & Tennenholtz, 2025; Wen et al., 2026). When users observe a ranked list of content, publishers are ranking-incentivized; that is, they might modify their content to increase their rankings (Kurland & Tennenholtz, 2022). A substantial body of work has studied content modification of such publishers in search and recommendation systems (Nachimovsky et al., 2024; Bardas et al., 2025; Mordo et al., 2025; Ye et al., 2025).

The emergence of GenAI systems, which often accompany their responses with citations (Kirsten et al., 2026), has changed the way users search for information (Chatterji et al., 2025). Publishers may be attribution-incentivized; they might modify their content to increase attribution-based exposure in GenAI systems (Chen et al., 2026b; Martinez, 2026). For instance, Aggarwal et al. (2024) propose a metric that jointly considers citations' position and the cited text to measure exposure. It has been

[2]We restrict to pure Nash equilibrium (PNE), as it corresponds to stationary content and therefore provides a natural notion of stability in a search ecosystem.

broadly adopted and has become a common practice for estimating exposure of publishers in GenAI systems (Chen et al., 2025; 2026a; Wu et al., 2026b; Zhou et al., 2026). Our work introduces, to the best of our knowledge, the first analysis of ecosystems with attribution-incentivized publishers.

3 PRELIMINARIES

We begin by introducing the game-theoretic definitions and solution concepts used throughout.

Game Theory Notation. An n-player **game** is a tuple $G = (N, (X_i)_{i\in N}, (u_i)_{i\in N})$, where $N := \{1, \ldots, n\}$ is the set of players, X_i is the action set of player i, and $u_i : X \to \mathbb{R}$ is player i's utility function. Here, $X := \times_{j\in N} X_j$ denotes the set of pure strategy profiles. For a profile $x \in X$, we write x_i for player i's action and $x_{-i} \in X_{-i} := \times_{j\in N\setminus\{i\}} X_j$ for the actions of all players except i.

Better-response, Equilibria, and Dynamics. Given a profile of opponents' actions $x_{-i} \in X_{-i}$ and two actions $x_i, x_i' \in X_i$ of player i, we say that x_i' is an **ϵ-better response** than x_i with respect to x_{-i} if $u_i(x_i', x_{-i}) > u_i(x_i, x_{-i}) + \epsilon$. This notion naturally induces the equilibrium concept used in our analysis. A pure strategy profile $x \in X$ is an **ϵ-pure Nash equilibrium** (ϵ-PNE) if, for every player $i \in N$, there is no action x_i' that is an ε-better response than x_i, with respect to x_{-i}. When $\epsilon = 0$, we refer to a 0-better response simply as a better response, and to a 0-PNE as a Pure Nash Equilibrium (PNE). A PNE represents a stable outcome of a game (Ben-Porat & Tennenholtz, 2018), in the sense that no publisher has an incentive to unilaterally change her strategy.

The notion of ϵ-better response induces **ϵ-better-response dynamics**, defined as a sequence in which, at every timestep $t \geq 1$, either $x^{(t-1)}$ is an ϵ-PNE and $x^{(t)} = x^{(t-1)}$, or there exists a player $i \in N$ such that $x_i^{(t)}$ is an ϵ-better response than $x_i^{(t-1)}$ with respect to $x_{-i}^{(t-1)}$, and $x_j^{(t)} = x_j^{(t-1)}$ for every $j \neq i$. We say that the dynamics *converges* if there exists a timestep T such that $x^{(T)}$ is an ϵ-PNE.

Potential games. A central class of games used in this paper is potential games. A game is a **potential game** if there exists a potential function $\phi : X \to \mathbb{R}$ such that for every player $i \in N$, every profile of opponents' actions $x_{-i} \in X_{-i}$, and every two actions $x_i', x_i'' \in X_i$, it holds that:

$$\phi(x_i', x_{-i}) - \phi(x_i'', x_{-i}) = u_i(x_i', x_{-i}) - u_i(x_i'', x_{-i}).$$

We note a celebrated result for potential games with better-response dynamics:

Theorem 1 (Monderer & Shapley (1996))**.** *Let G be a potential game and let $\epsilon > 0$. Then G admits at least one ϵ-PNE, and any ϵ-better-response dynamics in G converges.*

4 THE MODEL

Our model is a formalism of a setting in which content creators compete for exposure by selecting documents to publish. In response to a user's question, a mediator chooses which content to present. We focus on a novel and relatively understudied type of mediator: GenAI platforms.

A **generative publishers' game** consists of a set of publishers $N := \{1, 2, \ldots, n\}$, with $n \geq 2$, that strategically choose documents represented as p-dimensional vectors in the bounded domain[3] $\mathcal{V} := [0, 1]^p$. Specifically, each publisher $i \in N$ selects an action $x_i \in X_i := \mathcal{V}$, interpreted as the embedding of the content provided by that publisher. Each publisher has an initial document $x_0^i \in X_i$ and a cost factor $\lambda_i > 0$, which captures publisher i's cost of providing content that differs from her initial document, i.e., content drift entails a cost in terms of conveying her original intent. We use $\lambda > 0$ to denote a uniform cost factor, i.e., $\lambda_i = \lambda$ for all $i \in N$. Distances are measured by a continuous semi-metric $d : \mathcal{V} \times \mathcal{V} \to \mathbb{R}_+$, such that $d(a, b) \leq 1$ for all $a, b \in \mathcal{V}$. Given a strategy profile $x \in X$ and a user's question[4] $x^* \in \mathcal{V}$, the platform generates a response according to a generation function $y : X \times \mathcal{V} \to \mathcal{V}$.

In our model, aligned with modern real-world systems (Kirsten et al., 2026), users observe a generated response and attributed documents (e.g., in the form of citations). The platform employs an attribution

[3] As in prior models of search and recommendation ecosystems; e.g., Madmon et al. (2025b).

[4] The question serves as a proxy for the user's information need.

mechanism that maps content and a response to (cited) content. Formally, an attribution allocation function maps a strategy profile and a response to a distribution over publishers $r : X \times \mathcal{V} \to \Delta_n$. Let r_i, the i-th component of the distribution, be the attribution score of publisher i, which can be interpreted as her probability of being ranked highest in a citation list.

We abuse the notation and denote $y(x) \coloneqq y(x, x^*)$, $r(x) \coloneqq r(x, y(x))$, $d^*(x_i) \coloneqq d(x_i, y(x))$ and $d_i^0(x_i) \coloneqq d(x_i, x_0^i)$. The **utility** of publisher i under the profile x is:

$$u_i(x) \coloneqq r_i(x) - \lambda_i \cdot d_i^0(x_i). \tag{1}$$

That is, the utility of publisher i is the difference between her attribution score and the cost of providing content that differs from her initial document. We define the **publishers' welfare** $\mathcal{H}(x)$ and the **users' welfare** $\mathcal{U}(x)$.

$$\mathcal{H}(x) \coloneqq \sum_{i=1}^{n} u_i(x) = 1 - \sum_{i=1}^{n} \lambda_i \cdot d_i^0(x_i). \tag{2}$$

$$\mathcal{U}(x) \coloneqq \gamma \cdot \big(1 - d(x^*, y(x))\big) + (1-\gamma) \cdot \sum_{i=1}^{n} \big(1 - d^*(x_i)\big) \cdot r_i(x), \tag{3}$$

where $\gamma \in [0, 1]$ is the interpolation parameter. The publishers' welfare is the sum of their utilities. The users' welfare is a distinctive aspect of GenAI ecosystems, a linear interpolation of *responses' relevance*[5] to the question and *attributions' relevance* to the response, which are the first and second terms in Equation 3, respectively. Their weighting may differ between settings, domains, and users' underlying objectives. For example, in the medical domain, platforms may assign greater weight to attributions' relevance to ensure that responses are grounded with relevant external information.

We note that modern generation functions (e.g., LLMs) are complex and generally lack a tractable analytical structure. To enable theoretical analysis, we therefore consider a family of centroid-based generation functions. Our approach is motivated by Rocchio's model (1971), a classical centroid-based method in information retrieval for relevance feedback in vector spaces[6]. Formally:

$$y_{\alpha,z}(x) \coloneqq \alpha \cdot \frac{1}{k} \sum_{j \in S_k} x_j + (1-\alpha) \cdot z, \tag{4}$$

where $\alpha \in [0, 1]$ is an interpolation parameter that controls the relative impact of documents on generation (henceforth *retrieval weight*), k ($\leq n$) is a constant that accounts for the number of publishers' documents used in generation (henceforth *context size*), $S_k \subseteq N$ denotes the set of k publishers whose documents are closest to x^*, and $z \in \mathcal{V}$ represents the platform's posterior belief, which, given a question, could be an LLM's parametric knowledge (representation) or a user's information need approximation; e.g., Fan et al. (2025). In addition, we use the normalized squared Euclidean distance $d(a, b) = \frac{1}{p}\|a - b\|_2^2$ as our distance semi-metric.

We discuss two fundamentally different settings: $k < n$ and $k = n$. Although both settings can model GenAI systems, they represent different scenarios. The former setting represents ecosystems with a large or diverse set of publishers (e.g., the Web), in which systems mainly employ RAG frameworks to retrieve top-k documents presumed to be relevant. The latter setting may represent niche domains in which a small number of publishers compete for user engagement, or systems that rely on a fixed set of content creators. Each, as we demonstrate later, leads to substantially different outcomes.

Our model generalizes prior models of traditional systems, whose output is a ranked list of documents (Yao et al., 2023b; Madmon et al., 2025a;b). For instance, if we set $\alpha = 0$ and $z = x^*$ in Equation 4, then we arrive to Madmon et al.'s (2025a) model. We emphasize, however, a key distinction from these prior models. In ranking-based systems, publishers compete relative to an exogenous reference point (e.g., a query) and a ranking. In our model, publishers compete relative to a dynamic response (and a ranking), making the reference point endogenous (see Equation 4).

5 Learning Dynamics of Strategic Publishers

In this section, we present a theoretical analysis of better-response dynamics under various content attribution functions.

[5] We estimate the relevance of $a \in \mathcal{V}$ to $b \in \mathcal{V}$ by $1 - d(a, b)$.

[6] Centroids of document sets also effectively approximate desired text embeddings (Puduppully et al., 2023).

5.1 The Winner Takes All

The first attribution function we consider is based on the widely used winner-takes-all concept[7]. Naturally, the argmax function induces a deterministic attribution distribution that assigns all the mass to the presumably most relevant publisher. Recall, we estimate content's relevance using its distance from the response. Formally:

Definition 1. *The **argmax** attribution function r^* of publisher i is:*

$$r_i^*(x) \coloneqq \frac{\mathbb{I}\{i \in \mu(x)\}}{|\mu(x)|} \tag{5}$$

where $\mu(x) \coloneqq \operatorname{argmin}_{j \in N} d^*(x_j)$ is the set of publishers providing the closest content (most relevant) to the response $y(x)$ among all publishers and $\mathbb{I}$ denotes the indicator function.

Informally, the argmax attribution function assigns an attribution score of 1 to the most relevant publisher (in the case of ties, the distribution is uniform over the relevant publishers), and a score of 0 to others. Note that the argmax function yields optimal short-term attributions' relevance in the sense that it minimizes the attribution distance from the response for a fixed strategy profile x. However, it induces unstable ecosystems, in which dynamics may not converge, as we show in Appendix A (Observation 3). In fact, as we show in the following observation (whose proof is given in Appendix B), games may not even admit a PNE.

Observation 1. *Consider generative publishers' games induced by the argmax attribution function. For both $k < n$ and $k = n$ cases, there exists a set of games in which any instance possesses no PNE.*

The proof is essentially based on the discontinuity of the argmax function. This observation serves as evidence for the shortcomings of the argmax attribution function in terms of the ecosystem's stability. Specifically, the argmax induces a game in which better-response dynamics might not converge and a PNE might not exist. We next study the dynamics induced by a common continuous function.

5.2 The Softmax Function

The softmax function is widely used in machine learning to form a distribution. It therefore provides a natural way to smooth the discrete choice performed above.

Definition 2. *The **softmax** attribution function[8] $\tilde{r}$ of publisher i is:*

$$\tilde{r}_i(x) \coloneqq \frac{e^{-\beta \cdot d^*(x_i)}}{\sum_{j=1}^{n} e^{-\beta \cdot d^*(x_j)}}, \tag{6}$$

where $\beta > 0$ is an inverse temperature constant.

We start by showing in the following theorem, the proof of which can be found in Appendix B, that the generative game under the softmax function is not a potential game for the $k = n$ case. In Appendix A (Theorem 5) we prove a stronger result for the $k < n$ case. In particular, under any proportional function (as defined in Madmon et al., 2025b) — of which softmax is a special case — the game is not a potential game.

Theorem 2. *Consider generative publishers' games with context size $k = n$. For every $\beta > 0$, the softmax attribution function does not induce a potential game.*

Although Theorem 2 and Theorem 5 show that the softmax attribution function does not induce a potential game, this does not imply the divergence of better-response dynamics. In fact, empirical analysis in Section 6 suggests that with high likelihood, better-response dynamics do converge under the softmax function. However, the following observation, the proof of which is given in Appendix B, demonstrates dynamics that do not converge.

[7] See, for example, the probability ranking principle (PRP; Robertson, 1977), all-pay auctions (Baye et al., 1996), and TOP mechanisms (Ben-Porat & Tennenholtz, 2018).

[8] The generative publishers' game induced by the softmax attribution function extends Hirshleifer's well-known model of rent-seeking competition (Hirshleifer, 1989).

Observation 2. *Consider generative publishers' games induced by the softmax attribution function $\tilde{r}$. For both $k < n$ and $k = n$ cases, there exists a game set for which there exists $\beta_0 > 0$ such that for every $\beta \geq \beta_0$, there exists an infinite sequence of profitable deviations. In particular, for sufficiently small ε, ε-better-response dynamics may not converge.*

The result just stated indicates that under the softmax attribution function, stability is not guaranteed. Next, we discuss the linear attribution function, which leads to both stable and unstable ecosystems.

5.3 The Linear Relative Relevance Function

We adapt the notion of linear relative relevance from Madmon et al. (2025a), which is the difference between the average distance of all other publishers from the generated response $y(x)$ and the distance of publisher i from $y(x)$.

Definition 3. *The linear relative relevance of publisher $i \in N$ with respect to x is:*

$$\nu_i(x) \coloneqq \frac{1}{n-1}\Big(\sum_{j \in N \setminus \{i\}} d^*(x_j)\Big) - d^*(x_i). \tag{7}$$

Notably, the relative relevance of publisher i increases as other publishers become less relevant to $y(x)$ or she becomes more relevant to $y(x)$. To induce a valid probability distribution, a linear function of the form $r_i(x) \coloneqq m \cdot \nu_i(x) + b$ entails a slope $m \in (0, \frac{1}{n}]$ and a bias $b = \frac{1}{n}$. Consequently, we can naturally define the linear attribution function:

Definition 4. *The **linear** relative relevance attribution function $\hat{r}$ of publisher i is:*

$$\hat{r}_i(x) \coloneqq m \cdot \nu_i(x) + \frac{1}{n}, \qquad m \in (0, \frac{1}{n}]. \tag{8}$$

Note that the slope m controls the extent to which publishers are motivated to modify their content by varying the extent to which the function is uniform over publishers.

Madmon et al. (2025a) showed that linear ranking entails stability in ecosystems with ranking-based exposure. As discussed, this result does not naturally transfer to GenAI ecosystems. In fact, as we later show, this stability result does not hold for mechanisms with partial selection criteria (i.e., a context size of $k < n$). However, in the following theorem, the proof of which is given in Appendix B, we show that for mechanisms with context size $k = n$ and the linear attribution function, the induced game is a potential game.

Theorem 3. *Consider a generative publishers' game with the linear attribution function $\hat{r}$ and a generation using context size of $k = n$. Then, the induced game is an exact potential game with the potential function*

$$\Psi(x) = -(1-\alpha)m \sum_{i=1}^{n} d(x_i, z) - \frac{\alpha m(n-2)}{n(n-1)} \sum_{1 \leq p < q \leq n} d(x_p, x_q) - \sum_{i=1}^{n} \lambda_i \cdot d_i^0(x_i).$$

Consequently, for every $\varepsilon > 0$, every ε-better-response dynamics converges.

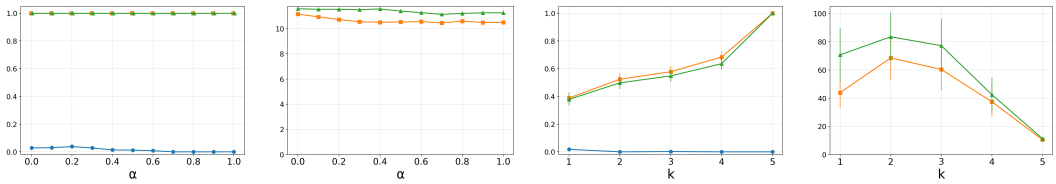
Since X_i is compact for every i, and since both $y(\cdot)$ and $d(\cdot, \cdot)$ are continuous, each utility function u_i is continuous in the strategy profile x. By Theorem 3, the game is an exact potential game, hence it has acyclic dynamics. Therefore, the conditions of Kukushkin's (2011) theorem hold, thus:

Corollary 1. *Under the conditions of Theorem 3, the induced publishers' game admits a PNE.*

In contrast, as we show in the following theorem (whose proof is presented in Appendix B), mechanisms with context size $k < n$ do not induce potential games. Notably, this case represents, for example, retrieval-based systems in which only a subset of content creators' documents are used in generation (e.g., RAG). Additionally, in Appendix A (Observation 4) we provide an example of a game in which better-response dynamics diverge.

Theorem 4. *Consider the class of generative publishers' games induced by the linear attribution function $\hat{r}$. For every $k < n$, this class contains games that are not potential games.*

All in all, whether the linear attribution function induces a stable ecosystem is a question of mechanism design. Specifically, it depends on the context size k. In other words, the ability of platforms to ensure stability under better-response dynamics is subject to their ability to use the set of all publishers for generation and attribution.



(a) Convergence ratio (b) Convergence rate (c) Convergence ratio (d) Convergence rate

Effect of the value of α **Effect of the value of k**

Figure 1: Stability analysis of the argmax, softmax, and linear attribution functions for various values of α and k, shown in blue circles, orange squares, and green triangles, respectively.

6 Empirical Results

In this section, we present an empirical analysis of learning dynamics using simulated environments. The goal is to model realistic better-response dynamics. We adopt a mechanism design perspective (Börgers et al., 2015) and study the impact of our attribution mechanisms (argmax, softmax, and linear) and novel design choices that emerged in the GenAI era (retrieval weight α and context size k) on both the stability and the welfare in the ecosystem.

As noted by Kurland & Tennenholtz (2022), empirically evaluating competitive search settings poses substantial challenges. Therefore, following prior work on learning dynamics (Madmon et al., 2025a;b), we use code-based simulations in the embedding space. Specifically, we employ the *discrete better-response dynamics* algorithm, which models realistic scenarios in which publishers iteratively modify their content to improve utility. The full algorithm and parameter settings are provided in Appendix C. In Appendix D we study the sensitivity of our results to the parameter values; the findings demonstrate the generality of the results reported below.

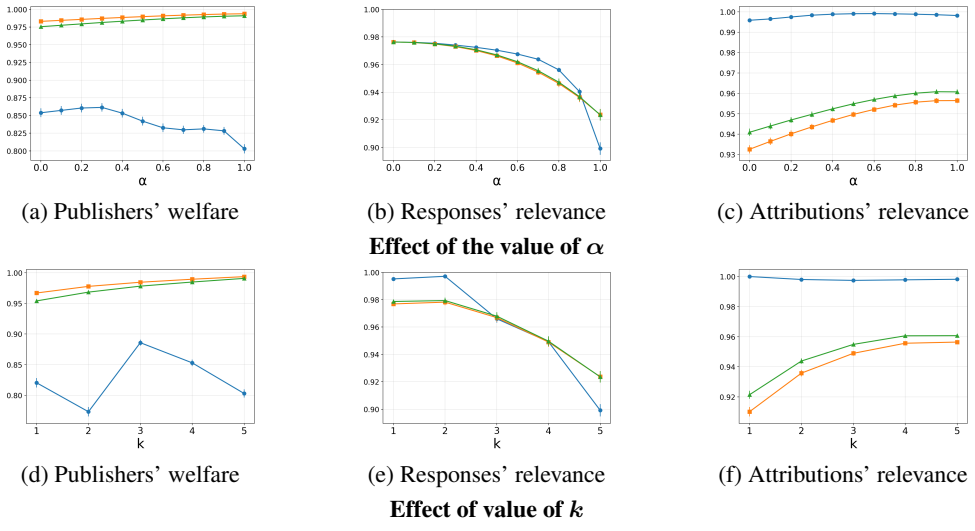
6.1 Stability Analysis

We begin with the stability analysis of the GenAI ecosystem. We report two stability measures: the *convergence ratio*, which is the proportion of simulations that converged[9], and the *convergence rate*, which is the average number of rounds until the simulation converged, among the converged instances. As we show below, since the dynamics under the argmax function rarely converged, we report the latter only for the softmax and linear functions. Figure 1 shows the stability results.

Figure 1a confirms the theoretical stability result for the linear function and highlights a substantial limitation of the argmax function: the vast majority of the dynamics it induces fail to converge. In contrast, the dynamics induced by the softmax function converges in all cases. Informally, both the softmax and linear functions appear to induce a stable ecosystem. We highlight a significant observation in Figure 1b; the convergence rate values remain relatively stable as the retrieval weight α varies. In other words, increasing the impact of publishers on the downstream response does not destabilize the ecosystem, neither in terms of the convergence ratio nor the convergence rate. Notably, the softmax function induces faster convergence than the linear function.

We can see in Figure 1c that the convergence ratio under the softmax and linear attribution functions increases with the value of the context size k, whereas under the argmax function it remains approximately 0. This suggests that utilizing more documents from the corpus for response generation induces a more stable ecosystem. This observation can likely be explained by the fact that as the value of k increases, the number of discontinuity points in the utility function decreases. Additionally, Figure 1d shows a parabolic trend for the convergence rate values. We point out that most of the mechanisms are statistically significantly indistinguishable, except for mechanisms with context size $k = n$ (recall that it is the only stable mechanism), for which both the softmax and linear attribution functions exhibit statistically significantly faster convergence than that of mechanisms with context size $k < n$. This serves as another case in point for the merits of generation with $k = n$.

[9]A simulation is said to converge if it has reached an ε-PNE.



(a) Publishers' welfare (b) Responses' relevance (c) Attributions' relevance

Effect of the value of α

(d) Publishers' welfare (e) Responses' relevance (f) Attributions' relevance

Effect of value of k

Figure 2: Welfare analysis of the argmax, softmax, and linear attribution functions for various values of α and k, shown in blue circles, orange squares, and green triangles, respectively.

6.2 Welfare Analysis

Since, as shown, many significant mechanisms — e.g., the argmax function and retrieval-based systems ($k < n$) — induce an unstable ecosystem, we turn to analyzing the ecosystem from the perspective of welfare. We focus on the measures defined in Section 4: *publishers' welfare*, *responses' relevance*, and *attributions' relevance*. If a simulation converged, we estimate the measures using the strategy profile to which the simulation converged. Otherwise, we average the measures over the last $M = 900$ rounds (Madmon et al., 2025a).

We can see in Figure 2a that as α increases, the publishers' welfare under the softmax and linear functions increases. Informally, as publishers gain greater influence over the system's output, they modify their content less. A likely explanation is that, for larger values of α, changes to a document have a stronger effect on the generated response. Thus, in a sense, α may be tuned to motivate publishers to explore new content. Moreover, the softmax function consistently yields higher publishers' welfare than the linear function, while both induce substantially higher values than those of the argmax function. This observation is due to the discontinuity of the argmax function, forcing publishers to apply extreme modifications to gain exposure.

We observe in Figures 2b and 2c that as the value of α increases, the relevance of the responses decreases, and the relevance of the attributions increases. That is, as publishers gain influence over the platform's outputs, the responses are less relevant, and, as expected, the attributions are more relevant. We note that the decrease in responses' relevance is an unwarranted effect of the competition; thus, in a way, α can be used to trade off between the negative and the positive consequences of strategic dynamics. Additionally, a key observation from Figures 2b and 2c is that the argmax function outperforms the softmax and linear functions in terms of both users' welfare measures. However, we point out that for $\alpha = 1$ (i.e., generation is conducted only with publishers' documents), the argmax induces lower responses' relevance than those of the softmax and linear functions.

Figure 2d shows that, under the softmax and linear functions, the publishers' welfare increases with k, while under argmax it exhibits non-monotonic behavior. Furthermore, we can see in Figure 2e that increasing k decreases the responses' relevance under all three attribution functions, with the effect being especially pronounced under argmax. This trend can be attributed to the fact that, as the value of k increases, less relevant documents (with respect to the question) affect generation. Yet, we point out that increasing the value of k from 1 to 2 increases the responses' relevance. In contrast, Figure 2f shows that increasing k increases the attributions' relevance under the softmax and linear functions, whereas under argmax it is nearly constant and nearly maximal. These observations serve as evidence that unstable mechanisms can yield improved welfare, as we further demonstrate below.

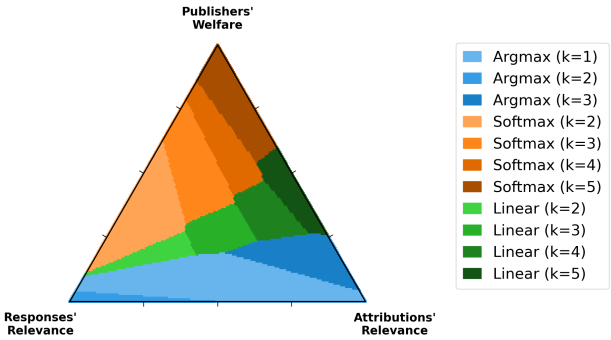


Figure 3: Optimal mechanism as a function of welfare measures distribution (weights). At each point, the color indicates which mechanism maximizes the social welfare. Different colors correspond to different attribution functions, and shades within the same color correspond to different values of k.

6.3 Mechanism Design

Having analyzed welfare under various mechanisms, we now address the fundamental question of *optimal mechanism design* (Börgers et al., 2015). Let $f : X \times \Delta^3 \to \mathbb{R}$ be a **social welfare** function that, given a strategy profile and a distribution (weighting) over the welfare measures used above, assigns a welfare score; specifically, a linear combination of the measures. We seek to characterize the mechanism that maximizes social welfare for every weighting. Notably, such weighting may vary with the setting, domain, and designer objectives. As the mechanism space, we consider our three attribution functions and the context size k[10].

Figure 3 presents the optimal mechanism for every distribution over our welfare measures. The first key observation is that the space is separated into clear regions of distinct optimal mechanism. Each attribution function has a substantial continuous region for which it is part of an optimal mechanism. In general, mechanisms with the argmax function are optimal when the publishers' welfare is of minor importance, with the softmax function when publishers' welfare is of greater importance, and with the linear function when the weights are more balanced between the welfare measures.

Additionally, we can see in Figure 3 that, in each attribution function region, the optimal value of k varies. For example, the optimal mechanism consists of higher values of k when publishers' welfare is of greater importance and of smaller values of k when responses' and attributions' relevance are of greater importance. Such an analysis can help a designer to derive guidelines for optimality. For instance, in cases where publishers' welfare is of greater importance, one should include a large number of publishers in generation.

7 Concluding Remarks

This work provides a first step toward understanding the learning dynamics of strategic, attribution-incentivized content creators in GenAI ecosystems. We acknowledge that these ecosystems are still evolving, and that the incentives of their stakeholders remain uncertain. Nevertheless, since GenAI systems increasingly include citations in their responses, a new channel of exposure to users has emerged for content creators. At the same time, users' utility takes on a new form, stemming from both GenAI systems' responses and the content accompanied by them.

Our results show that even in complete-information games, convergence to a stable state is a nontrivial, yet achievable, property. Specifically, several significant mechanisms induce an unstable ecosystem. This lays the groundwork for future research on learning dynamics in GenAI ecosystems, and in particular on whether less-informed publishers may lead to more stable ecosystems. In addition, our welfare analysis shows that unstable mechanisms may have advantages in terms of welfare. Indeed, the optimal mechanism depends on how social welfare is defined and weighted in the ecosystem. This enables future mechanism-design research aimed at jointly optimizing stability and social welfare.

[10]For brevity, we omit α from the mechanism design space.

AI USE STATEMENT

In this work, we used generative AI tools to assist with translation and assist in the writing of proofs. We have not used generative AI tools to generate synthetic data sets, help develop theoretical models or conceptual frameworks, formulate mathematical claims, provide critical ingredients for proving mathematical claims, propose or refine hypotheses, design or provide feedback on research methodology or experiments, interpret results, suggest experimental parameters, and implement methods, clean and reformat datasets, support qualitative and thematic data analysis are not applicable to this work. Additionally, we used generative AI tools to create or edit software code and for brainstorming. We have reviewed all AI-assisted work. We take responsibility for the final content of this work, including text, claims, or artifacts produced with the aid of generative AI.

REPRODUCIBILITY STATEMENT

All theoretical and empirical results in this paper can be reproduced from the materials described below. The full specification of our model, including all assumptions, is given in Section 4. Complete proofs of all theorems, observations, and corollaries stated in the main text can be found in Appendix B, and additional theoretical results, together with their proofs, appear in Appendix A. The code-based simulation algorithm and the full set of parameter values used in our simulations can be found in Appendix C. The source code for all simulations and for generating every figure reported in the paper can be found in the supplementary materials.

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A Additional Learning Dynamics Results

In this section, we provide additional theoretical results regarding stability and learning dynamics. We consider the model defined in Section 4 and the attribution functions described in Section 5 (argmax, softmax, and linear).

A.1 The Winner Takes All

As discussed in Section 5.1, we first show the shortcomings of the argmax attribution function in terms of ecosystem stability. In the following observation, we show that better-response dynamics may not converge in some games under the argmax function.

Observation 3. *For every $0 < \epsilon < \frac{3}{4}$, there exists a generative publishers' game for which there exists an infinite sequence of profitable deviations. That is, ε-better-response dynamics do not converge. Specifically, consider the class of games with:*

$$n = 3, \quad p = 1, \quad \lambda = 1, \quad x_1^0 = x_2^0 = x_3^0 = \frac{1}{2}, \quad r = r*,$$

and the generation function $y(x) = y_{1,z}(x) = \frac{1}{n}\sum_{j=1}^{n} x_j$ ($k = n$). Fix any $\rho \in \left(0, \frac{1}{4}\right)$ and let $x^{(0)} = \left(\frac{1}{2}, \frac{1}{2} + \rho, \frac{1}{2} + 2\rho\right)$. Define a sequence $(x^{(t)})_{t\geq 0}$ recursively as follows: at step t, the deviating publisher is $i_t \in \{1, 2, 3\}$ with cyclic order $i_t = 1, 2, 3, 1, 2, 3, \ldots$, and publisher i_t deviates to the midpoint of the other two publishers, i.e.,

$$x_{i_t}^{(t+1)} = \frac{x_j^{(t)} + x_\ell^{(t)}}{2}, \qquad \{i_t, j, \ell\} = \{1, 2, 3\}.$$

Then every deviation in this sequence is profitable. Hence $(x^{(t)})_{t\geq 0}$ is an infinite better-response path, and in particular the dynamics do not converge.

Proof. Let $t \geq 0$, and suppose publisher $i = i_t$ deviates while the other two publishers are j, ℓ. By construction we have $x_i^{(t+1)} = \frac{x_j^{(t)} + x_\ell^{(t)}}{2}$. Therefore,

$$y(x^{(t+1)}) = \frac{x_i^{(t+1)} + x_j^{(t)} + x_\ell^{(t)}}{3} = \frac{\frac{x_j^{(t)} + x_\ell^{(t)}}{2} + x_j^{(t)} + x_\ell^{(t)}}{3} = \frac{x_j^{(t)} + x_\ell^{(t)}}{2} = x_i^{(t+1)}.$$

Hence, after the deviation, publisher i is located exactly at the generated response:

$$d_i^*(x^{(t+1)}) = d\Big(x_i^{(t+1)}, y(x^{(t+1)})\Big) = 0.$$

Since the other two publishers are distinct, publisher i is the unique closest publisher to $y(x^{(t+1)})$, and thus under argmax receives full attribution:

$$r_i(x^{(t+1)}) = 1.$$

Next, note that all coordinates throughout the process remain in the interval $\left[\frac{1}{2}, \frac{1}{2} + 2\rho\right]$, because each deviation replaces one coordinate by the midpoint of the other two. Since $\rho < \frac{1}{4}$, it follows that for every t and every publisher m, $\left|x_m^{(t)} - \frac{1}{2}\right| \leq 2\rho < \frac{1}{2}$. Therefore, after publisher i deviates, her movement cost is strictly smaller than 1:

$$d_i^0(x_i^{(t+1)}) = \left(x_i^{(t+1)} - \frac{1}{2}\right)^2 \leq (2\rho)^2 < 0.25.$$

Since $\lambda = 1$, her post-deviation utility is:

$$u_i(x^{(t+1)}) = r_i(x^{(t+1)}) - \lambda d_i^0(x_i^{(t+1)}) = 1 - \left(x_i^{(t+1)} - \frac{1}{2}\right)^2 > 0.75.$$

On the other hand, before deviating, publisher i is not the winner in the cyclic construction, so $r_i(x^{(t)}) = 0$, and therefore $u_i(x^{(t)}) = -\left(x_i^{(t)} - \frac{1}{2}\right)^2 \leq 0$. Thus,

$$u_i(x^{(t+1)}) > 0.75 \geq u_i(x^{(t)}),$$

so the deviation is profitable.

Since this holds at every step, we obtain an infinite sequence of profitable deviations with utility gain of over 0.75. In particular, for any $0.75 > \epsilon > 0$, the above is an example of an ε-better-response dynamics that do not converge. □

Observation 3 shows that, under the argmax function, better-response dynamics may not converge even in a relatively simple publishers' game in the $k = n$ case. In the empirical results in Section 6, we observed the instability of the ecosystem with better-response dynamics for the $k < n$ case.

A.2 The Softmax Function

We now show that in the case where $k < n$, the softmax attribution function described in Section 5.2 does not induce a potential game. In fact, we show this for a broader family of attribution functions, the class of proportional attribution functions (Madmon et al., 2025b), defined as:

Definition 5. *We say an attribution function r is **proportional** if there exists a twice continuously differentiable function $g : [0, 1] \to \mathbb{R}_{++}$ such that:*

$$r_i(x) = \frac{g\big(d^*(x_i)\big)}{\sum_{j=1}^{n} g\big(d^*(x_j)\big)} \tag{9}$$

Notably, the softmax function is proportional.

Theorem 5. *Consider the class of generative publishers' games with a proportional attribution function $r_i(x)$ and $n > k \geq 1$.*

If this attribution function induces a potential game, then g is constant on $[0, 1]$. Hence the function is uniform:

$$r_i(x) \equiv \frac{1}{n}.$$

Proof. Assume by contradiction that there exists a non-constant twice continuously differentiable function $g : [0, 1] \to \mathbb{R}_{++}$ such that the proportional attribution rule

$$r_i(x) = \frac{g(d_i^*(x))}{\sum_{j=1}^{n} g(d_j^*(x))}$$

induces an exact potential game for every instance in the class.

We derive a contradiction by considering specific instances. Fix $p = 1$, $x^* = 0$, and $y(x) = y_{1,z}(x)$. Fix also an arbitrary cost factor $\lambda > 0$ and arbitrary initial documents.

On every open set in which the set of publishers' documents used in generation is fixed — i.e., no equal distances from x^* for the k-th and $k+1$-th distant documents — the utility functions are twice continuously differentiable. Therefore, by Theorem 4.5 of Monderer & Shapley (1996), a necessary condition for the induced game to be an exact potential game is

$$\frac{\partial^2 u_i(x)}{\partial x_j \partial x_i} = \frac{\partial^2 u_j(x)}{\partial x_i \partial x_j}.$$

for every pair of distinct publishers $i, j \in N$.

The content-modification cost of publisher i depends only on x_i. Therefore, it does not contribute to mixed partial derivatives with respect to the actions of two distinct publishers.

We distinguish between two cases.

Case 1: $1 \le k \le n-2$. Fix $\varepsilon, \rho > 0$ such that

$$0 < \varepsilon < 1 \qquad \text{and} \qquad 0 < \rho < 1 - \varepsilon.$$

Fix arbitrary constants $\bar{x}_{k+3}, \dots, \bar{x}_n \in (\varepsilon + \rho, 1)$, and consider profiles of the form:

$$x_1 = \cdots = x_k = \varepsilon, \qquad x_{k+1}, x_{k+2} \in (\varepsilon + \rho, 1),$$
$$x_\ell = \bar{x}_\ell \qquad \text{for every } \ell \in \{k+3, \dots, n\}.$$

When $n = k + 2$, the last condition is vacuous.

Since $x^* = 0$, publishers $1, \dots, k$ are exactly the k publishers closest to the user's question. Therefore,

$$y(x) = \frac{1}{k} \sum_{j=1}^{k} x_j = \varepsilon.$$

Consequently, $d_j^*(x) = 0$ for every $j \in \{1, \dots, k\}$. Define

$$s := (x_{k+1} - \varepsilon)^2 \qquad \text{and} \qquad t := (x_{k+2} - \varepsilon)^2.$$

For every $\ell \in \{k+3, \dots, n\}$, we have

$$d_\ell^*(x) = (\bar{x}_\ell - \varepsilon)^2,$$

which is constant with respect to x_{k+1} and x_{k+2}.

Define

$$A := kg(0) + \sum_{\ell=k+3}^{n} g\left((\bar{x}_\ell - \varepsilon)^2\right),$$

where the sum is interpreted as zero when $n = k + 2$. Since g is strictly positive, $A > 0$.

The denominator of the proportional attribution rule is

$$D(s,t) := A + g(s) + g(t).$$

Hence,

$$r_{k+1}(x) = \frac{g(s)}{D(s,t)} \qquad \text{and} \qquad r_{k+2}(x) = \frac{g(t)}{D(s,t)}.$$

Direct differentiation yields

$$\frac{\partial^2 u_{k+1}(x)}{\partial x_{k+2} \partial x_{k+1}} = \frac{4(x_{k+1} - \varepsilon)(x_{k+2} - \varepsilon) g'(s) g'(t)}{D(s,t)^3} \cdot \left(g(s) - A - g(t)\right),$$

and

$$\frac{\partial^2 u_{k+2}(x)}{\partial x_{k+1} \partial x_{k+2}} = \frac{4(x_{k+1} - \varepsilon)(x_{k+2} - \varepsilon) g'(s) g'(t)}{D(s,t)^3} \cdot \left(g(t) - A - g(s)\right).$$

By the mixed-partial condition, these expressions must be equal. Subtracting the second expression from the first gives

$$\frac{8(x_{k+1} - \varepsilon)(x_{k+2} - \varepsilon) g'(s) g'(t) \left(g(s) - g(t)\right)}{D(s,t)^3} = 0.$$

Since

$$x_{k+1} > \varepsilon, \qquad x_{k+2} > \varepsilon, \qquad D(s,t) > 0,$$

we obtain

$$g'(s) g'(t) \left(g(s) - g(t)\right) = 0.$$

We next show that this identity holds for every $s, t \in (0,1)$. Fix arbitrary $s, t \in (0,1)$. Choose $\varepsilon > 0$ sufficiently small such that

$$\varepsilon + \max\{\sqrt{s}, \sqrt{t}\} < 1,$$

and choose

$$0 < \rho < \min\{\sqrt{s}, \sqrt{t}\}.$$

Setting

$$x_{k+1} = \varepsilon + \sqrt{s} \qquad \text{and} \qquad x_{k+2} = \varepsilon + \sqrt{t}$$

produces the desired values of s and t. Thus, the identity holds for every $s, t \in (0, 1)$.

Suppose that there exists $s_0 \in (0, 1)$ such that $g'(s_0) \neq 0$. Since g' is continuous, there exists an open interval $I \subseteq (0, 1)$ containing s_0 such that

$$g'(s) \neq 0 \qquad \text{for every } s \in I.$$

For every $s, t \in I$, the identity implies

$$g(s) = g(t).$$

Therefore, g is constant on I, contradicting $g'(s_0) \neq 0$. It follows that

$$g'(s) = 0 \qquad \text{for every } s \in (0, 1).$$

Hence, g is constant on $[0, 1]$, contradicting the assumption that g is non-constant.

Case 2: $k = n-1$. In this case, there is only one publisher's document that is not used in generation, so the construction used in the previous case cannot be applied.

Let publisher n be the most distant publisher from the question. Fix an arbitrary $b_0 \in (0, 1)$ and choose $\varepsilon \in (0, 1 - b_0)$. There exist an open interval $B \subseteq (0, 1 - \varepsilon)$ containing b_0 and a constant $\delta > 0$ such that, for every $b \in B$ and $q \in (-\delta, \delta)$, the profile defined below belongs to $(0, 1)^n$ and publishers $1, \ldots, k$ are exactly the k publishers closest to $x^* = 0$.

For every $(q, b) \in (-\delta, \delta) \times B$, define the strategy profile $x(q, b) \in X$ by

$$x_1(q, b) = \varepsilon + kq, \qquad x_n(q, b) = \varepsilon + q + b,$$
$$x_j(q, b) = \varepsilon \qquad \text{for every } j \in \{2, \ldots, k\}.$$

When $k = 1$, the condition concerning publishers $2, \ldots, k$ is vacuous.

Indeed, for sufficiently small δ, we have

$$x_n(q, b) > x_j(q, b) \qquad \text{for every } j \in \{1, \ldots, k\}.$$

Since all coordinates are positive and $x^* = 0$, publisher n is farther from the question than publishers $1, \ldots, k$. Consequently, the set of publishers used in generation is fixed throughout this neighborhood.

For every such profile, the generated response is

$$y(x(q, b)) = \frac{1}{k} \sum_{j=1}^{k} x_j(q, b).$$

Substituting the definition of $x(q, b)$ gives

$$y(x(q, b)) = \frac{1}{k} \left(\varepsilon + kq + (k - 1)\varepsilon\right) = \varepsilon + q.$$

Therefore, the distances from the generated response are

$$d_1^*(x(q, b)) = \left(x_1(q, b) - y(x(q, b))\right)^2 = (k - 1)^2 q^2,$$
$$d_j^*(x(q, b)) = q^2 \qquad \text{for every } j \in \{2, \ldots, k\},$$
$$d_n^*(x(q, b)) = \left(x_n(q, b) - y(x(q, b))\right)^2 = b^2.$$

To simplify notation, define the restrictions of the attribution functions to this family of profiles by

$$R_i(q, b) := r_i(x(q, b)) \qquad \text{for every } i \in N.$$

Notice that r_i remains a function of strategy profiles. The function R_i is merely the composition of r_i with the profile-valued map $(q, b) \mapsto x(q, b)$.

Define

$$D(q,b) := g\left((k-1)^2q^2\right) + (k-1)g(q^2) + g(b^2).$$

It follows that

$$R_1(q,b) = \frac{g\left((k-1)^2q^2\right)}{D(q,b)} \qquad R_n(q,b) = \frac{g(b^2)}{D(q,b)}.$$

At $q = 0$, define

$$H(b) := R_1(0,b) = \frac{g(0)}{kg(0) + g(b^2)}.$$

At the profile $x(0,b)$, every publisher used in the generation process has a distance of zero from the generated response. Hence, all k publishers included in the context receive the same attribution score $H(b)$. Since the attribution scores sum to one,

$$R_n(0,b) = 1 - kH(b).$$

Both $R_1(q,b)$ and $R_n(q,b)$ are even functions of q, because q appears only through q^2. Therefore,

$$\frac{\partial R_1}{\partial q}(0,b) = 0 \qquad \text{and} \qquad \frac{\partial R_n}{\partial q}(0,b) = 0.$$

Differentiating these identities with respect to b yields

$$\frac{\partial^2 R_1}{\partial b \partial q}(0,b) = 0 \qquad \text{and} \qquad \frac{\partial^2 R_n}{\partial b \partial q}(0,b) = 0.$$

We now relate derivatives with respect to q and b to derivatives with respect to the original actions x_1 and x_n. On the two-dimensional family of profiles defined above we get

$$q = \frac{x_1 - \varepsilon}{k}, \qquad b = x_n - \varepsilon - \frac{x_1 - \varepsilon}{k}.$$

Consequently, the chain rule gives

$$\frac{\partial}{\partial x_1} = \frac{1}{k}\left(\frac{\partial}{\partial q} - \frac{\partial}{\partial b}\right), \qquad \frac{\partial}{\partial x_n} = \frac{\partial}{\partial b}.$$

The content-modification cost of each publisher depends only on that publisher's own action. Therefore, it does not contribute to mixed partial derivatives with respect to x_1 and x_n.

Evaluating the mixed partial derivative of publisher 1's utility at the profile $x(0,b)$ gives

$$\frac{\partial^2 u_1}{\partial x_n \partial x_1}(x(0,b)) = \frac{1}{k}\left(\frac{\partial^2 R_1}{\partial b \partial q}(0,b) - \frac{\partial^2 R_1}{\partial b^2}(0,b)\right) = -\frac{1}{k}H''(b).$$

Similarly, the mixed partial derivative of publisher n's utility is

$$\frac{\partial^2 u_n}{\partial x_1 \partial x_n}(x(0,b)) = \frac{1}{k}\left(\frac{\partial^2 R_n}{\partial q \partial b}(0,b) - \frac{\partial^2 R_n}{\partial b^2}(0,b)\right) = -\frac{1}{k}\frac{\mathrm{d}^2}{\mathrm{d}b^2}\left(1 - kH(b)\right) = H''(b).$$

As stated above, since the induced game is assumed to be an exact potential game, the mixed-partial condition implies

$$\frac{\partial^2 u_1}{\partial x_n \partial x_1}(x(0,b)) = \frac{\partial^2 u_n}{\partial x_1 \partial x_n}(x(0,b)).$$

Therefore,

$$-\frac{1}{k}H''(b) = H''(b).$$

Since k ≥ 1, we obtain $H''(b) = 0$. The point $b_0 \in (0,1)$ was arbitrary. Thus,

$$H''(b) = 0 \qquad \text{for every } b \in (0,1),$$

and H is affine on $(0,1)$.

Moreover,

$$H'(b) = -\frac{2b\, g(0) g'(b^2)}{\left(kg(0) + g(b^2)\right)^2}.$$

Since g' is continuous on $[0,1]$, $\lim_{b\to 0^+} H'(b) = 0$. Because H is affine on $(0,1)$, its derivative is constant. Consequently,

$$H'(b) = 0 \qquad \text{for every } b \in (0,1).$$

Hence, H is constant on $(0,1)$.

Finally,

$$H(b) = \frac{g(0)}{kg(0) + g(b^2)}.$$

Since $g(0) > 0$, the constancy of H implies that $g(b^2)$ is constant for every $b \in (0,1)$. Since b^2 ranges over $(0,1)$, g is constant on $(0,1)$. By continuity, g is constant on $[0,1]$, contradicting the assumption that g is non-constant. □

Theorem 5 implies, in particular, that any proportional attribution function — e.g., the softmax function — does not induce a potential game. One might naturally question whether the convergence of better-response dynamics under the softmax attribution function is still guaranteed, despite the game not being a potential game. However, as we showed in the main paper (Observation 2), this is not the case.

A.3 The Linear Relative Relevance Function

Here, we show an example of the non-convergence of better-response dynamics under the linear function in the $k < n$ case. Recall that in Section 5.3 we proved that, in this setting, the game is potentially not a potential game. The example below establishes the instability of mechanisms with the linear attribution function and generation based on $k < n$ publishers.

Observation 4. *There exists a generative publishers' game with $k < n$ and the linear attribution function for which better-response dynamics need not converge. In particular, there exists an infinite ε-better-response path for some $\varepsilon > 0$.*

Proof. Consider a generative publishers' game instance with:

$$n = 3, \quad p = 1, \quad x^* = \frac{1}{2}, \quad x_i^0 = \frac{1}{2}, \quad \lambda = \frac{1}{1000},$$

the generation function $y(x) = y_{1,z}(x) = \frac{1}{2}\sum_{j=1}^{2} x_j$ ($k = 2$), and the linear attribution function $r = \hat{r}$ with slope $m = \frac{1}{3}$.

Consider the following eight profiles:

$$A = \left(0, \frac{1}{5}, \frac{1}{5}\right), \quad B = \left(0, \frac{2}{5}, \frac{1}{5}\right), \quad C = \left(0, \frac{3}{5}, \frac{1}{5}\right), \quad D = \left(0, \frac{4}{5}, \frac{1}{5}\right), \quad E = \left(0, \frac{4}{5}, \frac{2}{5}\right),$$
$$F = \left(1, \frac{4}{5}, \frac{2}{5}\right), \quad G = \left(1, \frac{1}{5}, \frac{2}{5}\right), \quad H = \left(1, \frac{1}{5}, \frac{1}{5}\right).$$

We claim that the following cycle is a better-response path:

$$A \xrightarrow{2} B \xrightarrow{2} C \xrightarrow{2} D \xrightarrow{3} E \xrightarrow{1} F \xrightarrow{2} G \xrightarrow{3} H \xrightarrow{1} A.$$

For each step, the deviating publisher's increase in relative relevance and utility is as follows:

Step	Deviator	$\Delta\nu_i$	Δu_i
$A \to B$	2	$\frac{1}{50}$	$\frac{253}{37500}$
$B \to C$	2	$\frac{1}{50}$	$\frac{1}{150}$
$C \to D$	2	$\frac{1}{50}$	$\frac{247}{37500}$
$D \to E$	3	$\frac{2}{25}$	$\frac{1003}{37500}$
$E \to F$	1	$\frac{1}{5}$	$\frac{1}{15}$
$F \to G$	2	$\frac{9}{50}$	$\frac{3}{50}$
$G \to H$	3	$\frac{2}{25}$	$\frac{997}{37500}$
$H \to A$	1	$\frac{3}{5}$	$\frac{1}{5}$

All utility gains are strictly positive. Therefore each step is a better response. Moreover,

$$\min \Delta u_i = \frac{247}{37500} > 0.$$

Hence, for every

$$0 < \varepsilon < \frac{247}{37500},$$

each step in the cycle is an ε-better response. Repeating the cycle indefinitely yields an infinite ε-better-response path, so the dynamics do not converge. □

B Omitted Proofs

Proof of Observation 1. We divide the proof into two parts: the $k < n$ case and the $k = n$ case. We start with the $k = n$ case.

Let $n \geq 5$, let $\lambda \in \left(\frac{4}{n}, 1\right]$, and consider a generative publishers' game G in which:

$$p = 1, \quad x_1^0 = 1, \quad x_2^0 = \cdots = x_n^0 = 0,$$

the generation function $y(x) = y_{1,z}(x) = \frac{1}{n}\sum_{j=1}^{n} x_j$ $(k = n)$, an arbitrary $x^* \in [0, 1]$, and the argmax attribution function $r = r^*$.

Assume by contradiction that x^{eq} is a PNE, and write $y := y(x^{eq}) = \frac{1}{n}\sum_{j=1}^{n} x_j^{eq}$.

We first rule out the case where all strategies are equal. If $x_1^{eq} = \cdots = x_n^{eq} = a$, then under the argmax, each publisher gets attribution probability of $\frac{1}{n}$. Hence:

$$u_1(x^{eq}) = \frac{1}{n} - \lambda(1-a)^2, \qquad u_i(x^{eq}) = \frac{1}{n} - \lambda a^2 \quad (i \geq 2).$$

For this profile to be a PNE, all these utilities must be nonnegative (otherwise, at least one publisher has a profitable deviation to her initial document), so we would need

$$a \leq \frac{1}{\sqrt{n\lambda}} \qquad \text{and} \qquad 1 - a \leq \frac{1}{\sqrt{n\lambda}}.$$

Thus,

$$1 \leq \frac{2}{\sqrt{n\lambda}},$$

i.e.,

$$\lambda \leq \frac{4}{n},$$

contrary to $\lambda > \frac{4}{n}$. Therefore x^{eq} is not an equal strategies profile.

Next, in PNE under the argmax, every non-winning publisher[11] must play her initial document. For example, if publisher 1 is not a winner and $x_1^{eq} \neq 1$, then deviating to 1 yields a utility of at least 0, which is strictly better than her current utility $-\lambda(1 - x_1^{eq})^2 < 0$.

We now claim that publisher 1 cannot be a winner. Suppose otherwise. Since the profile is not all equal, there is at least one non-winner, and every non-winner is at her initial document. In particular, every losing publisher is at 0.

Also, a winning publisher cannot lie above y: if some winner $i \geq 2$ satisfies $x_i^{eq} > y$, then by moving slightly toward y, publisher i becomes strictly closer to the new average than every other publisher, while strictly decreasing her movement cost. Hence this would be a profitable deviation. By the same argument, if publisher 1 is a winner then necessarily $x_1^{eq} \geq y$.

Therefore, if publisher 1 is a winner, then all winning publishers $i \geq 2$ lie at some point $y - \rho$ and publisher 1 lies at $y + \rho$, for some $\rho \geq 0$. Let $t \geq 1$ be the number of winning publishers. Since all losers are at 0, the generation function gives:

$$ny = (y + \rho) + (t - 1)(y - \rho),$$

[11] Here, a winning publisher is any publisher receiving a positive attribution score.

hence

$$(n-t)y = (2-t)\rho.$$

Notably, if $t \geq 3$ the generation response would be closer to $y - \rho$ (i.e., publisher 1 is not the winner). Therefore, it holds that $t \leq 2$.

Suppose first that $t = 1$. Let publisher 1 be the unique winning publisher. Since all other publishers are located at 0, the generated response satisfies $y = \frac{x_1}{n}$. Hence, the distance of publisher 1 from the generated response is

$$\rho = |x_1 - y| = (n-1)y,$$

whereas the distance of every other publisher from the generated response is y. Because publisher 1 is the unique winner, it must be at least as close to the generated response as every other publisher. Therefore,

$$\rho \leq y.$$

Combining the two relations gives

$$(n-1)y \leq y.$$

Since $n \geq 5$, this implies $y = 0$, and consequently $x_1 = 0$. Thus, all publishers choose the same strategy, contradicting the case under consideration. Therefore, $t \neq 1$.

If $t = 2$, the left-hand side is nonnegative ($n - t > 0$) and the right-hand side is 0, thus $y = 0$. It holds that $y - \rho \in [0, 1]$ so $\rho = 0$, contrary to the non-equal strategy profile. So publisher 1 is not a winner.

Hence, publisher 1 is a non-winner (i.e., all winners are high-index publishers), therefore,

$$x_1^{eq} = 1.$$

We next claim that every winner must in fact be located exactly at y. Suppose some winning high-index publisher is located at $a < y$.

If there is at least one non-winner high-index publisher, then that player is located at 0. Let i be such a player, and let

$$m := \frac{1 + \sum_{j \neq 1,i}^{n} x_j^{eq}}{n-1},$$

be the average of the other $n-1$ publishers. Since the current winners are all at the same point $a < y$, we have

$$m = \frac{ny}{n-1} > y > a.$$

If publisher i deviates to m, then the new generated answer is exactly m, so publisher i becomes the unique argmax winner. Her new utility is:

$$u_i(m, x_{-i}^{eq}) = 1 - \lambda m^2 > 0,$$

because $m < 1$ and $\lambda \leq 1$. But currently $u_i(x^{eq}) = 0$, contradiction.

If, on the other hand, all high-index publishers are winners, then there are at least two winners at the same point $a < y$. Any one of them can move slightly toward y, become the unique winner, and improve her ranking payoff from $\frac{1}{n-1}$ to 1, while changing her cost by an arbitrarily small amount. This is again a profitable deviation, contradiction.

Therefore every winner must be exactly at y.

So x^{eq} must be of the form:

$$(1, \underbrace{y, \ldots, y}_{t \text{ times}}, \underbrace{0, \ldots, 0}_{n-t-1 \text{ times}})$$

for some $t \in \{1, \ldots, n-2\}$. The case $t = n-1$ would imply

$$y = \frac{1 + (n-1)y}{n} = 1,$$

which would make the equal strategies profile, already ruled out.

Now pick a losing high-index publisher i, so $x_i^{eq} = 0$. Let $m := \frac{1+ty}{n-1}$ the average of the other $n-1$ publishers. We get

$$m = \frac{ny}{n-1}.$$

In particular, $m > y$ and $0 < m < 1$. If publisher i deviates to m, then the generated response becomes exactly m, so publisher i becomes the unique argmax winner. Hence,

$$u_i(m, x_{-i}^{eq}) = 1 - \lambda m^2 > 0 = u_i(x^{eq}),$$

a profitable deviation.

This contradiction shows that no PNE exists and completes the proof for the case of $k = n$. We now turn to the $k < n$ case.

Let $\lambda > \frac{1}{3}$ and consider a generative publishers' game G in which:

$$n = 3, \quad p = 1, \quad x_1^0 = 1, \quad x_2^0 = x_3^0 = 0, \quad x^* = 1,$$

the generation function $y(x) = y_{1,z}(x) = \frac{1}{2}\sum_{j=1}^{2} x_j$ ($k = 2 < n$), and the argmax attribution function $r = r^*$.

Assume by contradiction that $x^{eq} = (x_1^{eq}, x_2^{eq}, x_3^{eq})$ is a PNE of G.

We first claim that $x_1^{eq} = 1$. Indeed, if publisher 1 is not among the two publishers closest to $x^* = 1$, then $r_1(x^{eq}) = 0$, whereas by deviating to $\hat{x}_1 = 1$, publisher 1 becomes one of the (at least) two closest publishers to $x^* = 1$, obtains an attribution probability of at least $\frac{1}{3}$, and incurs no cost. If publisher 1 is already among the two publishers closest to $x^* = 1$, then deviating to $\hat{x}_1 = 1$ does not decrease her attribution probability. In all cases, the deviation strictly decreases publisher 1's cost unless already $x_1^{eq} = 1$. Hence $x_1^{eq} = 1$.

We now consider the remaining two coordinates.

- $x_2^{eq} = x_3^{eq} = a$. If $a < 1$, we obtain

  $$y(x^{eq}) = \frac{1+a}{2}.$$

  All three publishers are at the same distance from $y(x^{eq})$, so each receives an attribution probability of $\frac{1}{3}$. Thus

  $$u_2(x^{eq}) = \frac{1}{3} - \lambda a^2.$$

  By deviating to $\hat{x}_2 = a + \varepsilon$ for sufficiently small $\varepsilon > 0$, publisher 2 becomes the unique non-player-1 member of the generating set, and the attributed publishers are exactly publishers 1 and 2. Hence

  $$u_2(\hat{x}_2, x_{-2}^{eq}) = \frac{1}{2} - \lambda(a+\varepsilon)^2 > \frac{1}{3} - \lambda a^2 = u_2(x^{eq}),$$

  a contradiction. If $a = 1$, then

  $$u_2(x^{eq}) = \frac{1}{3} - \lambda < 0,$$

  whereas by deviating to $\hat{x}_2 = 0$, publisher 2 gets utility 0. Hence, this case is not a PNE either.

- $x_2^{eq} \neq x_3^{eq}$. Without loss of generality, assume

  $$x_2^{eq} > x_3^{eq}.$$

  Since $x_1^{eq} = 1$, publisher 2 is the unique second-closest publisher to $x^* = 1$, and therefore

  $$y(x^{eq}) = \frac{1 + x_2^{eq}}{2}.$$

  We now compare distances to the generated response:

  $$|1 - y(x^{eq})| = |x_2^{eq} - y(x^{eq})| = \frac{1 - x_2^{eq}}{2},$$

while

$$|x_3^{eq} - y(x^{eq})| = \frac{1 + x_2^{eq} - 2x_3^{eq}}{2} > \frac{1 - x_2^{eq}}{2},$$

because $x_2^{eq} > x_3^{eq}$. Hence, publishers 1 and 2 are the unique closest publishers to the generated response, and

$$u_2(x^{eq}) = \frac{1}{2} - \lambda(x_2^{eq})^2, \qquad u_3(x^{eq}) = -\lambda(x_3^{eq})^2.$$

If $x_3^{eq} > 0$, then publisher 3 can deviate to $\hat{x}_3 = 0$, therefore,

$$u_3(\hat{x}_3, x_{-3}^{eq}) = 0 > -\lambda(x_3^{eq})^2 = u_3(x^{eq}),$$

contradiction. Hence necessarily $x_3^{eq} = 0$.

We are therefore left with a candidate equilibrium of the form $(1, x_2^{eq}, 0)$ with $x_2^{eq} > 0$. At this profile, publisher 2 gets

$$u_2(x^{eq}) = \frac{1}{2} - \lambda(x_2^{eq})^2.$$

If publisher 2 deviates to $\hat{x}_2 = 0$, the new profile is $(1, 0, 0)$. Regardless of how ties are broken, the generated response is

$$y(\hat{x}_2, x_{-2}^{eq}) = \frac{1}{2},$$

because the second selected generator is located at 0. All three publishers are then at a distance of $\frac{1}{2}$ from the generated response, so each receives an attribution probability of $\frac{1}{3}$. Hence, the deviation yields utility

$$u_2(\hat{x}_2, x_{-2}^{eq}) = \frac{1}{3}.$$

Therefore, if x^{eq} were a PNE, we would have

$$\frac{1}{2} - \lambda(x_2^{eq})^2 \geq \frac{1}{3},$$

i.e.,

$$\lambda(x_2^{eq})^2 \leq \frac{1}{6}.$$

In particular,

$$(x_2^{eq})^2 \leq \frac{1}{6\lambda} < \frac{1}{2\lambda},$$

so

$$x_2^{eq} < \frac{1}{\sqrt{2\lambda}}.$$

Choose any

$$b \in \left(x_2^{eq},\, \min\left\{1, \frac{1}{\sqrt{2\lambda}}\right\}\right).$$

Such a b exists by the strict inequality above. If publisher 3 deviates to $\hat{x}_3 = b$, then publisher 3 becomes the unique second-closest publisher to $x^* = 1$, so the generating set becomes $\{1, 3\}$ and

$$y(x_1^{eq}, x_2^{eq}, \hat{x}_3) = \frac{1 + b}{2}.$$

Now publishers 1 and 3 are the unique closest publishers to this generated response, because $b > x_2^{eq}$. Hence

$$u_3(x_1^{eq}, x_2^{eq}, \hat{x}_3) = \frac{1}{2} - \lambda b^2 > 0.$$

Since currently $u_3(x^{eq}) = 0$, this is a profitable deviation, contradiction.

All possible cases lead to a contradiction. Therefore, G possesses no PNE. □

Proof of Theorem 2. Assume that there exists $\beta > 0$ such that the softmax attribution rule $\tilde{r}$ induces an exact potential game. In particular, consider a generative publishers' game with the following parameters:

$$n = 3, \quad p = 1, \quad y(x) = y_{1,z}(x) := \frac{1}{n}\sum_{j=1}^{n} x_j,$$

an arbitrary $\lambda > 0$, and arbitrary initial documents $x_1^0, x_2^0, x_3^0 \in [0, 1]$. By assumption, this is a potential game and, thus, admits an exact potential function, which we denote by Φ^β.

Since the utilities are twice continuously differentiable, by Theorem 4.5 of Monderer & Shapley (1996), the existence of an exact potential implies that

$$\frac{\partial^2 u_i}{\partial x_j \partial x_i} = \frac{\partial^2 u_j}{\partial x_i \partial x_j} \qquad \text{for every } i, j \in N.$$

Now fix $s \in (0, 1)$ and choose $t > 0$ such that:

$$0 < t < \min\{s, 1 - s\}.$$

For the profile $x = (s + t, s, s - t)$, denote

$$g := e^{-\beta t^2}, \qquad D := 1 + 2g.$$

Note that $y(x) = s$, so we get

$$d_1^* = t^2, \qquad d_2^* = 0, \qquad d_3^* = t^2,$$

and hence

$$\tilde{r}_1(x) = \tilde{r}_3(x) = \frac{g}{D}, \qquad \tilde{r}_2(x) = \frac{1}{D}.$$

Since the cost term $d_i^0(x_i)$ depends only on x_i, its mixed cross-partials vanish, and therefore:

$$\frac{\partial^2 u_1}{\partial x_2 \partial x_1} - \frac{\partial^2 u_2}{\partial x_1 \partial x_2} = \frac{\partial^2 \tilde{r}_1}{\partial x_2 \partial x_1} - \frac{\partial^2 \tilde{r}_2}{\partial x_1 \partial x_2}.$$

Now

$$d_1^* = \left(\frac{2x_1 - x_2 - x_3}{3}\right)^2, \quad d_2^* = \left(\frac{2x_2 - x_1 - x_3}{3}\right)^2, \quad d_3^* = \left(\frac{2x_3 - x_1 - x_2}{3}\right)^2.$$

Evaluating at $x = (s + t, s, s - t)$ gives

$$\partial_{x_1} d_1^* = \frac{4t}{3}, \qquad \partial_{x_2} d_1^* = -\frac{2t}{3}, \qquad \partial^2_{x_1 x_2} d_1^* = -\frac{4}{9},$$

$$\partial_{x_1} d_2^* = 0, \qquad \partial_{x_2} d_2^* = 0, \qquad \partial^2_{x_1 x_2} d_2^* = -\frac{4}{9},$$

$$\partial_{x_1} d_3^* = \frac{2t}{3}, \qquad \partial_{x_2} d_3^* = \frac{2t}{3}, \qquad \partial^2_{x_1 x_2} d_3^* = \frac{2}{9}.$$

For the softmax rule,

$$\partial_{x_j} \tilde{r}_i = -\beta \tilde{r}_i \left(\partial_{x_j} d_i^* - \sum_{m=1}^{3} \tilde{r}_m \, \partial_{x_j} d_m^*\right).$$

Thus, if we set

$$B_1 := \sum_{m=1}^{3} \tilde{r}_m \, \partial_{x_1} d_m^*, \qquad B_2 := \sum_{m=1}^{3} \tilde{r}_m \, \partial_{x_2} d_m^*,$$

then at $x = (s + t, s, s - t)$,

$$B_1 = \frac{g}{D} \cdot \frac{4t}{3} + \frac{1}{D} \cdot 0 + \frac{g}{D} \cdot \frac{2t}{3} = \frac{2gt}{D},$$

$$B_2 = \frac{g}{D} \cdot \left(-\frac{2t}{3}\right) + \frac{1}{D} \cdot 0 + \frac{g}{D} \cdot \frac{2t}{3} = 0.$$

Hence,

$$\partial_{x_2} \tilde{r}_1 = -\beta \frac{g}{D} \left(-\frac{2t}{3} - 0\right) = \frac{2\beta g t}{3D},$$
$$\partial_{x_1} \tilde{r}_2 = -\beta \frac{1}{D} \left(0 - \frac{2gt}{D}\right) = \frac{2\beta g t}{D^2}.$$

Differentiating once more,

$$\partial^2_{x_2 x_1} \tilde{r}_1 = -\beta \left[\partial_{x_2} \tilde{r}_1 \big(\partial_{x_1} d_1^* - B_1\big) + \tilde{r}_1 \big(\partial^2_{x_2 x_1} d_1^* - \partial_{x_2} B_1\big)\right],$$
$$\partial^2_{x_1 x_2} \tilde{r}_2 = -\beta \left[\partial_{x_1} \tilde{r}_2 \big(\partial_{x_2} d_2^* - B_2\big) + \tilde{r}_2 \big(\partial^2_{x_1 x_2} d_2^* - \partial_{x_1} B_2\big)\right].$$

Using

$$\partial_{x_2} B_1 = \partial_{x_1} B_2 = \frac{4\beta g t^2}{9D} - \frac{2(2+g)}{9D},$$

one obtains:

$$\partial^2_{x_2 x_1} \tilde{r}_1 = \frac{2\beta g(3g - 4\beta t^2)}{9D^2}, \qquad \partial^2_{x_1 x_2} \tilde{r}_2 = \frac{2\beta g(3 + 2\beta t^2)}{9D^2}.$$

Therefore,

$$\left(\frac{\partial^2 u_1}{\partial x_2 \partial x_1} - \frac{\partial^2 u_2}{\partial x_1 \partial x_2}\right)\Bigg|_{x=(s+t,s,s-t)} = -\frac{2\beta g}{3(1+2g)^2}\big(1 + 2\beta t^2 - g\big).$$

Since $\beta > 0$ and $t > 0$, we have $0 < g(\beta) = e^{-\beta t^2} < 1$, and therefore $1 + 2\beta t^2 - g(\beta) > 0$. It follows that

$$-\frac{2\beta\, g(\beta)}{3(1+2g(\beta))^2}\Big(1 + 2\beta t^2 - g(\beta)\Big) \neq 0.$$

Hence,

$$\frac{\partial^2 u_1}{\partial x_2 \partial x_1} \neq \frac{\partial^2 u_2}{\partial x_1 \partial x_2}$$

at the profile $x = (s+t, s, s-t)$, contradicting a necessary condition for the existence of an exact potential. We conclude that no exact potential can exist when $\beta > 0$. Therefore, if the softmax rule induces an exact potential game, it must be that $\beta = 0$.

Finally, when $\beta = 0$, the attribution function becomes uniform across publishers:

$$\tilde{r}_i^0(x) = \frac{1}{\sum_{j=1}^n 1} = \frac{1}{n} \qquad \text{for all } i \in N.$$

□

Proof of Observation 2. We divide the proof into two parts: the $k < n$ case and the $k = n$ case. We start with the $k = n$ case.

Fix any $t \in \big(0, \frac{1}{2}\big)$ and $\lambda > 0$ such that $\lambda t^2 < \frac{1}{2}$ and $\lambda(1-t)^2 < \frac{1}{2}$. Consider a generative publishers' game in which:

$$n = 3, \quad p = 1, \quad x_1^0 = 1, \quad x_2^0 = x_3^0 = 0, \quad r = \tilde{r},$$

and the generation function $y(x) = y_{1,z}(x) = \frac{1}{n}\sum_{j=1}^n x_j$ $(k = n)$.

Consider the following strategy profiles:

$$A = (t, 0, 0),\ B = (1, 0, 0),\ C = (1, 0, t),\ D = (t, 0, t).$$

These profiles form a cyclic better-response dynamics, with deviators $A \xrightarrow{1} B \xrightarrow{3} C \xrightarrow{1} D \xrightarrow{3} A$.

Let

$$\Delta_{A\to B}(\beta) := u_1(B) - u_1(A), \quad \Delta_{B\to C}(\beta) := u_3(C) - u_3(B),$$
$$\Delta_{C\to D}(\beta) := u_1(D) - u_1(C),$$
$$\Delta_{D\to A}(\beta) := u_3(A) - u_3(D).$$

For $A = (t, 0, 0)$, we have $y(A) = t/3$, so

$$d_1^*(A) = \left(t - \frac{t}{3}\right)^2 = \frac{4t^2}{9},$$

$$d_2^*(A) = d_3^*(A) = \left(0 - \frac{t}{3}\right)^2 = \frac{t^2}{9}.$$

Hence,

$$\tilde{r}_1(A) = \frac{e^{-4\beta t^2/9}}{e^{-4\beta t^2/9} + 2e^{-\beta t^2/9}} = \frac{1}{1 + 2e^{\beta t^2/3}}.$$

For $B = (1, 0, 0)$, we have $y(B) = 1/3$, so:

$$d_1^*(B) = \left(1 - \frac{1}{3}\right)^2 = \frac{4}{9},$$

$$d_2^*(B) = d_3^*(B) = \left(0 - \frac{1}{3}\right)^2 = \frac{1}{9}.$$

Hence,

$$\tilde{r}_1(B) = \frac{e^{-4\beta/9}}{e^{-4\beta/9} + 2e^{-\beta/9}} = \frac{1}{1 + 2e^{\beta/3}}.$$

Using the above we get

$$\Delta_{A\to B}(\beta) = \frac{1}{1 + 2e^{\beta/3}} - \frac{1}{1 + 2e^{\beta t^2/3}} + \lambda(1-t)^2 \xrightarrow[\beta\to\infty]{} \lambda(1-t)^2 > 0.$$

At B, publisher 3 has softmax weight

$$\tilde{r}_3(B) = \frac{1}{2 + e^{-\beta/3}} \xrightarrow[\beta\to\infty]{} \frac{1}{2}.$$

At $C = (1, 0, t)$, since

$$d_3^*(C) < d_2^*(C) < d_1^*(C) \qquad \text{for every } t \in \left(0, \frac{1}{2}\right),$$

publisher 3 is the unique closest publisher to the generated response, and therefore

$$\tilde{r}_3(C) \xrightarrow[\beta\to\infty]{} 1.$$

Thus,

$$\Delta_{B\to C}(\beta) \xrightarrow[\beta\to\infty]{} 1 - \lambda t^2 - \frac{1}{2} = \frac{1}{2} - \lambda t^2 > 0.$$

At C, publisher 1 is not a closest publisher, so

$$\tilde{r}_1(C) \xrightarrow[\beta\to\infty]{} 0.$$

At $D = (t, 0, t)$, publishers 1 and 3 are tied for being closest to the generated response, while publisher 2 is farther, hence

$$\tilde{r}_1(D) = \frac{1}{2 + e^{-\beta t^2/3}} \xrightarrow[\beta\to\infty]{} \frac{1}{2}.$$

Therefore,

$$\Delta_{C\to D}(\beta) \xrightarrow[\beta\to\infty]{} \frac{1}{2} - \lambda(1-t)^2 > 0.$$

Finally, at both A and D, publisher 3 has the same attribution probability

$$\tilde{r}_3(A) = \tilde{r}_3(D) = \frac{1}{2 + e^{-\beta t^2/3}},$$

while her movement cost equals 0 at A and λt^2 at D. Hence,

$$\Delta_{D\to A}(\beta) = \lambda t^2 > 0 \qquad \text{for every } \beta > 0.$$

Since the first three utility gaps converge to strictly positive limits, there exists $\beta_0 > 0$ such that all four deviations are profitable for every $\beta \geq \beta_0$. Fix any such β, and define:

$$\epsilon := \frac{1}{2} \min\Big\{\Delta_{A\to B}(\beta), \Delta_{B\to C}(\beta), \Delta_{C\to D}(\beta), \Delta_{D\to A}(\beta)\Big\} > 0.$$

Then every step in the cycle is an ϵ-better response. Repeating the cycle yields an infinite ϵ-better-response dynamic. This completes the proof for the case of $k = n$. We now turn to the $k < n$ case.

Fix $\lambda > \frac{1}{6}$, and choose $b, c \in (0, 1)$ such that:

$$0 < b < c < 1, \qquad \lambda b^2 < \frac{1}{6}, \qquad \frac{1}{6} < \lambda c^2 < \frac{1}{2}.$$

Consider a generative publishers' game with:

$$n = 3, \quad p = 1, \quad x^* = 1, \quad x_1^0 = 1, \quad x_2^0 = x_3^0 = 0,$$

the generation function $y(x) = y_{1,z}(x) := \frac{1}{2}\sum_{j=1}^{2} x_j$ ($k = 2$), and the softmax attribution function $r = \tilde{r}_\beta$. Then there exists $\beta_0 > 0$ such that for every $\beta \geq \beta_0$, the following four profiles form a cyclic better-response path:

$$A = (1, 0, 0),\ B = (1, b, 0),\ C = (1, b, c),\ D = (1, 0, c),$$

with deviators:

$$A \xrightarrow{2} B \xrightarrow{3} C \xrightarrow{2} D \xrightarrow{3} A.$$

Hence, there exists an infinite sequence of profitable deviations, and in particular, for sufficiently small $\varepsilon > 0$, ε-better-response dynamics may not converge.

We analyze the four profiles one by one. At the profile $A = (1, 0, 0)$, publisher 1 is certainly among the two publishers closest to $x^* = 1$, and the second selected publisher is one of $2, 3$. In either case, the generated response is:

$$y(A) = \frac{1 + 0}{2} = \frac{1}{2}.$$

Therefore, all three publishers are at the same distance from the generated response,

$$d_1^*(A) = d_2^*(A) = d_3^*(A) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

Hence, under the softmax attribution rule, we have

$$\tilde{r}_1(A) = \tilde{r}_2(A) = \tilde{r}_3(A) = \frac{1}{3},$$

for every $\beta > 0$. In particular, $u_2(A) = u_3(A) = \frac{1}{3}$.

Next, consider $B = (1, b, 0)$. Since $0 < b < 1$, the two publishers closest to $x^* = 1$ are 1 and 2; therefore

$$y(B) = \frac{1 + b}{2}.$$

The distances to the generated response are:

$$d_1^*(B) = d_2^*(B) = \left(\frac{1 - b}{2}\right)^2, \qquad d_3^*(B) = \left(\frac{1 + b}{2}\right)^2.$$

Since $d_3^*(B) > d_1^*(B) = d_2^*(B)$, it follows that

$$\lim_{\beta\to\infty} \tilde{r}_2(B) = \frac{1}{2}, \qquad \lim_{\beta\to\infty} \tilde{r}_3(B) = 0.$$

Hence,

$$\lim_{\beta\to\infty} \big(u_2(B) - u_2(A)\big) = \left(\frac{1}{2} - \lambda b^2\right) - \frac{1}{3} = \frac{1}{6} - \lambda b^2 > 0,$$

that is, for sufficiently large β publisher 2 has a profitable deviation from profile A to profile B.

Now consider $C = (1, b, c)$. Since $0 < b < c < 1$, the two publishers closest to $x^* = 1$ are 1 and 3, and thus:

$$y(C) = \frac{1+c}{2}.$$

The distances to the generated response are

$$d_1^*(C) = d_3^*(C) = \left(\frac{1-c}{2}\right)^2, \quad d_2^*(C) = \left(\frac{1+c}{2} - b\right)^2.$$

Because $c > b$, we have

$$\frac{1+c}{2} - b > \frac{1-c}{2},$$

and therefore $d_2^*(C) > d_1^*(C) = d_3^*(C)$. Thus, it follows that

$$\lim_{\beta\to\infty} \tilde{r}_3(C) = \frac{1}{2}, \qquad \lim_{\beta\to\infty} \tilde{r}_3(B) = 0,$$

and hence

$$\lim_{\beta\to\infty} \big(u_3(C) - u_3(B)\big) = \left(\frac{1}{2} - \lambda c^2\right) - 0 = \frac{1}{2} - \lambda c^2 > 0,$$

that is, for sufficiently large β publisher 3 has a profitable deviation from profile B to profile C.

Next, consider $D = (1, 0, c)$. Again the two publishers closest to $x^* = 1$ are 1 and 3, so

$$y(D) = \frac{1+c}{2}.$$

The distances to the generated response are

$$d_1^*(D) = d_3^*(D) = \left(\frac{1-c}{2}\right)^2, \qquad d_2^*(D) = \left(\frac{1+c}{2}\right)^2.$$

Hence,

$$\lim_{\beta\to\infty} \tilde{r}_2(C) = 0, \qquad \lim_{\beta\to\infty} \tilde{r}_2(D) = 0,$$

and therefore

$$\lim_{\beta\to\infty} \big(u_2(D) - u_2(C)\big) = 0 - (-\lambda b^2) = \lambda b^2 > 0,$$

that is, for sufficiently large β publisher 2 has a profitable deviation from profile C to profile D.

Finally, comparing D and A, we have already seen that $u_3(A) = \frac{1}{3}$ for every $\beta > 0$, while:

$$\lim_{\beta\to\infty} u_3(D) = \frac{1}{2} - \lambda c^2.$$

Thus, we get:

$$\lim_{\beta\to\infty} \big(u_3(A) - u_3(D)\big) = \frac{1}{3} - \left(\frac{1}{2} - \lambda c^2\right) = \lambda c^2 - \frac{1}{6} > 0,$$

that is, for sufficiently large β publisher 3 has a profitable deviation from profile D to profile A.

We have shown that the four utility differences

$$u_2(B) - u_2(A),\ u_3(C) - u_3(B),\ u_2(D) - u_2(C),\ u_3(A) - u_3(D)$$

all converge, as $\beta \to \infty$, to strictly positive quantities. Therefore, there exists $\beta_0 > 0$ such that for every $\beta \geq \beta_0$, all four deviations remain profitable. In other words, $A \xrightarrow{2} B \xrightarrow{3} C \xrightarrow{2} D \xrightarrow{3} A$ is a cyclic better-response path. Hence, there exists an infinite sequence of profitable deviations, and in particular, for sufficiently small $\varepsilon > 0$, ε-better-response dynamics may not converge. □

Proof of Theorem 4. Assume by contradiction that the linear attribution function $\hat{r}$ induces a potential game for the $k < n$ case. Thus, it induces a potential game for a generative publishers' game with:

$$p = 1, \qquad x^* = 0, \qquad \lambda > 0,$$

the generation function $y(x) = y_{1,z}(x) = \frac{1}{k}\sum_{j=1}^{k} x_j$ where $k < n$, and arbitrary initial documents.

Fix parameters ε, ρ such that

$$0 < \varepsilon < 1, \qquad 0 < \rho < 1 - \varepsilon.$$

Now consider the open set:

$$U := \Big\{ x \in [0,1]^n : \; x_1, \ldots, x_k \in (0, \varepsilon), \; x_{k+1}, \ldots, x_n \in (\varepsilon + \rho, 1) \Big\}.$$

Since $x^* = 0$, for every $x \in U$ the k closest publishers to the user's question x^* are exactly publishers $1, \ldots, k$. Hence, on U it holds that:

$$y(x) = \frac{1}{k}\sum_{j=1}^{k} x_j.$$

In particular, on U, the function y is twice continuously differentiable, it does not depend on x_{k+1}, and

$$\frac{\partial y}{\partial x_1}(x) = \frac{1}{k} \qquad \forall x \in U.$$

By assumption, this game is a potential game. Since the utilities are twice continuously differentiable on U, Theorem 4.5 of Monderer & Shapley (1996) implies that for every $x \in U$ we have:

$$\frac{\partial^2 u_1}{\partial x_{k+1} \partial x_1}(x) = \frac{\partial^2 u_{k+1}}{\partial x_1 \partial x_{k+1}}(x).$$

We now compute these two cross-derivatives.

Because y is independent of x_{k+1} on U, for every $j \neq k+1$ the term: $d_j^*(x) = (x_j - y(x))^2$ is independent of x_{k+1}. Therefore,

$$\frac{\partial^2 d_j^*}{\partial x_{k+1} \partial x_1}(x) = 0 \qquad \forall j \neq k+1.$$

For publisher $k+1$, $d_{k+1}^*(x) = (x_{k+1} - y(x))^2$. Differentiating first with respect to x_1, we obtain

$$\frac{\partial d_{k+1}^*}{\partial x_1}(x) = -2(x_{k+1} - y(x))\frac{\partial y}{\partial x_1}(x) = -\frac{2}{k}(x_{k+1} - y(x)).$$

Differentiating now with respect to x_{k+1}, and using again that y does not depend on x_{k+1}, gives

$$\frac{\partial^2 d_{k+1}^*}{\partial x_{k+1} \partial x_1}(x) = -\frac{2}{k}.$$

Recall that

$$\nu_i(x) = \frac{1}{n-1}\sum_{j \neq i} d_j^*(x) - d_i^*(x).$$

Hence,

$$\frac{\partial^2 \nu_1}{\partial x_{k+1} \partial x_1}(x) = \frac{1}{n-1}\frac{\partial^2 d_{k+1}^*}{\partial x_{k+1} \partial x_1}(x) = -\frac{2}{k(n-1)},$$

whereas

$$\frac{\partial^2 \nu_{k+1}}{\partial x_{k+1} \partial x_1}(x) = -\frac{\partial^2 d_{k+1}^*}{\partial x_{k+1} \partial x_1}(x) = \frac{2}{k}.$$

The cost terms depend only on a player's own action, so their mixed partial derivatives vanish. Therefore,

$$\frac{\partial^2 u_1}{\partial x_{k+1} \partial x_1}(x) = m\frac{\partial^2 \nu_1}{\partial x_{k+1} \partial x_1}(x) = -\frac{2m}{k(n-1)},$$

$$\frac{\partial^2 u_{k+1}}{\partial x_1 \partial x_{k+1}}(x) = m\frac{\partial^2 \nu_{k+1}}{\partial x_{k+1}\partial x_1}(x) = \frac{2m}{k}.$$

Since $m > 0$, $k \geq 1$, $n > k$ these two expressions are not equal. This contradicts the necessary cross-partial equality for potential games.

Therefore, the game is not a potential game. Since this game belongs to the above class, we conclude that the linear attribution function does not induce a potential game for generative publishers' games.

□

An interesting question that arises is whether better-response dynamics in generative publishers' games with context size of $k < n$ converge, even though such games are not potential games. As we showed in Section 6, that is not the case. To conclude, generative publishers' games with the linear attribution function converge under any ϵ-better-response dynamic in the case of $k = n$; moreover, a PNE exists.

Proof of Theorem 3. Let $i \in N$, $x_{-i} \in X_{-i}$, $x_i, x_i' \in X_i$, and denote $x := (x_i, x_{-i})$,$x' := (x_i', x_{-i})$. Our goal is to show that:

$$\Psi(x') - \Psi(x) = u_i(x') - u_i(x).$$

Recall that $u_i(x) = m\nu_i(x) + \frac{1}{n} - \lambda d_i^0(x_i)$.

Let

$$S_{-i} := \sum_{j \neq i} x_j.$$

For fixed x_{-i}, we can write the generated response as

$$y_{\alpha,z}(x) = \frac{\alpha}{n}x_i + \frac{\alpha}{n}S_{-i} + (1-\alpha)z.$$

For brevity, define

$$\eta := \frac{\alpha}{n}, \qquad h_{-i} := \eta S_{-i} + (1-\alpha)z.$$

Then $y_{\alpha,z}(x) = \eta x_i + h_{-i}$.

By definition of the linear relative relevance and the notations above, we obtain

$$\nu_i(x) = \frac{1}{p(n-1)}\sum_{j \neq i}\|x_j - \eta x_i - h_{-i}\|_2^2 - \frac{1}{p}\|(1-\eta)x_i - h_{-i}\|_2^2.$$

We now isolate the terms that depend on x_i. Expanding the first term gives

$$\begin{aligned}&\frac{1}{p(n-1)}\sum_{j \neq i}\|x_j - \eta x_i - h_{-i}\|_2^2 \\ &= C_1(x_{-i}) - \frac{2\eta}{p(n-1)}x_i^\top \sum_{j \neq i}(x_j - h_{-i}) + \frac{\eta^2}{p}\|x_i\|_2^2,\end{aligned}$$

where $C_1(x_{-i})$ is independent of x_i. Similarly for the second term:

$$\frac{1}{p}\|(1-\eta)x_i - h_{-i}\|_2^2 = \frac{(1-\eta)^2}{p}\|x_i\|_2^2 - \frac{2(1-\eta)}{p}x_i^\top h_{-i} + C_2(x_{-i}),$$

where $C_2(x_{-i})$ is independent of x_i. Hence,

$$\begin{aligned}\nu_i(x) = C_i(x_{-i}) &- \frac{1-2\eta}{p}\|x_i\|_2^2 \\ &- \frac{2\eta}{p(n-1)}x_i^\top \sum_{j \neq i}(x_j - h_{-i}) + \frac{2(1-\eta)}{p}x_i^\top h_{-i},\end{aligned}$$

for some term $C_i(x_{-i})$ that is independent of x_i.

Since

$$\sum_{j\neq i}(x_j - h_{-i}) = S_{-i} - (n-1)h_{-i},$$

we get:

$$\nu_i(x) = C_i(x_{-i}) - \frac{1-2\eta}{p}\|x_i\|_2^2 - \frac{2\eta}{p(n-1)}x_i^\top S_{-i} + \frac{2}{p}x_i^\top h_{-i}.$$

Substituting

$$\eta = \frac{\alpha}{n}, \qquad h_{-i} = \frac{\alpha}{n}S_{-i} + (1-\alpha)z,$$

yields

$$\nu_i(x) = C_i(x_{-i}) - \frac{1-\frac{2\alpha}{n}}{p}\|x_i\|_2^2 + \frac{2\alpha(n-2)}{kn(n-1)}x_i^\top S_{-i} + \frac{2(1-\alpha)}{p}x_i^\top z. \tag{10}$$

Now define

$$Q_i(x_i; x_{-i}) := (1-\alpha)d(x_i, z) + \frac{\alpha(n-2)}{n(n-1)}\sum_{j\neq i} d(x_i, x_j).$$

Expanding Q_i, we obtain

$$Q_i(x_i; x_{-i}) = \frac{1-\alpha}{p}\left(\|x_i\|_2^2 - 2x_i^\top z + \|z\|_2^2\right) + \frac{\alpha(n-2)}{kn(n-1)}\sum_{j\neq i}\left(\|x_i\|_2^2 - 2x_i^\top x_j + \|x_j\|_2^2\right).$$

Collecting only the terms that depend on x_i, we get

$$Q_i(x_i; x_{-i}) = C_i'(x_{-i}) + \frac{1-\frac{2\alpha}{n}}{p}\|x_i\|_2^2 - \frac{2\alpha(n-2)}{kn(n-1)}x_i^\top S_{-i} - \frac{2(1-\alpha)}{p}x_i^\top z, \tag{11}$$

where $C_i'(x_{-i})$ is independent of x_i. Comparing (10) with (11), we obtain

$$\nu_i(x) = C_i''(x_{-i}) - Q_i(x_i; x_{-i}),$$

for some term $C_i''(x_{-i})$ independent of x_i. Therefore,

$$\nu_i(x') - \nu_i(x) = -\left[Q_i(x_i'; x_{-i}) - Q_i(x_i; x_{-i})\right].$$

That is,

$$\nu_i(x') - \nu_i(x) = -(1-\alpha)\left[d(x_i', z) - d(x_i, z)\right] - \frac{\alpha(n-2)}{n(n-1)}\sum_{j\neq i}\left[d(x_i', x_j) - d(x_i, x_j)\right].$$

Now consider the proposed potential. Since only player i changes her action, all terms that do not involve x_i cancel in the following difference:

$$\begin{aligned}\Psi(x') - \Psi(x) = &-(1-\alpha)m\left[d(x_i', z) - d(x_i, z)\right] \\ &- \frac{\alpha m(n-2)}{n(n-1)}\sum_{j\neq i}\left[d(x_i', x_j) - d(x_i, x_j)\right] - \lambda_i\left[d_i^0(x_i') - d_i^0(x_i)\right].\end{aligned}$$

Using the identity for $\nu_i(x') - \nu_i(x)$ derived above, this becomes

$$\Psi(x') - \Psi(x) = m\left[\nu_i(x') - \nu_i(x)\right] - \lambda_i\left[d_i^0(x_i') - d_i^0(x_i)\right].$$

Finally, since

$$u_i(x) = m\nu_i(x) + \frac{1}{n} - \lambda_i d_i^0(x_i),$$

we conclude that

$$\Psi(x') - \Psi(x) = u_i(x') - u_i(x).$$

Thus, Ψ is an exact potential function.

The existence of an ε-PNE and the convergence of every ε-better-response dynamics follow from Theorem 1. □

C The Discrete Better-Response Simulation

We use the discrete better-response dynamics simulation described in detail by Madmon et al. (2025a).

Simulation Algorithm. The simulation starts from an initial strategy profile (i.e., initial documents) and proceeds in rounds. In each round, we identify the publishers who can improve their utility by more than a threshold ε through a single document modification, where the modification is restricted to a predefined finite set of directions D and step sizes S. If at least one such publisher exists, one improving publisher is selected uniformly at random. Among all feasible modifications for that publisher, the simulation selects a utility-maximizing modification; if several modifications are tied for maximal utility, one of them is chosen uniformly at random. The selected publisher's document is then updated accordingly, while all other publishers' documents remain unchanged.

The simulation stops early if no publisher can improve her utility by more than ε using any feasible modification (i.e., the simulation has converged). In this case, the relevant metrics at the final strategy profile are reported. If the simulation reaches the maximal number of rounds T, we return the averages of the utilities over the final M rounds.

Simulation Details. As noted above, a simulation is said to converge if it has reached an ε-PNE in no more than T iterations. In accordance with prior work (Madmon et al., 2025a), we set $T = 1000$, $\varepsilon = 10^{-6}$, and perform 500 simulations to construct a bootstrap confidence interval with a confidence level of 95%. To ensure reproducibility, we use a seed of 36.

We consider simulations with $n = 5$ players (which is the common practice for the number of retrieved documents in RAG (Xu et al., 2024; Yu et al., 2024)), a cost factor $\lambda = 0.5$ (to balance the trade offs induced by it, as we show in Appendix E, where we study the impact of λ), a dimension $p = 2$, the generation function $y(x) = y_{\alpha,z}(x)$, and draw the question x^* and initial documents x_i^0 uniformly from $\mathcal{V}$. As in prior work (Madmon et al., 2025a), we set $\beta = 1$ for the softmax function and $m = \frac{1}{n}$ for the linear function. As stated, in Appendix D we study the effect of n, p, β, m, and the distribution of the question and the initial documents.

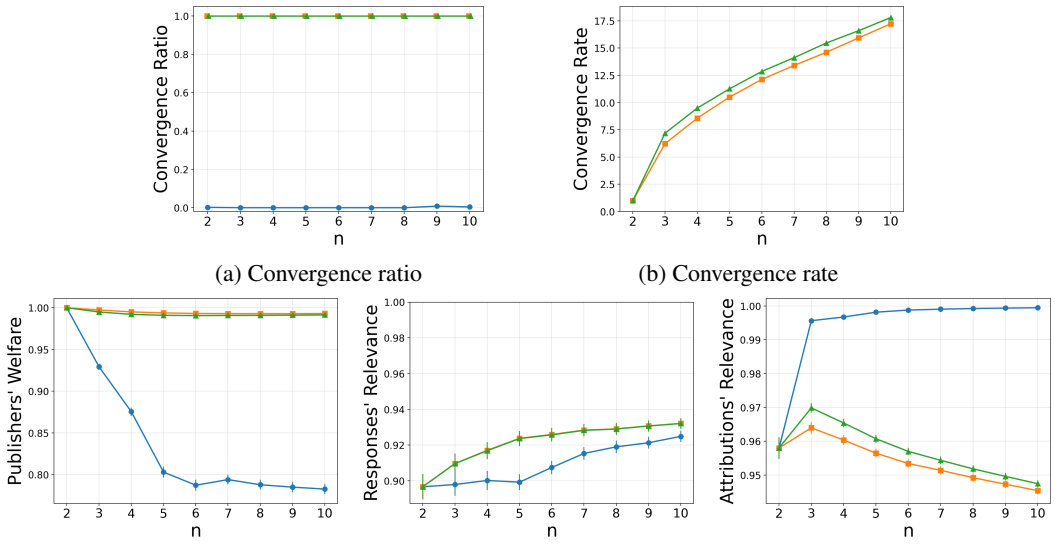
For the analysis of the retrieval weight α, we draw z — the posterior on the user's information need — from a normal distribution with expectation $\mu = x^*$ and set $k = n$. For the analysis of the context size k, we set $\alpha = 1$ to eliminate the effect of the platform's knowledge z and focus on the dynamics among publishers.

D Sensitivity Analysis of Our Experimental Results

This section provides supplementary empirical analyses that extend the results reported in Section 6. We assess the sensitivity of the result patterns reported in Section 6 to the hyperparameter values chosen and explore novel trends. We study the effect of the number of publishers n, the documents' representation dimension p, the hyperparameters β and m of the softmax and linear distributions, and the distribution of the user's question and initial documents on the stability and welfare in the ecosystem. We use the algorithm, evaluation measures, and hyperparameters as described in Section 6 for α analysis.

The Number of Players. Figure 4 reports the stability and welfare measures for various numbers of players n (from 2 to 10). We highlight that in all measures and for all $n \neq 2$, the relative performance (as observed in Section 6) between our three attribution functions is preserved. For $n = 2$, all functions perform similarly in terms of convergence rate and welfare measures, since both publishers always have identical distances from the generated response, resulting in a degenerate game in that sense.

In terms of the convergence ratio, the results presented in Section 6 hold for all values of n. We can see in Figure 4a that the softmax and linear functions induce stability (i.e., convergence ratio of 1). The argmax function induces an unstable dynamics, with a convergence ratio of approximately 0. As expected, in Figure 4b we observe that more players induce slower convergence. In fact, the convergence rate increases linearly with the number of players.



(a) Convergence ratio (b) Convergence rate

(c) Publishers' welfare (d) Responses' relevance (e) Attributions' relevance

Figure 4: Stability and welfare analysis of the argmax, softmax, and linear attribution functions for various values of players n, shown in blue circles, orange squares, and green triangles, respectively. The top charts report stability measures: convergence ratio and convergence rate. The bottom charts report welfare measures: publishers' welfare, responses' relevance, and attributions' relevance.

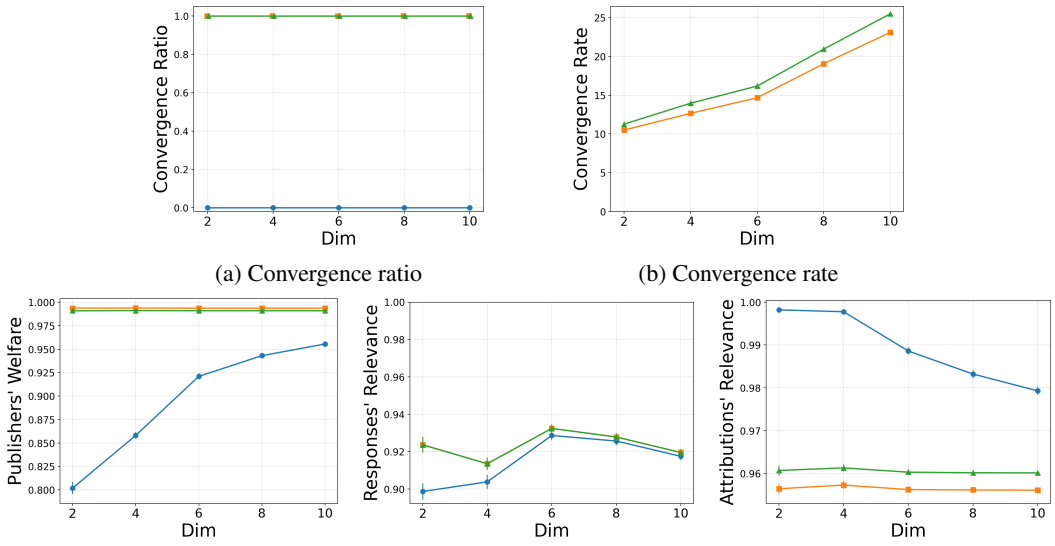
In terms of publishers' welfare (Figure 4c), the numbers under the softmax and linear functions remain relatively stable, whereas under the argmax function, they decline and stabilize for $n > 5$. This indicates that while the softmax and linear functions maintain the welfare of publishers as more publishers participate in the competition, the argmax function maintains a substantially lower welfare for the higher values of n.

As the number of content creators increases, we observe increased performance in terms of the responses' relevance (Figure 4d). This suggests that GenAI ecosystems with a large number of content creators produce more relevant responses to users than those with a small number. Moreover, for $n > 3$, the attributions' relevance (Figure 4e) under the softmax and linear functions decreases and, under the argmax function, increases. In fact, there is a substantially increased gap in relevance between the argmax function and the softmax and the linear functions. Notably, the attributions' relevance is almost optimal under the argmax function. These observations are important for mechanism designers aiming to ground responses in external sources or to refer users to external sources for additional information (e.g., using citations).

Documents' Representation Dimension. Figure 5 reports the stability and welfare measures for various values of the dimension of documents' representation in the embedding space p. We acknowledge that, due to resource limitations, our experiments are conducted using much smaller dimensions than those used by state-of-the-art LLMs (e.g., 768 by BERT (Warner et al., 2025)). We hasten to point out that our goal is to identify trends, not concrete conclusions regarding a specific dimension.

Figure 5 reports stability and welfare measures for various values of embedding dimensions p. A key observation is that for all values of p, the relative performance between our three attribution functions is preserved. In terms of convergence ratio, as one may expect, we can see in Figure 5a that it is not affected by the dimension value; specifically, the dynamics induced by the softmax and the linear functions converge for every value of p. Since, as the value of p increases, convergence requires additional coordinate-wise adjustments, we observe in Figure 5b that the convergence rate increases with the value of p.

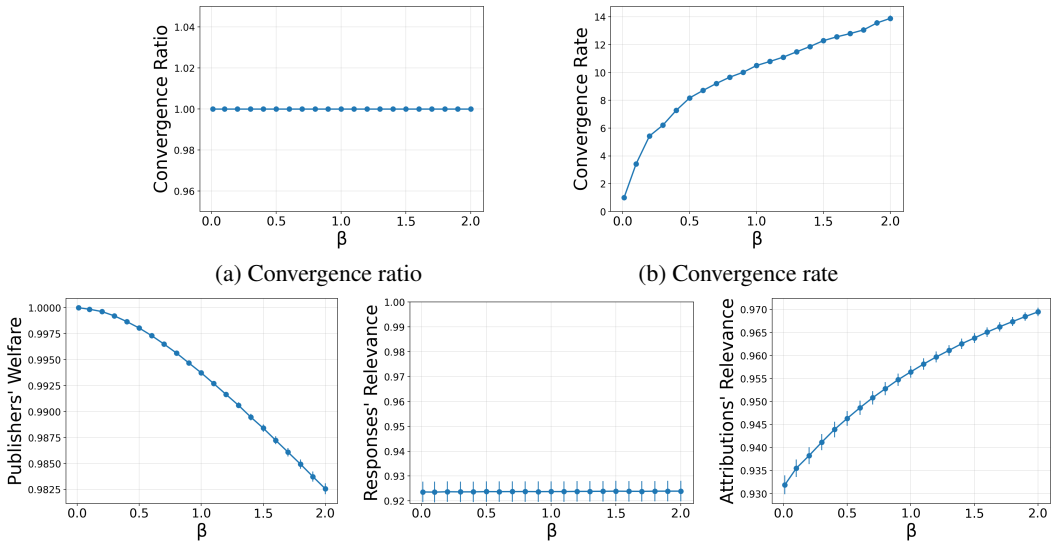
In terms of welfare, Figures 5c, 5d, and 5e show that all measures admit diminishing gaps between the values induced by the argmax function and those induced by the softmax and linear functions.



(a) Convergence ratio (b) Convergence rate

(c) Publishers' welfare (d) Responses' relevance (e) Attributions' relevance

Figure 5: Stability and welfare analysis of the argmax, softmax, and linear attribution functions for various values of the documents' representation dimension p, shown in blue circles, orange squares, and green triangles, respectively. The top charts report stability measures: convergence ratio and convergence rate. The bottom charts report welfare measures: publishers' welfare, responses' relevance, and attributions' relevance.

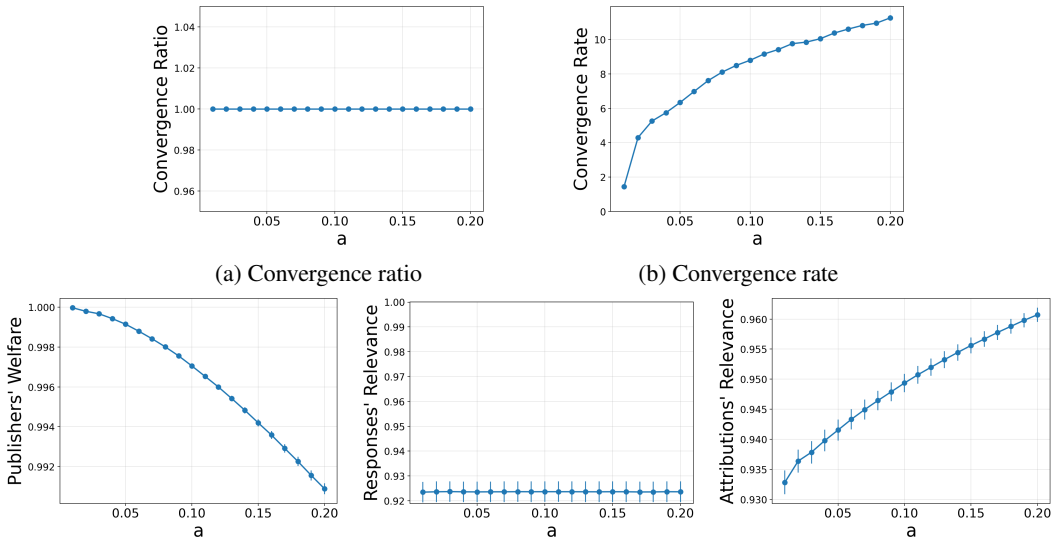


(a) Convergence ratio (b) Convergence rate

(c) Publishers' welfare (d) Responses' relevance (e) Attributions' relevance

Figure 6: Stability and welfare analysis of the softmax attribution functions for various values of the softmax function distribution's inverse-temperature β. The top charts report stability measures: convergence ratio and convergence rate. The bottom charts report welfare measures: publishers' welfare, responses' relevance, and attributions' relevance.

Note that since the diminishing gaps may be a result of the normalized Euclidean distance, our focus is on relative performance between the functions. The gap between the values induced by the softmax function and the linear function remains approximately the same. This observation suggests that, for high values of the dimension p, welfare may be approximately the same across different mechanisms; however, this possibility requires further investigation.



(a) Convergence ratio (b) Convergence rate

(c) Publishers' welfare (d) Responses' relevance (e) Attributions' relevance

Figure 7: Stability and welfare analysis of the linear attribution functions for various values of the linear relative relevance distribution's slope m. The top charts report stability measures: convergence ratio and convergence rate. The bottom charts report welfare measures: publishers' welfare, responses' relevance, and attributions' relevance.

Distributions' Hyperparameters. We now turn to examine the effect of distributional hyperparameters induced by the softmax and linear functions. Specifically, the inverse-temperature β of the softmax distribution and the slope m of the linear distribution.

Figure 6 presents the stability and welfare measures under the softmax attribution function for various values of β, ranging from 0.01[12] to 2 in increments of 0.1. We observe in Figure 6a that the convergence ratio values remain constant for every β, i.e., the ecosystem is stable under the softmax function for any examined value of temperature. As expected, Figure 6b shows that the convergence rate increases with the inverse-temperature β. This can be addressed by the fact that larger values of β lead to a more winner-focused attribution score, which strengthens the incentives of publishers to deviate at each round.

Furthermore, we identify that while the values of the responses' relevance in Figure 6d are approximately constant with β, in Figure 6c and Figure 6e, a trade-off exists between publishers' welfare and attributions' relevance. For smaller β values, publishers whose documents are closer to the responses receive a higher attribution score (and, in turn, utility) compared to larger β values. Consequently, publishers must apply more substantial modifications to their content to increase their utility, which in turn leads to fewer modifications (i.e., higher publishers' welfare) overall and less relevant attributions (i.e., lower attributions' relevance). We emphasize that the inverse-temperature value used in Section 6, $\beta = 1$, induces the standard softmax function and provides a natural balance in this trade-off.

Figure 7 presents the stability and welfare measures under the linear attribution function for various values of m, ranging from 0.01 to 0.2 (maximum value to induce an appropriate distribution in games with $n = 5$) in increments of 0.01. The observed trends are the same as those presented for the inverse-temperature β of the softmax function above. However, in the case of the slope m, the reason is different: smaller values induce a more uniform distribution over publishers, thus publishers are less motivated to modify their documents.

Question and Initial Documents Distributions. We proceed to study the impact of the distribution of the user's question and the initial documents on the stability and welfare in the generative ecosystem.

[12]The smallest value of β is larger than 0 since $\beta = 0$ entails a constant distribution over publishers; i.e., publishers' action does not affect their attribution score.

Recall that in Section 6 we draw both uniformly at random from $\mathcal{V}$. Here, we consider questions and initial documents that are drawn from combinations of a uniform distribution and a multivariate normal distribution. For the multivariate normal distribution, we consider an isotropic Gaussian with a mean μ of 0.5 and a covariance matrix Σ of $0.1 \cdot \mathcal{I}_p$.

In Section 6, Figures 1 and 2 present the stability and welfare results using a uniform distribution over $\mathcal{V}$. Figure 8 presents the results with the other combinations of uniform and normal distributions as stated above. The key observation is that the pattern of results as k changes remains the same for all configurations. For example, consider the convergence ratio, for which the argmax function is constantly around 0, while the softmax and linear functions increase with k, meeting in a ratio of 1 when $k = n$. Additionally, although we see different value ranges for the convergence rate, the patterns as k varies, and the relative numbers between the softmax and linear functions remain the same.

We can see in Figure 2 and Figure 8 that while the relative values of the publishers' welfare and the attributions' relevance induced by the attribution functions are maintained, some variation arises in terms of the responses' relevance. Specifically, in Figure 8b the argmax function statistically significantly outperforms both the softmax and linear functions for $k < n$, whereas in the other configurations this holds only for $k \leq 2$. However, we hasten to point out that the pattern of values of the attribution functions and the relative performance between them for most values of k remain consistent.

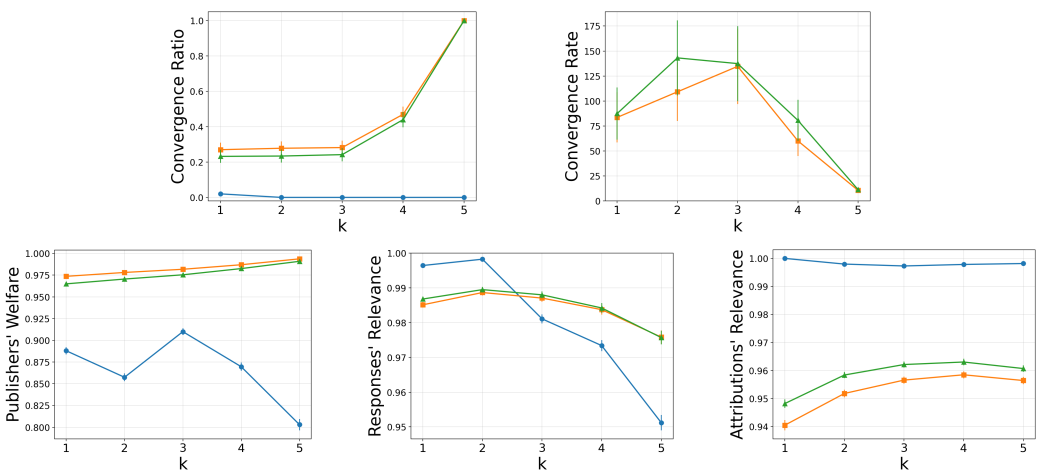
E Cost Factor Analysis

In this section, we empirically study the effect of the cost factor λ on the stability of the ecosystem and the social welfare, as described in Section 6. Specifically, we use the same discrete better response dynamics simulation and hyperparameters we used for α analysis in Section 6, while varying the value of the cost factor λ. Figure 9 reports the resulting outcomes as a function of λ, starting from 0.01 and increasing in increments of 0.1. We observe that, under our three attribution functions, both the convergence ratio values (Figure 9a) and responses' relevance values (Figure 9d) remain stable across all λ values. In contrast, the convergence rate (Figure 9b), the publishers' welfare (Figure 9c), and the attributions' relevance (Figure 9e) exhibit more notable trends.

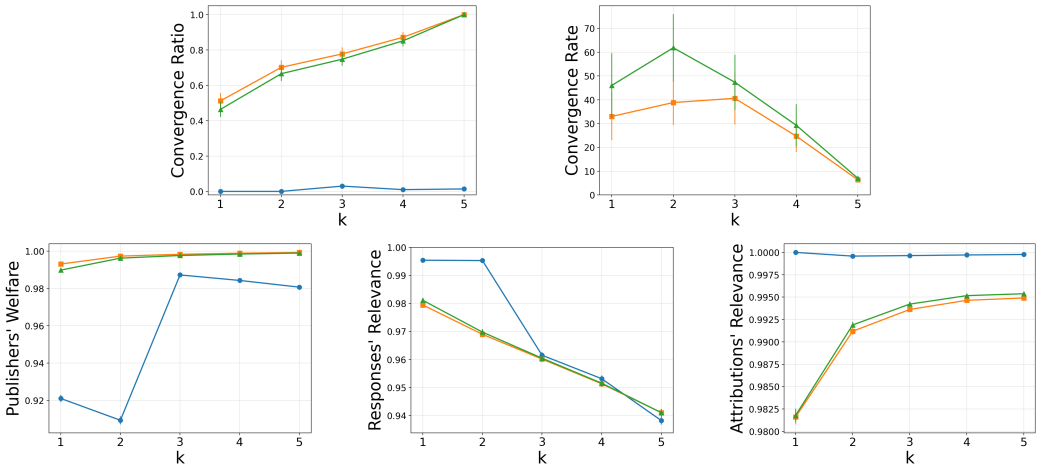
We emphasize that the observed stability in response relevance is not a trivial outcome. One might have expected relevance to vary as publishers face different incentives to modify their content. Instead, our results suggest that generative platforms can preserve response relevance across heterogeneous publisher types, including publishers that differ in the cost of deviating from their original content. This capability has previously been shown to be non-trivial (Ohayon et al., 2026), where creators' effort can be interpreted as a manageable cost factor.

The convergence rate decreases as λ increases, likely because for small values of λ, publishers are more incentivized to modify their initial content, resulting in a longer interaction. The publishers' welfare decreases consistently under the argmax attribution function, while it remains relatively stable under the softmax and linear functions. In other words, higher content-modification costs (i.e., larger values of λ) lead to increased modifications under the argmax function. This may be attributed to the deviation that publishers have to perform from their original content to gain exposure under the argmax function. As expected, when publishers are less incentivized to modify their initial content (large λ), the attributions' relevance decreases consistently under the softmax and linear attribution functions. Surprisingly, it remains stable under the argmax function. This stability is likely due to the discontinuity of argmax, which allows a publisher to obtain the entire exposure allocation by exceeding its competitors by only a small margin.

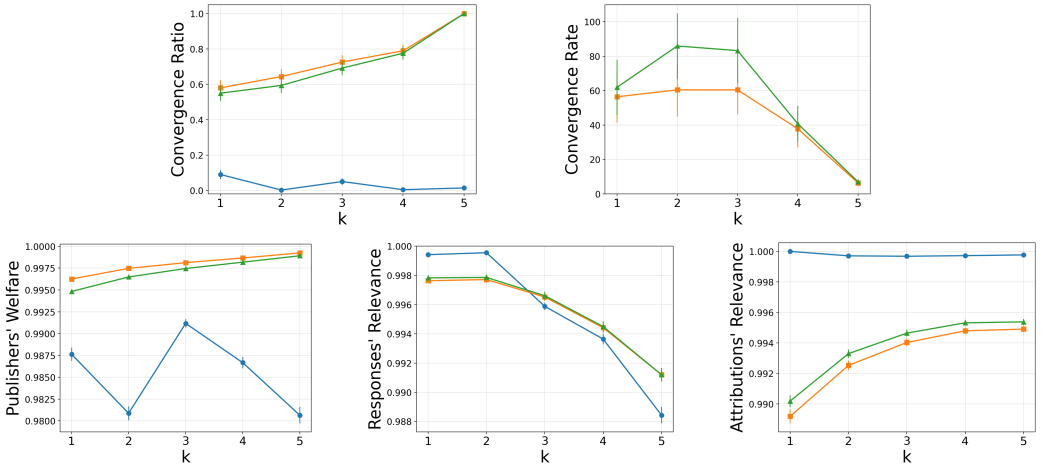
Additionally, Figure 9 shows that, for the vast majority of measures and configurations, the relative performance between the three attribution functions remains consistent across all values of λ. For example, for any value of λ, the publishers' welfare is highest under the softmax function and lowest under the argmax function.



(a) $x^* \sim \mathcal{N}(\mathbf{0.5}_p, 0.1 \cdot \mathcal{I}), \quad x_i^0 \sim \mathcal{U}(\mathcal{V})$

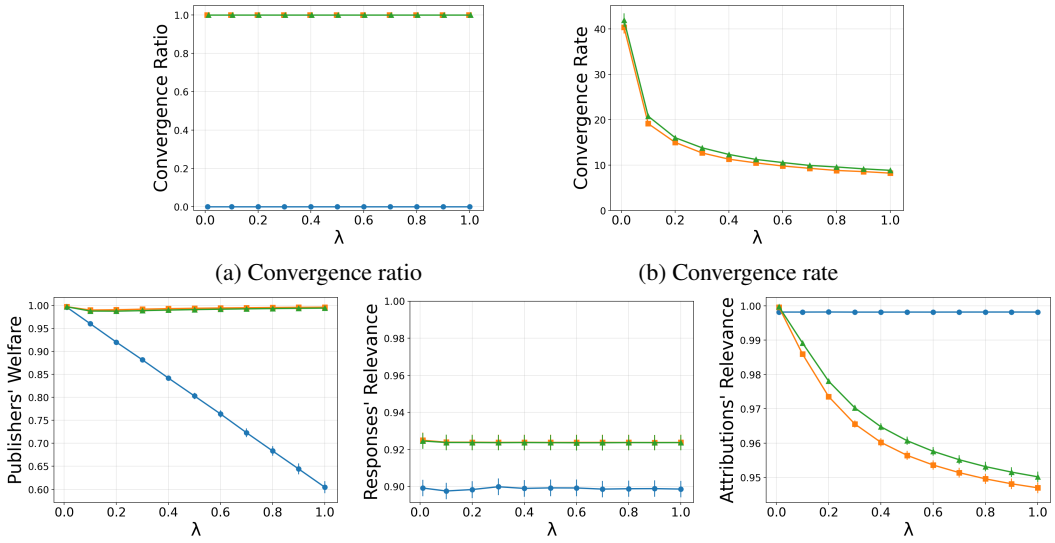


(b) $x^* \sim \mathcal{U}(\mathcal{V}), \quad x_i^0 \sim \mathcal{N}(\mathbf{0.5}_p, 0.1 \cdot \mathcal{I})$



(c) $x^* \sim \mathcal{N}(\mathbf{0.5}_p, 0.1 \cdot \mathcal{I}), \quad x_i^0 \sim \mathcal{N}(\mathbf{0.5}_p, 0.1 \cdot \mathcal{I})$

Figure 8: Stability and welfare analysis of the argmax, softmax, and linear attribution functions for uniform and normal distribution combinations of the user's question x^* and initial documents x^0, shown in blue circles, orange squares, and green triangles, respectively. For each distribution configuration, the top charts report stability measures: convergence ratio and convergence rate. The bottom charts report welfare measures: publishers' welfare, responses' relevance, and attributions' relevance.



(a) Convergence ratio (b) Convergence rate

(c) Publishers' welfare (d) Responses' relevance (e) Attributions' relevance

Figure 9: Stability and welfare analysis of the argmax, softmax, and linear attribution functions for various values of cost factors λ, shown in blue circles, orange squares, and green triangles, respectively. The top charts report stability measures: convergence ratio and convergence rate. The bottom charts report welfare measures: publishers' welfare, responses' relevance, and attributions' relevance.