LEAN FINDER V2: SEMANTIC SEARCH FOR MATHLIB THAT DISAMBIGUATES USER QUERIES

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Paper under double-blind review

ABSTRACT

We introduce **Lean Finder V2**, a semantic search framework for Mathlib that disambiguates user queries through *complementary model-level adversarial training and input-level query rewriting*. Human mathematicians and coding agents rarely search Mathlib by providing detailed context. In practice, they query short properties, notation, aliases, or hypotheses, leaving much of the intended mathematics implicit. In contrast, existing Mathlib search engines unrealistically assume context-rich queries. The scarcity of real-world short queries paired with ground-truth Mathlib statements makes this ambiguity even more intractable. We argue that this is essentially a **data** problem. Through adversarial training, we scale up the generation of labeled short and ambiguous queries, and learn to disambiguate user queries at both model-level fine-tuning and input-level query rewriting. On the human-curated MathlibQR benchmark, our Lean Finder V2 sets a new state of the art, with at least 11% relative improvement over the previous best Mathlib semantic search engines. On top of our state-of-the-art retriever, optimized query rewriting further improves MathlibQR retrieval. On unlabeled queries from human and coding-agent interactions, Lean Finder V2 is ranked as the strongest system. Integrated with Claude Code, Lean Finder V2 improves proof success on downstream theorem-proving task. We will release our code, model, and data upon acceptance.

1 INTRODUCTION

Advances in the Lean proof assistant and its central library Mathlib are turning formal mathematics into a collaborative and verifiable workflow (De Moura et al., 2015; Moura & Ullrich, 2021). Spanning a broad range of mathematics, Mathlib now contains more than 400,000 theorems and definitions across roughly 10,000 files (Lean Prover Community, 2026). At this scale, effective theorem search is essential for both human and automated formalization, enabling users and systems to efficiently identify and reuse relevant results from Mathlib's large and growing library, and its quality shapes end-to-end proof success (Lu et al., 2026; Gao et al., 2026; Avigad & Massot, 2020). Alongside rapid progress in specialized Lean provers (Ren et al., 2025; Lin et al., 2025), agentic systems have achieved tremendous success from competition mathematics to frontier research. Many of these systems rely on Mathlib search engines to locate reusable results during proof construction (Liu et al., 2026; Varambally et al., 2026; Ju et al., 2026; Dressler, 2025).

Despite this progress, current theorem retrieval faces a central bottleneck: **theorem-search queries from *both* mathematicians and coding agents are inherently ambiguous**. Human mathematicians and coding agents rarely search by restating a complete theorem. They use short properties, notation, aliases, or hypotheses, and different users use different terms referring to the same mathematical result, leaving much of the intended mathematics implicit. Such everyday queries are challenging precisely because they expose wide *ambiguity gaps*, yet existing systems leave these gaps unaddressed. Lu et al. (2026) synthesizes realistic queries from intent clusters mined from Lean discussions, but the resulting data are static and generated independently of its retriever's current failure modes. Other Lean search systems likewise rely largely on fixed, relatively detailed data, such as informalizations, formal statements, or proof states (Gao et al., 2026; 2024; Asher, 2025; Yang et al., 2023). We call such inputs *context-rich* : they preserve much of the source declaration's detail, unlike concise real-world queries that express the same intent through a few salient cues. Figure 1(a) quantifies this

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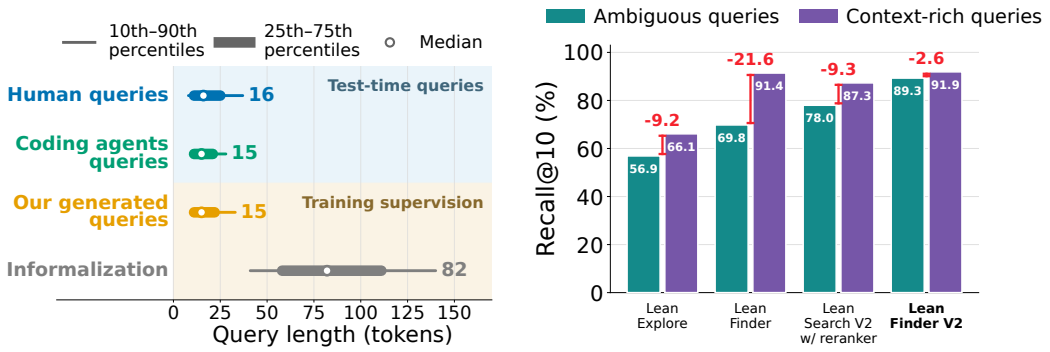

(a) Query-length distributions.

(b) Recall@10 across query types.

Figure 1: Real-world queries expose the ambiguity gap. (a) Human and coding-agent queries contain far less explicit context than statement informalizations which are commonly used to train Lean search engines, as shown by the length gap. Our generated queries used to train retrievers better match the length of real-world queries. (b) Every existing Lean search engine performs worse on ambiguous queries compared to context-rich ones, with Lean Finder V2 closing the gap to the smallest 2.6% and having the best performance on both query types. This further demonstrates previous Lean search engines leave the ambiguity gap unaddressed.

mismatch: human and coding-agent queries have median lengths of 16 and 15 tokens (i.e., ambiguous), respectively, compared with 82 tokens (i.e., context-rich) for the statement informalizations commonly used for training previous Mathlib search engines. This gap is visible even when a shorter query remains semantically aligned with the same target theorem:

One target, two levels of query ambiguity

Target statement:

```
theorem Function.Even.const
    {α β : Type*} [Neg α] (b : β) :
    Function.Even (fun _ : α ↦ b)
```

Context-rich query: *"for a type α with a negation and any type β, every constant function $f\colon \alpha \to \beta$, $f(x) = b$ for some $b \in \beta$, satisfies $f(-x) = f(x)$ for all $x \in \alpha$, i.e., constant functions are even"*

Ambiguous query: *"constant function invariant under input negation"*

Both queries ask for the same mathematical fact, and the ambiguous query remains faithful to it. Yet leaving the Mathlib concept of an even function implicit moves the target from rank one to rank seven. Therefore, we ask our core question:

How to disambiguate Mathlib queries, at both model level and input level?

In this paper, we introduce **Lean Finder V2**, a semantic search framework for Mathlib that excels on ambiguous queries while retaining strong performance on context-rich queries. Lean Finder V2 addresses the **data bottleneck** caused by the **scarcity of labeled ambiguous queries**. We synthesize ambiguous query-declaration pairs as training data to improve the retriever through **model-level adversarial training**, and the same synthetic data is used in **input-level prompt optimization** for query rewriting. At the model level, an RL-trained attacker produces challenging and ambiguous queries for Mathlib declarations, with a *difficulty reward* that targets weaknesses of the current retriever and an *alignment reward* that keeps each query faithful to its source declaration. These synthetic query-declaration pairs improve the retriever, whose feedback recalibrates query difficulty. At the input level, we use GEPA (Agrawal et al., 2026) to optimize a query-rewriting prompt on the same synthetic data with a ranking-based objective, making the intent of ambiguous queries explicit before retrieval. **Lean Finder V2 sets a new state of the art on MathlibQR** (Gao et al., 2026), with **14% relative improvement** in Recall@10 over the previous best semantic search engine (Figure 1(b)). On top of the state-of-the-art retriever, the optimized rewriting prompt further improves retrieval on MathlibQR. When integrated with Claude Code, Lean Finder V2 also improves downstream proof success.

We summarize our contributions as follows:

1. Lean Finder V2 sets a new state of the art on MathlibQR (Gao et al., 2026) while retaining strong performance on context-rich queries (Lu et al., 2026), achieving 14% relative gains over the previous-best semantic search system in Recall@10 on MathlibQR. On unlabeled queries for Mathlib drawn from real user interactions, Lean Finder V2 is ranked as most helpful 18+% more often than the previous best system by frontier LLM judges.
2. Through retriever-guided adversarial training, we construct a training corpus of over 600K labeled ambiguous query-declaration pairs. The same corpus powers both model-level fine-tuning and input-level query-rewriting prompt optimization to disambiguate user queries. We will release this corpus to support future research.
3. We present a study of input-side query rewriting. Using the same ambiguous query-declaration pairs for prompt optimization, GEPA-optimized query rewriting further improves MathlibQR retrieval for Lean Finder V2.
4. Integrated with Claude Code, Lean Finder V2 improves downstream theorem proving, achieving a proof success rate of 39.3% compared to the next-best 36.0% on FATE-H (Jiang et al., 2026).

2 Related Work

Prior Lean retrieval systems improve access to Mathlib (Gao et al., 2024; 2026; Asher, 2025; Lu et al., 2026; Kurgan et al., 2026), and are often incorporated into agentic systems (Varambally et al., 2026; Ju et al., 2026; Liu et al., 2026). Outside Lean, reinforcement learning (RL) has primarily optimized queries against fixed retrievers to improve retrieval accuracy (Hsu et al., 2024; Jiang et al., 2025), rather than using generated queries as supervision to adapt the retriever itself. Lean Finder V2 instead adapts the retriever to real-world ambiguous queries through adversarial training: a generator produces declaration-faithful queries that challenge the current retriever, these query-declaration pairs retrain the retriever, and the updated retriever provides difficulty feedback for the next-round generator training. Complementarily, input-level query rewriting makes implicit mathematical context explicit before retrieval. Extended related work is provided in Appendix A.

3 Methods

Lean Finder V2 disambiguates user queries by addressing a *data* bottleneck: labeled ambiguous queries are scarce, and existing training data are context-rich and static (Section 1). We therefore generate ambiguous queries directly from Mathlib declarations through an adversarial training process that targets the current retriever's weaknesses, while ensuring each query remains faithful to its source. Importantly, the generated ambiguous queries *enables disambiguation at two complementary levels*. At the *model* level, we train the retriever on the synthetic query-declaration pairs to recover the intended declaration directly from salient cues. At the *input* level, we use the same synthetic pairs to optimize a query-rewriting prompt that makes implicit intent explicit before retrieval. Section 3.1 describes the adversarial training that generates the data and fine-tunes the retriever, and Section 3.2 reuses the data for input-level query rewriting.

3.1 Model-level Disambiguation Through Adversarial Training

The goal of model-level adversarial training is to synthesize high-quality ambiguous query-declaration pairs for retriever training. It couples a retrieval model with a query generator. The generator produces challenging and ambiguous queries that remain aligned with their source Mathlib declarations. We train the retriever on these query-declaration pairs, and its feedback guides the next round of query generator training.

First, we initialize the adversarial training process by training a retrieval model on large-scale, context-rich data, establishing broad Mathlib coverage before adapting it to ambiguous queries (Section 3.1.1). *Second*, we optimize the generator with a retriever-guided reward that combines retrieval difficulty with query-declaration alignment, encouraging faithful queries that challenge the current retriever (Section 3.1.2). *Finally*, we train the retriever on the generated query-declaration pairs and repeat the

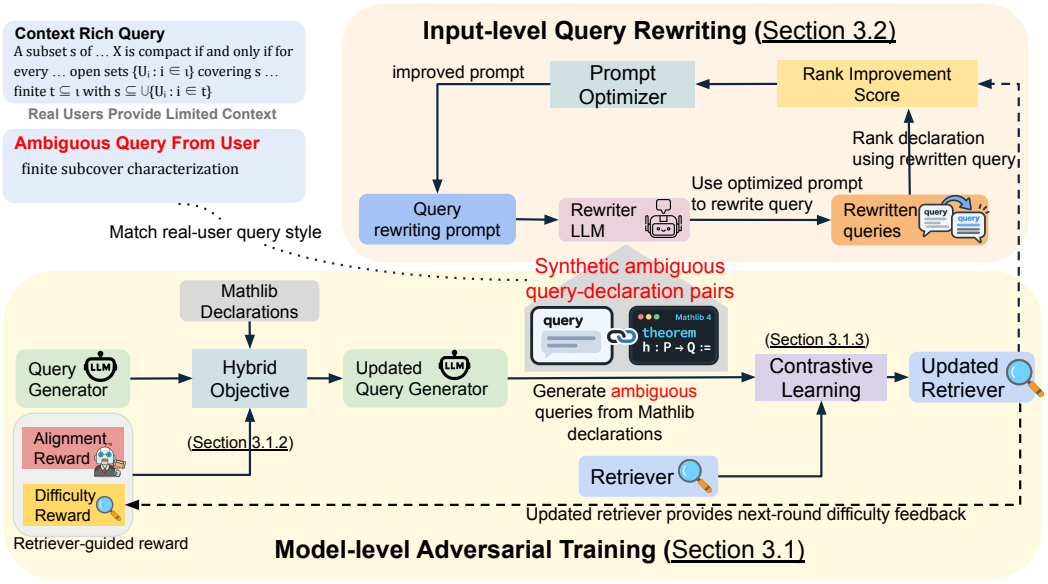

Figure 2: Lean Finder V2 disambiguates Mathlib queries at two complementary levels. Model-level adversarial training (Section 3.1) uses a hybrid objective that combines reinforcement learning and supervised fine-tuning with retriever-guided difficulty rewards to incentivize the query generator LLM to generate faithful, challenging, and ambiguous queries for Mathlib declarations. Contrastive learning on these query-declaration pairs improves the retriever, which supplies the query-difficulty feedback for the next round training of the generator. Input-level query rewriting (Section 3.2) uses the same generator-produced query-declaration pairs as the training data for optimizing the query-rewriting prompt, guided by the rank-improvement feedback based on the retriever. The optimized query rewriting prompt makes implicit mathematical context explicit before retrieval.

process with the updated retriever (Section 3.1.3). Figure 2 provides an overview of the framework, and Algorithm 1 summarizes the training procedure.

3.1.1 Initializing and Fine-tuning the Retriever

We initialize the retrieval model from Qwen3-Embedding-8B (Zhang et al., 2025) and further fine-tune it on two sources of large-scale context-rich data. First, we use synthetic user-query and Mathlib declaration pairs from Lu et al. (2026), which capture multiple user intents summarized from Zulip discussions. Second, we curate training pairs from the informalization datasets from Lu et al. (2026) and LeanSearch V2 (Gao et al., 2026). Together, they form a training set of more than one million query-declaration pairs.

We fine-tune the retriever using masked InfoNCE with in-batch negatives. To reduce false-negative supervision, we mask any candidate whose cosine similarity exceeds that of the labeled positive by more than a margin m. This prevents highly plausible matches from being incorrectly treated as negatives. The full objective is given in Appendix F.2. This stage produces the initial retriever, which is used to guide the query-generator RL training (Section 3.1.2).

3.1.2 Scaling Up the Generation of Ambiguous Queries via Adversarial Attacks

The initial retriever from Section 3.1.1 provides the difficulty signal that guides the training of the query generator, whose role is to create ambiguous queries tailored to the retriever's weaknesses. These queries serve as "adversarial" samples for adapting the retriever to ambiguous inputs. We use a hybrid objective that combines on-policy reward optimization against the current retriever with off-policy supervision from expert deployment queries, anchoring generation in real-world query styles.

Given a Mathlib declaration d, the generator $\pi_\theta(\cdot \mid d)$, parameterized by θ, samples a group of candidate search queries $\{q_i\}_{i=1}^{G}$. For the on-policy component, we use Group Relative Policy

Optimization (GRPO) (Shao et al., 2024). Each candidate q_i receives a retriever-guided reward r_i that combines its alignment with d, difficulty for the current retriever, and length. We define this reward in Equation 4. The normalized advantage is:

$$\widehat{A}_i = \frac{r_i - \text{mean}(r_1, \ldots, r_G)}{\text{std}(r_1, \ldots, r_G) + \epsilon_A}. \tag{1}$$

We use the following GRPO loss:

$$\mathcal{L}_{\text{GRPO}}(\theta) = -\,\mathbb{E}\left[\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|q_i|}\sum_{s=1}^{|q_i|}\left[\min\Big(\rho_{i,s}(\theta)\widehat{A}_i, \text{clip}(\rho_{i,s}(\theta), 1-\epsilon, 1+\epsilon)\,\widehat{A}_i\Big) - \beta D_{\text{KL}}^{i,s}\right]\right], \tag{2}$$

where s indexes token positions in query q_i, $D_{\text{KL}}^{i,s}$ is the per-token KL penalty to the reference policy, and $\rho_{i,s}(\theta)$ is the token-level policy ratio,

$$\rho_{i,s}(\theta) = \frac{\pi_\theta(q_{i,s} \mid d, q_{i,<s})}{\pi_{\theta_{\text{old}}}(q_{i,s} \mid d, q_{i,<s})}. \tag{3}$$

Retriever-guided reward. For a generated query q targeting Mathlib declaration d, let k be the rank of d under the current retriever and let L be the query length measured by the retriever tokenizer. The reward is

$$r(q,d) = \begin{cases} 0, & \text{if the output format is invalid,} \\ \max\{0,\, j(q,d) f_{\text{rank}}(k) - \lambda_{\text{len}} p_{\text{len}}(L)\}, & \text{otherwise,} \end{cases} \tag{4}$$

where $j(q,d) \in \{0,1\}$ is an alignment gate, $f_{\text{rank}}(k)$ measures query difficulty relative to the current retriever, and $p_{\text{len}}(L)$ is the query overlong penalty, with λ_{len} controlling the penalty strength. The LLM judge based alignment gate prevents the generator from creating difficulty by producing a query unrelated to the target declaration. Because the gate multiplies the difficulty reward, alignment is a prerequisite rather than a property that can be traded against difficulty: an unrelated query receives zero reward regardless of its retrieval rank. We define the rank-based difficulty reward below. The difficulty term $f_{\text{rank}}(k)$ selects queries in a useful rank region:

$$f_{\text{rank}}(k) = \begin{cases} 0, & k \le k_{\min} \text{ or } k > k_{\text{cut}}, \\ \dfrac{\log(k/k_{\min})}{\log(k_{\text{sat}}/k_{\min})}, & k_{\min} < k \le k_{\text{sat}}, \\ 1, & k_{\text{sat}} < k \le k_{\text{cut}}. \end{cases} \tag{5}$$

Queries whose target appears within the top $k_{\min}$ receive no difficulty reward because the retriever already ranks the target highly. The reward then grows logarithmically and saturates across a useful challenging region. A target outside the retrieval cutoff receives zero reward, preventing arbitrary retrieval failure from being treated as useful difficulty. We set $k_{\min} = 5$, $k_{\text{sat}} = 30$, and $k_{\text{cut}} = 100$. Appendix H provides justifications for these chosen thresholds. To prevent the generator from increasing retrieval difficulty simply by appending unnecessary text, we apply a capped soft length penalty $p_{\text{len}}(L)$ on the generated query if it becomes too long. Appendix F.1 provides the exact definition for $p_{\text{len}}(L)$.

Hybrid objective with on-policy and supervised optimization. To anchor on-policy learning in real-world query styles, we curate the supervised dataset from a corpus of interactions by human users and coding agents with Lean search engines during formalization projects that rely on Mathlib. The diverse queries submitted during the interactions provide initial supervision for generating queries that reflect how both human mathematicians and coding agents search in practice. We use the standard SFT loss $\mathcal{L}_{\text{SFT}}(\theta)$ on this dataset. Following the global loss-mixing strategy of CHORD (Zhang et al., 2026), we minimize $\mathcal{L}_{\text{hybrid}}(\theta) = (1-\mu)\mathcal{L}_{\text{GRPO}}(\theta) + \mu\mathcal{L}_{\text{SFT}}(\theta)$, where μ decreases over training, shifting the generator from supervised anchoring toward on-policy learning.

3.1.3 Improving the Retriever With Generated Queries

After training the generator against the current retriever, we generate queries from Mathlib declarations. We use the generated query-declaration pairs alongside the context-rich data from Section 3.1.1 to retrain the retriever with the same masked InfoNCE objective. The updated retriever then provides the rank signal for the next round of generator training. Algorithm 1 provides the pseudocode for the training process.

3.2 Input-level Disambiguation Through Optimized Query Rewriting

Complementary to the model-level training in Section 3.1, we can also disambiguate queries at the *input* level (before retrieval) by using an LLM to rewrite them and expose their implicit mathematical context. Specifically, using the retriever's ranking as feedback, we optimize the prompt given to the rewriting LLM. This single optimized prompt is then reused to rewrite all queries.

Importantly, it is precisely our synthesized ambiguous query-declaration pairs (generated in Section 3.1.2) that make this rewriting possible. Beyond retriever training (Section 3.1.3), the *same synthetic data* can also be *reused* to discover a rewriting strategy (i.e., the prompt to the rewriting LLM) that exposes implicit mathematical context. This is why we emphasize that the same data bottleneck connects both model-level and input-level disambiguation.

We use GEPA (Agrawal et al., 2026) to optimize a rewriting prompt initialized from a simple seed prompt. We split a subset of the synthetic query-declaration pairs from Section 3.1.2 into feedback and Pareto sets. In each optimization step, GEPA selects a candidate prompt to rewrite a minibatch of queries from the feedback set, and the fixed Lean Finder V2 retriever ranks each target declaration. The resulting ranks yield a scalar score s (defined in Equation 6) and textual feedback comparing the target ranks for the original query, seed-prompt rewrite, and current rewrite (feedback template is in Appendix K). A reflection model (DeepSeek-V4.1-Flash (DeepSeek-AI, 2026)) uses this feedback to revise the prompt. GEPA retains a revision only if it improves the mean score on the same minibatch, then evaluates it on every Pareto-set example and adds it to the candidate prompt pool. When selecting the next prompt to revise from the pool, GEPA favors candidates that perform best on more Pareto-set examples. After optimization, it returns the prompt with the highest average Pareto-set score.

The score s rewards a rewrite both for placing the target declaration near the top of the retrieved list and for placing it at a better rank than the original query and the seed-prompt rewrite do. For each query-declaration pair (q_i, d_i) and rewriting prompt p, let $rank_i(p)$ denote the rank of the target declaration d_i after rewriting q_i with p; $rank_i^{\text{orig}}$ and $rank_i^{\text{seed}}$ denote its ranks for the original query and seed-prompt rewrite. The per-example scalar score $s_i(p)$ is

$$s_i(p) = \frac{\lambda_{\text{abs}}}{\log_2(rank_i(p)+1)} + \lambda_{\text{orig}} \frac{rank_i^{\text{orig}}}{rank_i^{\text{orig}} + rank_i(p)} + \lambda_{\text{seed}} \frac{rank_i^{\text{seed}}}{rank_i^{\text{seed}} + rank_i(p)}. \tag{6}$$

The three terms reward absolute retrieval quality, improvement over the original query, and improvement over the seed-prompt rewrite, respectively, and $\lambda_{\text{abs}}, \lambda_{\text{orig}}, \lambda_{\text{seed}}$ are weights that balance them. See Appendix F.4 for details on how we use GEPA. The optimized prompt is shown in Appendix K.

4 Experiments

We initialize the retrieval model from Qwen3-Embedding-8B (Zhang et al., 2025) and the query generator from Qwen3-4B (Yang et al., 2025), with GPT-OSS-120B (OpenAI, 2025) as the alignment judge, and DeepSeek-V4.1-Flash (DeepSeek-AI, 2026) for both GEPA prompt optimization and query rewriting. We perform two rounds of alternating generator-retriever training. In each round, we optimize the query generator on 2,000 Mathlib statements across diverse modules to encourage broad coverage of mathematical domains while keeping training tractable. Full model training and prompt optimization details are in Appendix F, and prompts are in Appendix K.

We evaluate Lean Finder V2 on both concise and context-rich query benchmarks, comparing against existing Lean search engines (Section 4.1). We test retrieval quality on real-world unlabeled queries collected from human and coding-agent interactions with Lean search engines (Section 4.2). We then evaluate input-side disambiguation by comparing our GEPA-optimized query rewriting against existing query-rewriting methods from Lean search engines on concise queries (Section 4.3). We assess downstream application by comparing Claude Code equipped with Lean Finder V2 against the same agent with no semantic search tools or a prior retrieval system (Section 4.4). Finally, Appendix B reports ablation studies that isolate the contributions of the training signals.

4.1 Lean Finder V2 Is the Strongest Retriever Across Query Types

Setup. We evaluate concise and ambiguous query retrieval on the human-curated MathlibQR benchmark (the 810-query fair set) (Gao et al., 2026), and context-rich query retrieval on the 2,526-

query test set of Lu et al. (2026). We ensure that the ground-truth declaration for every evaluation query is present in the corpus of every evaluated Lean search engine. **We ensure that neither any ground-truth declaration from the test sets nor any query associated with such a declaration exists in the training corpus at any stage of training.**

We report three checkpoints in our series: *Lean Finder V2 Round 0*, trained only on existing context-rich data (Section 3.1.1); *Lean Finder V2 Round 1*, which adds queries from the first RL-trained generator (Section 3.1.2); and *Lean Finder V2*, the final model after two rounds. We compare them with Lean Finder (Lu et al., 2026), LeanSearch V2 (Gao et al., 2026), LeanExplore (Asher, 2025), TheoremGraph (Kurgan et al., 2026), and Claude Code (Sonnet 5) (Anthropic, 2026). For TheoremGraph, we report the MathlibQR results from Kurgan et al. (2026); since its evaluation uses a cutoff of 10, Recall@50 and Recall@100 are not reported. We instruct Claude Code to return only ten declarations per query due to cost, so Recall@50 and Recall@100 are also not reported. Appendix C.1 provides the full protocol.

Metrics. Following prior evaluations of Lean search engines (Lu et al., 2026; Gao et al., 2026; Asher, 2025), we report nDCG@k and Recall@k. nDCG@k rewards placing the target declaration near the top of the first k results, while Recall@k measures the proportion of queries for which the target appears within those results.

Table 1: Retrieval performance on the 810-query MathlibQR fair set of short, human-written queries. Systems in comparison include LeanExplore, LeanSearch V2, Claude Code (Sonnet 5), TheoremGraph, Lean Finder, and three Lean Finder V2 variants (Round 0, Round 1, and the final model). Lean Finder V2 denotes the final model. As TheoremGraph uses cutoff of 10 in Kurgan et al. (2026), its Recall@50 and Recall@100 are not reported. Claude Code returns only ten candidates, so its Recall@50 and Recall@100 are also not reported. We also report the model size of each system in comparison. Best results in each metric column are **bold**.

		nDCG@k			Recall@k		
System	Model size	1	5	10	10	50	100
LeanExplore	109M	0.246	0.358	0.393	0.569	0.743	0.789
LeanSearch V2 (retriever-only)	8B	0.340	0.472	0.494	0.657	0.790	0.830
LeanSearch V2 (retriever+reranker)	8B + 8B	0.470	0.601	0.623	0.780	0.847	0.858
Claude Code (Sonnet 5)	–	0.527	0.629	0.641	0.747	–	–
TheoremGraph	8B	0.391	0.534	0.558	0.733	–	–
Lean Finder	7B	0.370	0.514	0.533	0.698	0.824	0.875
Lean Finder V2 Round 0	8B	0.463	0.590	0.611	0.760	0.864	0.901
Lean Finder V2 Round 1	8B	0.520	0.657	0.682	0.842	0.925	0.951
Lean Finder V2	8B	**0.558**	**0.707**	**0.732**	**0.893**	**0.956**	**0.973**

Table 2: Retrieval performance on the 2,526-query context-rich test set. Systems in comparison include LeanExplore, LeanSearch V2, Claude Code (Sonnet 5), Lean Finder, and three Lean Finder V2 variants (Round 0, Round 1, and the final model). Lean Finder V2 denotes the final model. Claude Code returns only ten candidates, so its Recall@50 and Recall@100 are not reported. Best results in each column are **bold**.

	nDCG@k			Recall@k		
System	1	5	10	10	50	100
LeanExplore	0.180	0.376	0.406	0.661	0.837	0.872
LeanSearch V2 (retriever-only)	0.414	0.569	0.596	0.783	0.907	0.942
LeanSearch V2 (retriever+reranker)	0.506	0.669	0.691	0.873	0.954	0.964
Claude Code (Sonnet 5)	0.245	0.345	0.363	0.485	–	–
Lean Finder	0.544	0.720	0.742	0.914	0.980	0.989
Lean Finder V2 Round 0	0.563	0.720	0.743	0.910	0.967	0.981
Lean Finder V2 Round 1	0.569	**0.728**	0.743	0.916	**0.985**	**0.993**
Lean Finder V2	**0.573**	**0.728**	**0.749**	**0.919**	**0.985**	0.992

Tables 1 and 2 show that Lean Finder V2, which is a retriever-only system, achieves the best result in every reported metric on MathlibQR, and performs competitively on context-rich query benchmark, outperforming LeanSearch V2, previous best two-stage retriever+reranker Lean semantic search engine. We discuss model sizes in Appendix G. We attribute Claude Code's strong MathlibQR performance to Sonnet 5 combining pretrained knowledge of Mathlib naming with iterative search

over Mathlib documentation to infer Mathlib terminology from ambiguous and concise queries. Context-rich queries instead require fine-grained matching of a detailed mathematical description against many closely related Mathlib declarations, which directly favors dense retrievers trained on paired informalization-declaration data over general-purpose agentic search.

Lean Finder V2 Round 0, trained only on context-rich data, performs strongly on context-rich queries and competitively on ambiguous queries, but performs worse than LeanSearch V2 with reranking on MathlibQR. Adversarial training closes this gap by producing large-scale synthetic ambiguous queries: training on these pairs improves every MathlibQR metric from Round 0 to the final model, while context-rich performance remains consistently strong, demonstrating the quality and usefulness of our synthetic data. Appendix B reports ablation studies that further isolate the contributions of this training procedure. Finally, Appendix E shows that reranking further improves retrieval performance on MathlibQR and context-rich queries on top of the strong Lean Finder V2 performance.

4.2 Lean Finder V2 Is Preferred on Unlabeled Real-World Queries

Ground-truth declarations are unavailable for most queries in practice. We therefore complement labeled benchmarks with preference evaluation on 200 queries collected from human and coding-agent interactions with Lean search engines. These evaluation queries are drawn from interactions held out from the generator's SFT data in Section 3.1.2. Each retrieval system retrieves five declarations per query. Following prior work (Lu et al., 2026; Gao et al., 2026; Asher, 2025), we use LLM judges to determine the quality and helpfulness of retrieval from different Lean search systems. Two frontier models, GPT-5.6 Sol and Claude Opus 5, independently rank the five systems by the helpfulness of their retrieved declarations to the query. We ensure that the identity of each system is not visible to LLM judges, and the order of systems is shuffled per query to avoid positional bias. Appendix D gives the evaluation details and validates the reliability of both LLM judges against the human-curated MathlibQR labels.

Table 3: LLM-judged retrieval preference on 200 unlabeled deployment queries. GPT-5.6 Sol and Claude Opus 5 independently rank LeanExplore, LeanSearch V2, Claude Code (Sonnet 5), Lean Finder, and Lean Finder V2 for each query. Rank 1 is reported as count (percentage); lower mean rank is better. Best results are **bold**.

	GPT-5.6 Sol		Claude Opus 5	
System	Mean rank ↓	Rank 1	Mean rank ↓	Rank 1
LeanExplore	3.740	14 (7.0%)	3.735	8 (4.0%)
LeanSearch V2 (retriever+reranker)	2.380	52 (26.0%)	2.305	64 (32.0%)
Claude Code (Sonnet 5)	3.775	32 (16.0%)	3.920	22 (11.0%)
Lean Finder	3.125	32 (16.0%)	3.095	30 (15.0%)
Lean Finder V2	**1.980**	**70 (35.0%)**	**1.945**	**76 (38.0%)**

Table 3 shows that both judges place Lean Finder V2 first overall: its mean rank is 1.980 under GPT-5.6 Sol and 1.945 under Claude Opus 5, and it receives the most rank-1 selections from each judge. The two judges choose the same rank-1 system on 89.0% of queries. Thus, the gain on labeled benchmarks extends to the open-ended queries that humans and coding agents issue during deployment.

4.3 Optimized Query Rewriting Improves Top-Ranked Retrieval

On MathlibQR, we compare GEPA-optimized rewriting (Agrawal et al., 2026) against its seed prompt, no rewriting, LeanSearch[1] and LeanDex[2] augmentations. All configurations use the same Lean Finder V2 retriever. For each query-rewriting method, we independently generate three rewrites per query and report mean and standard deviation in all metrics; no-rewriting configuration is evaluated once so no standard deviation is reported. We report the same retrieval metrics from Section 4.1. See Appendix C.2 for the full protocol.

Table 4 shows that GEPA-optimized rewriting achieves the strongest top-ranked retrieval, raising nDCG@1 from 0.558 to 0.588 over the original queries. With Recall@100 already at 0.973, rewriting

[1] https://leansearch.net/

[2] https://leandex.projectnumina.ai/

Table 4: Query-rewriting performance on the 810-query MathlibQR fair set. We compare GEPA-optimized rewriting with no rewriting , LeanSearch and LeanDex augmentation, and the GEPA seed prompt rewriting. Each rewriting method produces three query sets, with the Lean Finder V2 retriever fixed; results report their mean and standard deviation, while no-rewriting is evaluated once. Best results in each column are **bold**.

	nDCG@k			Recall@k		
Input Rewrite Config.	1	5	10	10	50	100
Original query (no rewrite)	0.558	0.707	0.732	0.893	0.956	0.973
LeanSearch augmentation	0.475 ± 0.006	0.612 ± 0.004	0.637 ± 0.004	0.808 ± 0.004	0.908 ± 0.005	0.936 ± 0.005
LeanDex augmentation	0.520 ± 0.013	0.665 ± 0.009	0.688 ± 0.009	0.852 ± 0.008	0.944 ± 0.005	0.962 ± 0.005
GEPA seed rewriting	0.532 ± 0.002	0.674 ± 0.004	0.695 ± 0.005	0.854 ± 0.010	0.942 ± 0.004	0.963 ± 0.004
GEPA-optimized rewriting	$\mathbf{0.588} \pm 0.004$	$\mathbf{0.722} \pm 0.003$	$\mathbf{0.741} \pm 0.002$	$\mathbf{0.895} \pm 0.002$	$\mathbf{0.964} \pm 0.007$	$\mathbf{0.975} \pm 0.003$

mainly resolves which retrieved candidate the user intends, moving the ground-truth declaration into the top ranks that users inspect first. The improvement that the GEPA-optimized rewriting brings further demonstrates the value of our synthetic ambiguous query-declaration pairs for addressing real-world queries from a data perspective: the same synthetic data is used for both training the retrieval model and for optimizing a rewriting prompt, and each improves retrieval on human-written MathlibQR queries. LeanSearch, LeanDex, and the seed prompt reduce performance at every cutoff. We find that their rewrites tend to add unnecessary context and shift focus away from the user's intent. We present a case study in Appendix J. In addition, with reranking added on top of query rewriting, we achieve further improvement, reaching 0.610 for nDCG@1. Appendix E provides more details for the reranking+rewriting configuration.

4.4 Lean Finder V2 Helps the Most on Theorem Proving Task

We evaluate Claude Code (Sonnet 5) on FATE-H (Jiang et al., 2026), which contains difficult formal algebra problems, using LeanExplore, LeanSearch V2 with reranking, Lean Finder, Lean Finder V2, and no additional semantic search tool. All settings share the same prompt template and per-problem budget. Table 5 reports mean pass rates and standard deviations over three runs, counting only compiling, sorry-free proofs; Appendix C.3 details the protocol.

Table 5: Theorem-proving performance on FATE-H benchmark. We compare Claude Code (Sonnet 5) with LeanExplore, Lean Finder, LeanSearch V2, Lean Finder V2, or no semantic retrieval tool. Results are mean pass rates and standard deviations over three runs under matched agent settings and budgets; best result is **bold**.

Retrieval tool	Pass rate (%)
No semantic retrieval tool	34.0 ± 1.0
LeanExplore	27.3 ± 0.6
Lean Finder	30.7 ± 1.2
LeanSearch V2 (retriever+reranker)	36.0 ± 1.0
Lean Finder V2	$\mathbf{39.3} \pm 1.2$

Lean Finder V2 achieves the highest pass rate at 39.3%. The no semantic search tool baseline is also strong and outperforms LeanExplore and Lean Finder while relying on Sonnet 5's internal Mathlib knowledge and repository grep. This suggests that, under a fixed agent budget, retrieval improves theorem proving only when it returns sufficiently relevant results to justify the additional effort required to inspect them and invoke the tool. Only LeanSearch V2 and Lean Finder V2 outperform Claude Code with no semantic search tool. The result shows that Lean Finder V2's training strategy is consistent with queries issued during agentic proof development and translates its retrieval advantage into stronger end-to-end theorem proving.

5 Conclusion

We introduce Lean Finder V2, a semantic search framework designed around how human mathematicians and coding agents actually search Mathlib: with ambiguous queries that leave much of their intent implicit. Lean Finder V2 aligns the retriever with real-world queries through complementary model-level adversarial training and input-level query-rewriting prompt optimization. The same synthetic ambiguous query-declaration pairs power both optimization processes. Comprehensive evaluations demonstrate its superior performance over existing search systems. By making Mathlib more accessible through the concise and ambiguous queries used in practice, Lean Finder V2 provides critical infrastructure for autonomous agents in formal mathematics and scalable human-AI collaboration.

AI USE STATEMENT

In this work, we used generative AI tools to generate synthetic query-declaration training pairs, serve as LLM judge for data processing and RL training, assist with limited implementation of the experimental pipeline, and correct minor grammatical and typographical errors in writing. We did not use generative AI tools for other tasks requiring disclosure. We reviewed and tested all AI-assisted implementation and reviewed all AI-assisted writing corrections. We take responsibility for the final content of this work, including text, claims, data, and other artifacts produced with the aid of generative AI.

ETHICS STATEMENT

This work focuses on improving retrieval in Mathlib through adversarial training and query rewriting. We do not identify and anticipate risks arising from the proposed methods.

REPRODUCIBILITY STATEMENT

Sections 3.1 and 3.2 describe the training and query-rewriting methods, including the reward functions. Appendix F provides model choices, training objectives, hyperparameters, and pseudocode for the generator, retriever, and reranker; Appendix C documents detailed retrieval and downstream evaluation protocols.

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A Extended Related Work

Lean and Mathlib Retrieval. Semantic search and retrieval for Lean and Mathlib have advanced rapidly, with a growing number of systems designed to help humans and agents locate relevant declarations in the library. LeanDojo (Yang et al., 2023) introduced ReProver, which combines premise retrieval with tactic generation, while LeanExplore (Asher, 2025), LeanDex (Liu et al., 2026), and LeanSearch (Gao et al., 2024) support semantic search over declarations and input query rewriting. Lean Finder (Lu et al., 2026) synthesized queries from user intents mined from Lean discussions and aligned retrieval with preference feedback. LeanSearch V2 (Gao et al., 2026) used hierarchy-informed informalizations with embedding-based retrieval and reranking. These systems either train or simply embed using context-rich informalizations or synthetic data derived from Mathlib declarations, yet queries from humans or agents are usually short and ambiguous. None of the existing systems adapt to the real world ambiguous queries. Lean Finder V2 overcomes this limitation by scaling up generation of short and ambiguous queries with adversarial training, and optimizing the query-rewriting prompt to make implicit mathematical intent explicit before retrieval.

Adversarial and RL-Based Retrieval. Reinforcement-learning-based approaches have increasingly been used to improve retrieval systems. Early work such as IRGAN (Wang et al., 2017) formulated retrieval as a minimax game in which a generative model selects difficult documents for fixed queries to challenge a discriminative ranker, while AR2 (Zhang et al., 2022) similarly co-trained a dense retriever and cross-encoder ranker through progressively harder negative documents. More recent work moves optimization to the query side. LeReT (Hsu et al., 2024) and DeepRetrieval (Jiang et al., 2025) trained LLMs through RL to generate or rewrite queries that directly optimize retrieval metrics, while keeping the retriever fixed. Reinforced-IR (Li et al., 2025) focused on generating hypothetical-document augmentations. Jagatap et al. (2025) used RL to generate synthetic product queries for classification models. Lean Finder V2 instead uses RL to generate concise and ambiguous queries that the current retriever fails on yet that remain faithful to their target declarations, and retrains the retriever on them.

Lean Provers and Agentic Systems. Recent advances in Lean theorem proving have improved formal reasoning through stronger provers and agentic system designs. DeepSeek-Prover-V2 (Ren et al., 2025) used recursive subgoal decomposition to construct cold-start reasoning data before reinforcement learning, while Goedel-Prover-V2 (Lin et al., 2025) combined scaffolded data synthesis with verifier-guided self-correction. GAR (Wang et al., 2026) further introduced an adversarial reinforcement-learning framework in which a problem composer adapts task difficulty to the evolving capability of a solver. Beyond standalone provers, systems such as Numina-Lean-Agent (Liu et al., 2026) and Hilbert (Varambally et al., 2026) integrated theorem proving with retrieval, decomposition, and verification, while research-level agents including Archon (Ju et al., 2026) and MechMathAgent (Cao et al., 2026) orchestrated multiple reasoning components with the Lean compiler in the loop. These agents typically use infrastructure such as Lean-LSP-MCP (Dressler, 2025) to interact with the Lean compiler and auxiliary tools, including theorem-search engines for locating reusable Mathlib results. As a result, retrieval is becoming critical infrastructure for agentic formal reasoning. Lean Finder V2 focuses on improving this search infrastructure itself: it trains the retrieval model to better handle concise, ambiguous queries which are common from coding agents through adversarially generated supervision whose difficulty adapts to the retriever's evolving capability.

B Ablation Studies

We conduct detailed ablation studies to answer two questions about adversarial query generation. First, do the alignment and difficulty rewards each contribute to effective synthetic data? Second, does another round of generator RL improve over generation from the base model, a fixed RL-trained generator, or simply sampling more queries? All comparisons use the same source declarations, context-rich data, retriever initialization, and matched retriever-update budgets. We evaluate both concise-query on MathlibQR (Gao et al., 2026) and context-rich benchmark (Lu et al., 2026).

B.1 Contribution of Alignment and Difficulty Rewards

Setup. We isolate the two RL reward components after one generator-retriever round. The alignment-only, difficulty-only, and full-reward variants start from the same generator and Round 0 retriever, use matched generator-training steps, and receive the same query-sampling budget. We then train their retrievers under matched training budgets. Round 0 adds no generated concise queries and serves as the context-rich-only control. This setup isolates the effect of each reward component from generator compute, sampling volume, and retriever compute.

Table 6: Ablation of the query-generator reward after one generator-retriever training round on the 810-query MathlibQR fair set. Round 0 uses only context-rich data; the Round 1 variants differ only in the reward used to train the query generator. Generator steps, sampling budgets, and retriever steps are matched. Best result in each column is bold.

	nDCG@k			Recall@k		
Configuration	1	5	10	10	50	100
Round 0	0.463	0.590	0.611	0.760	0.864	0.901
Round 1 (difficulty only)	0.453	0.575	0.602	0.762	0.870	0.919
Round 1 (alignment only)	0.463	0.598	0.623	0.780	0.884	0.910
Round 1 (full reward)	**0.520**	**0.657**	**0.682**	**0.842**	**0.925**	**0.951**

Table 7: Ablation of the query-generator reward after one generator-retriever training round on the context-rich test set. The configurations and matched training budgets are identical to Table 6. Best result in each column is bold.

	nDCG@k			Recall@k		
Configuration	1	5	10	10	50	100
Round 0	0.563	0.720	**0.743**	0.910	0.967	0.981
Round 1 (difficulty only)	0.530	0.689	0.711	0.879	0.956	0.975
Round 1 (alignment only)	0.552	0.718	0.738	0.903	0.968	0.980
Round 1 (full reward)	**0.569**	**0.728**	**0.743**	**0.916**	**0.985**	**0.993**

Results. The full reward is the only configuration that improves every MathlibQR metric over Round 0, and it outperforms both single-reward variants in every column (Table 6). Difficulty alone degrades retrieval at the most important top-rank metrics and fall below Round 0. This pattern indicates that difficulty without alignment can reward queries that are hard for the wrong reason, producing mismatched query-declaration supervision that disrupts ranking. Alignment alone avoids this failure mode but yields only modest, uneven gains.

The context-rich results reinforce this conclusion (Table 7). Alignment alone remains close to Round 0 across all metrics, whereas difficulty alone reduces top rank metrics substantially (nDCG@10 from 0.743 to 0.711 and Recall@10 from 0.910 to 0.879). The full reward instead achieves the strongest overall results across both query types: alignment keeps queries faithful to their declarations, and difficulty turns those faithful queries toward current retriever gaps.

B.2 RL-Trained Generation Versus Additional Sampling

Setup. We next separate the effects of RL training, sampling volume, and an additional round of generator optimization. Every final retriever starts from the same base checkpoint, uses the same context-rich data, and receives a matched number of optimization steps. Round 0 adds no generated concise queries. In each branch, Round 1 adds one query per declaration in total and Round 2 uses two queries per declaration in total.

The frozen base-generator control samples both queries from Qwen3-4B before RL. The fixed RL-generator control samples both from the generator trained in the first RL round. Expert-queries SFT-generator control samples both from the generator trained with only SFT loss on the expert queries described in Section 3.1.2. Lean Finder V2 shares the same Round 1 generator, but applies

another round of RL before generating the second query. We repeat the shared Round 1 result in the fixed-control and Lean Finder V2 table blocks so that their Round 2 outcomes can be compared directly.

Table 8: Ablation of generator RL training on the 810-query MathlibQR fair set. Round 1 uses one generated query per declaration, and Round 2 uses two in total. All retrievers receive a matched number of optimization steps. Best result in each column is **bold**.

	nDCG@k			Recall@k		
Round	1	5	10	10	50	100
Round 0	0.463	0.590	0.611	0.760	0.864	0.901
Frozen base-generator control						
Round 1	0.472	0.603	0.628	0.791	0.895	0.921
Round 2	0.469	0.597	0.620	0.789	0.898	0.919
Expert-queries SFT-generator control						
Round 1	0.475	0.610	0.632	0.794	0.904	0.919
Round 2	0.473	0.607	0.633	0.795	0.896	0.914
Fixed RL-generator control						
Round 1	0.520	0.657	0.682	0.842	0.925	0.951
Round 2	0.515	0.657	0.676	0.830	0.921	0.954
Lean Finder V2						
Round 1	0.520	0.657	0.682	0.842	0.925	0.951
Round 2	**0.558**	**0.707**	**0.732**	**0.893**	**0.956**	**0.973**

Table 9: Ablation of generator RL training on the context-rich test set. The configurations are identical to Table 8, and all retrievers receive a matched number of optimization steps. Best result in each column is **bold**.

	nDCG@k			Recall@k		
Round	1	5	10	10	50	100
Round 0	0.563	0.720	0.743	0.910	0.967	0.981
Frozen base-generator control						
Round 1	0.559	0.710	0.729	0.894	0.967	0.981
Round 2	0.552	0.705	0.725	0.890	0.965	0.981
Expert-queries SFT-generator control						
Round 1	0.565	0.712	0.730	0.898	0.969	0.980
Round 2	0.562	0.715	0.735	0.904	0.970	0.982
Fixed RL-generator control						
Round 1	0.569	0.728	0.743	0.916	0.985	0.993
Round 2	**0.573**	**0.729**	0.742	0.917	0.982	0.990
Lean Finder V2						
Round 1	0.569	0.728	0.743	0.916	**0.985**	**0.993**
Round 2	**0.573**	0.728	**0.749**	**0.919**	**0.985**	0.992

Results. RL training is more effective than generic query generation: from Round 0 to Round 1, it raises MathlibQR nDCG@10 from 0.628 to 0.682 and Recall@10 from 0.791 to 0.842 (Table 8). The SFT-only generator performs similarly to the frozen base generator across both rounds, with small top-rank gains, while the RL-trained generators achieve higher retrieval scores. Sampling a second query from any fixed generator adds no material gain, while applying another round of RL before generating the second query raises nDCG@10 to 0.732 and Recall@10 to 0.893. Lean Finder V2 Round 2 also achieves the best or close-to-best result in every context-rich metric (Table 9), preserving strong retrieval beyond concise queries.

C Detailed Evaluation Setup

C.1 Retrieval Evaluation

We submit the original benchmark query to every system and evaluate its returned retrievals against the ground-truth Mathlib declaration. For LeanExplore, we obtain results from its public service at `https://www.leanexplore.com/`. We omit LeanDex (Liu et al., 2026) from this comparison because we find that it uses the LeanExplore retriever and adds query rewriting, which we evaluate separately in Section 4.3. For LeanSearch V2, we run inference using the authors' released code at `https://github.com/frenzymath/LeanSearch-v2`, evaluating both its retriever-only and retriever-plus-reranker configurations. For Lean Finder, we run inference using the authors' released code at `https://github.com/delta-lab-ai/lean-finder`. For TheoremGraph (Kurgan et al., 2026), we report the numbers found in Kurgan et al. (2026), and since its evaluation uses a cutoff of 10, Recall@50 and Recall@100 are not reported.

For the general-agent baseline, we use Claude Code with Sonnet 5 and permit online search over Mathlib documentation and source pages. We explicitly disallow Lean search-engine websites and tools so that the agent cannot delegate retrieval to one of the specialized systems under comparison. To control cost, we process ten independent queries in each Claude Code invocation, and require a separate ranked list of exactly ten Mathlib declaration names for every query. Ten candidates provide a fixed top-10 retrieval that supports direct computation of nDCG@1, nDCG@5, nDCG@10, and Recall@10. Because the agent returns no candidates beyond rank 10, we do not report Recall@50 or Recall@100 for Claude Code. The cost of running Claude Code with Sonnet 5 is approximately $60 for the two benchmarks.

C.2 Query-Rewriting Evaluation

All rewriting methods receive the same original MathlibQR query, and their rewritten output replaces that original query as the input to Lean Finder V2. GEPA is initialized with the seed prompt provided in Appendix K. This seed is the short, pre-optimization instruction that initializes GEPA and defines the rewriting task. The GEPA-optimized prompt is provided in Appendix K. We obtain the LeanSearch augmentation from its public augmentation endpoint at `https://leansearch.net/augment`. For LeanDex, we send the query to `https://leandex.projectnumina.ai/api/v1/search` using the `q` parameter and `generate_query=true`, then use the returned `agent_query` as the rewrite. Each resulting query is evaluated against the same ground-truth declaration and with the same fixed Lean Finder V2 model. For every rewriting method, we independently generate the complete rewritten query set three times and report the mean and standard deviation of the retrieval metrics across these runs. The no-rewrite condition bypasses generation and is evaluated once on the original queries.

C.3 Downstream Theorem-Proving Evaluation

FATE-H contains 100 challenging formal algebra problems drawn from research literature, PhD qualifying examinations, and textbooks. We evaluate all five retrieval conditions on these problems using Claude Code with Sonnet 5. Each retrieval engine is exposed to the agent through the same tool interface, while the agent prompt and execution environment remain fixed. We impose a $1 token-cost cap on every problem for all conditions due to cost. The agent is not permitted to change the original theorem statement. We count a problem as passed only when the produced Lean file is sorry-free, compiles in the benchmark environment, and proves the original statement within the cost cap.

We perform three independent runs for each retrieval condition and report the mean pass rate and sample standard deviation in percentage points across runs. Across a single complete 100-problem run, token costs are approximately $90 with Lean Finder V2, $92 with LeanSearch V2, $94 with no semantic search tool, $97 with Lean Finder, and $98 with LeanExplore.

D LLM-JUDGE EVALUATION AND VALIDATION

Deployment-query protocol. We sample 200 queries from interactions with Lean search engines, covering queries issued by both humans and coding agents. Unlike MathlibQR, these queries do not come with a designated target declaration. For every query, Lean Finder V2, LeanSearch V2, Lean Finder, LeanExplore, and Claude Code each return five declarations. To ensure fair comparison, we randomly order the five systems' retrievals for every query, and mask the identities of each system. GPT-5.6 Sol and Claude Opus 5 independently inspect the query and the five result lists, then rank the systems from most to least helpful. We average the ordinal ranks across queries and separately report how often each system is ranked first. The two judges choose the same rank-1 system on 89.0% of queries, showing that the preference for Lean Finder V2 is consistent across judge families.

Validation of LLM judge reliability against human-curated labels. We validate the same judges on all 810 queries in the MathlibQR fair set (Gao et al., 2026) to ensure the reliability of LLM judges. For each query, we construct a five-way choice from the top Lean Finder V2 retrievals and ensure that the human-labeled declaration is present. When it is absent, we replace one retrieved declaration with the ground truth; the five choices are then shuffled. Each judge sees only the query and each choice's formal name, declaration kind, and Lean type. It does not see the benchmark label, original retrieval rank, retrieval score, or which choice was inserted. Agreement is the fraction of queries for which the judge selects the human-labeled declaration. We report the result in Table 10.

Table 10: Agreement of GPT-5.6 Sol and Claude Opus 5 with human-curated labels on the 810-query MathlibQR fair set. Each judge selects one of five candidate declarations; we report agreement counts and rates.

Judge	Agreements	Agreement (%)
GPT-5.6 Sol	738/810	91.11
Claude Opus 5	745/810	91.98

Both judges recover the human-selected MathlibQR declaration on more than 91% of queries despite choosing among the retriever's strongest confusions. This independent agreement supports the reliability of the preference evaluation conducted with the same LLM judges. It also provides a second validation of MathlibQR itself: human curation and two frontier model families converge on the same labels, making widespread false-positive benchmark labels unlikely.

E RERANKER TRAINING AND RESULTS

In this section, we study reranking on top of our strong Lean Finder V2 retrieval model. We fine-tune Qwen3-Reranker-8B to estimate the relevance of Mathlib declarations to a given query. The queries used for training include the large-scale context-rich queries described in Section 3.1.1 as well as the synthetic short queries described in Section 3.1.2. For each query, we use the ground-truth declaration as a positive example and randomly sample negative examples from the remaining Mathlib declarations. The model jointly processes each query-declaration pair and predicts its relevance using the output logits, $z_{\texttt{yes}}$ and $z_{\texttt{no}}$, for the `yes` and `no` tokens. Specifically, the relevance score is

$$s_\theta(q, d) = \sigma(z_{\texttt{yes}} - z_{\texttt{no}}), \tag{7}$$

where σ denotes the sigmoid function. We minimize binary cross-entropy over the training pairs:

$$\mathcal{L}(\theta) = -\frac{1}{|\mathcal{D}|} \sum_{(q,d,y)\in\mathcal{D}} [y \log s_\theta(q, d) + (1 - y) \log(1 - s_\theta(q, d))], \tag{8}$$

where $y = 1$ indicates a positive pair and $y = 0$ indicates a sampled negative.

Table 11 and Table 12 show that adding the reranker further improves the ranking quality on top of Lean Finder V2's strong performance on both MathlibQR and context-rich queries. With the GEPA-optimized query rewriting and reranking both applied to Lean Finder V2, we achieve even more improvements across all metrics (except for Recall@10), reaching 0.610 for nDCG@1 on MathlibQR.

Table 11: Retrieval performance on the 810-query MathlibQR fair set of short, human-written queries. Systems include LeanExplore, LeanSearch V2, Claude Code (Sonnet 5), Lean Finder, Lean Finder V2 with and without our reranker, and Lean Finder V2 with both our reranker and GEPA-optimized query rewriting. Lean Finder V2 denotes the retriever-only model. Claude Code returns only ten candidates, so its Recall@50 and Recall@100 are not reported. Best results in each column are bold.

	nDCG@k			Recall@k		
System	1	5	10	10	50	100
LeanExplore	0.246	0.358	0.393	0.569	0.743	0.789
LeanSearch V2 (retriever-only)	0.340	0.472	0.494	0.657	0.790	0.830
LeanSearch V2 (retriever+reranker)	0.470	0.601	0.623	0.780	0.847	0.858
Claude Code (Sonnet 5)	0.527	0.629	0.641	0.747	–	–
Lean Finder	0.370	0.514	0.533	0.698	0.824	0.875
Lean Finder V2	0.558	0.707	0.732	**0.893**	0.956	0.973
Lean Finder V2 (w/ reranker)	0.575	0.714	0.735	0.890	0.968	0.980
Lean Finder V2 (w/ reranker+gepa rewrite)	**0.610**	**0.726**	**0.746**	0.890	**0.970**	**0.981**

Table 12: Retrieval performance on the 2,526-query context-rich test set. Systems in comparison include LeanExplore, LeanSearch V2, Claude Code (Sonnet 5), Lean Finder, and Lean Finder V2 with and without our reranker. Lean Finder V2 denotes the retriever-only model. Claude Code returns only ten candidates, so its Recall@50 and Recall@100 are not reported. Best results in each column are bold.

	nDCG@k			Recall@k		
System	1	5	10	10	50	100
LeanExplore	0.180	0.376	0.406	0.661	0.837	0.872
LeanSearch V2 (retriever-only)	0.414	0.569	0.596	0.783	0.907	0.942
LeanSearch V2 (retriever+reranker)	0.506	0.669	0.691	0.873	0.954	0.964
Claude Code (Sonnet 5)	0.245	0.345	0.363	0.485	–	–
Lean Finder	0.544	0.720	0.742	0.914	0.980	0.989
Lean Finder V2	0.573	0.728	0.749	0.919	0.985	0.992
Lean Finder V2 (w/ reranker)	**0.607**	**0.760**	**0.779**	**0.939**	**0.987**	**0.995**

F Training Details

We describe the training details of our generator, retrieval, reranker, and our prompt optimization process in this section. All experiments are conducted on NVIDIA 6000Ada and L40S GPUs.

F.1 Generator Training Details

We initialize the generator from Qwen3-4B (Yang et al., 2025) and use GPT-OSS-120B (OpenAI, 2025) as the alignment judge. We apply GRPO to 2,000 Mathlib statements in each round. We train for one epoch with learning rate 5×10^{-7}, group size $G = 8$, sampling temperature 1.0, $\beta = 0.001$, and a 1% warmup. The first RL round additionally mixes expert deployment queries through the CHORD objective. Here, μ has peak value 0.9 and valley value 0.05, with no warmup and 30 decay steps. We use the expert data only in this first round because the two supervision sources serve different roles. The expert queries provide a prior over the phrasing, brevity, and search patterns observed in deployment, which the rank and alignment rewards are not designed to encode. After the first round, the generator enters the next round from this deployment-aligned checkpoint, while the updated retriever defines a new difficulty landscape. We therefore remove the fixed expert objective in the second round and let the on-policy reward adapt the generator to the updated retriever. The alignment gate and length penalty remain active, preserving target fidelity and concise query form during this adaptation. Each round generates over 300K queries, and the final combined synthetic query-declaration pairs from adversarial training reaches over 600K.

We now define our query overlong penalty $p_{\text{len}}(L)$, which is used as part of the RL reward function in Section 3.1.2. For query length L, measured with the retriever tokenizer, the soft length penalty is

$$p_{\text{len}}(L) = \begin{cases} 0, & L \le L_{\max}, \\ \dfrac{L - L_{\max}}{L_{\text{buf}}}, & L_{\max} < L \le L_{\max} + L_{\text{buf}}, \\ 1, & L > L_{\max} + L_{\text{buf}}. \end{cases} \tag{9}$$

We set $L_{\max} = 40$, $L_{\text{buf}} = 10$, and $\lambda_{\text{len}} = 0.15$. The penalty is zero through 40 tokens, grows linearly from 40 to 50 tokens, and remains at its maximum thereafter. Appendix H motivates these settings using the query-length distributions of human users and coding agents.

For the expert dataset used in SFT component of the hybrid objective, we use records from human and coding-agents interactions with current Lean search engines through lean-lsp-mcp (Dressler, 2025), including Lean Finder (Lu et al., 2026) and LeanSearch (Gao et al., 2024; 2026). Since those queries don't have ground truth label, we use an LLM-assisted label discovery process. We first use an existing search engine (Lean Finder) to retrieve the top 50 Mathlib declarations for the real world queries, and employ an LLM-judge (Opus 5) to determine the correct ground truth Mathlib declaration. If no ground truth is found in the top 50 retrievals, then we skip the query. We keep a total of 2,000 queries-declaration pairs as the expert dataset. The reliability of Opus 5 as the LLM judge in replicating human preference on quality of retrieval is validated in Appendix D, and previous research showed that LLM judges closely match human judgments of which retrieved Mathlib statements are useful (Lu et al., 2026). Using a frontier LLM judge is the only practical way to reliably scale the data preparation process. We also filter the expert queries used for SFT against the 200 held-out queries used for LLM-judge-based evaluation (Section 4.2) and all other datasets used in evaluation (context-rich queries and MathlibQR), using exact-match deduplication and MinHash-LSH-based near-duplicate detection to ensure fairness.

F.2 Retrieval Training Details

We initialize our retriever with Qwen3-Embedding-8B (Zhang et al., 2025), and fine-tune using masked InfoNCE. We use a per-device batch size of 32, learning rate 10^{-5}, 5% warmup, $\tau = 0.1$, and $m = 0.1$. Each round combines the context-rich data with the accumulated concise queries generated up to that round. For a controlled comparison, all retriever variants receive matched optimization steps.

We show the masked InfoNCE loss here. Let B denote the mini-batch size, q_i the i-th query, and c_i^+ its ground-truth positive declaration. The candidate set $\mathcal{C}_B = \{c_j^+\}_{j=1}^B$ contains all positive declarations in the mini-batch. We write $\text{Sim}(\cdot,\cdot)$ for cosine similarity, τ for the softmax temperature, and m for the false-negative margin. The indicator $\mathbb{I}(q_i, c)$ is zero when $\text{Sim}(q_i, c) > \text{Sim}(q_i, c_i^+) + m$, masking a candidate that scores higher than the labeled positive by more than m, and one otherwise. The masked InfoNCE objective is

$$\mathcal{L}_{\text{ret}} = -\frac{1}{B}\sum_{i=1}^{B} \log \frac{\exp\big(\text{Sim}(q_i, c_i^+)/\tau\big)}{\sum_{c \in \mathcal{C}_B} \mathbb{I}(q_i, c)\exp(\text{Sim}(q_i, c)/\tau)}. \tag{10}$$

F.3 Reranker Training Details

We train the reranker models for one epoch using LoRA (Hu et al., 2021) (rank $r = 8$, $\alpha = 32$) on all linear layers, with a learning rate of 5×10^{-5}, a warmup ratio of 0.05, and a per-device batch size of 64.

F.4 Prompt Optimization Training Details

Optimization procedure. We optimize the rewriting prompt using GEPA (Agrawal et al., 2026). Starting from the seed rewriting prompt, the optimizer maintains a pool of candidate prompts. The training data are split into a feedback set $\mathcal{D}_{\text{feedback}}$ and a Pareto set $\mathcal{D}_{\text{pareto}}$. At each optimization step, GEPA samples a candidate prompt that is best-performing on at least one example in $\mathcal{D}_{\text{pareto}}$, and evaluates it on a minibatch from $\mathcal{D}_{\text{feedback}}$. The resulting scalar scores and textual retrieval-grounded

feedback are provided to a reflection model, which proposes a revised prompt. More details on the scalar score and the retrieval-grounded feedback are provided in the following two paragraphs. The revision is retained only if it improves the average minibatch score; retained candidates are then evaluated on the $\mathcal{D}_{\text{pareto}}$ and added to the candidate pool. Candidate selection favors non-dominated prompts that achieve the best score on one or more examples in $\mathcal{D}_{\text{pareto}}$, with higher sampling probability for candidates that are best on more examples. After the optimization budget is exhausted, we return the candidate with the highest average score on $\mathcal{D}_{\text{pareto}}$.

Scalar score. Let (q_i, d_i) denote each query-declaration pair used for prompt optimization. For a rewriting prompt p, let $rank_i(p)$ denote the rank of the target declaration d_i when the fixed retriever receives the rewrite of q_i under p; $rank_i^{\text{orig}}$ and $rank_i^{\text{seed}}$ denote its ranks for the original query and seed-prompt rewrite, respectively. For each example, we return the scalar score

$$s_i(p) = \lambda_{\text{abs}} \frac{1}{\log_2(rank_i(p)+1)} + \lambda_{\text{orig}} \frac{rank_i^{\text{orig}}}{rank_i^{\text{orig}} + rank_i(p)} + \lambda_{\text{seed}} \frac{rank_i^{\text{seed}}}{rank_i^{\text{seed}} + rank_i(p)}. \quad (11)$$

The first term rewards high absolute target rank, while the latter two reward improvement over the original query and the seed-prompt rewrite. We set $\lambda_{\text{abs}} = 0.5$ and $\lambda_{\text{orig}} = \lambda_{\text{seed}} = 0.25$, placing greater emphasis on absolute retrieval quality while weighting improvements over the two baselines equally.

Retrieval-grounded feedback. The natural-language diagnostic feedback for each rewrite contains the target Lean declaration and its informal description, the original query, the current rewrite, the original and current target ranks, the seed-prompt rewrite and its target rank, the scalar score, and the top-10 retrieved declarations. We also provide principles of reflection, which ask the reflection model to compare the current, original, and seed outcomes, analyze both gains and regressions, and propose a reusable behavioral change.

Hyperparameters. We use 200 training examples, split evenly between $\mathcal{D}_{\text{feedback}}$ and $\mathcal{D}_{\text{pareto}}$, so that $|\mathcal{D}_{\text{feedback}}| = |\mathcal{D}_{\text{pareto}}| = 100$. We use a reflection minibatch size of 3 and an optimization budget of $B = 1{,}000$, representing the maximum number of scalar score evaluations. Here, the minibatch size determines how many feedback examples are used to construct each reflective prompt update, while the Pareto-set size determines how many examples are used to evaluate and select prompt candidates.

Training data. We construct the optimization data from a subset of the concise query-declaration pairs produced by the RL-trained generator in Section 3.1.2. Each optimization example contains the original query, the target declaration, its rank based on the original query, the seed-prompt-rewritten query as well as its retrieval rank.

F.5 Pseudocode for Training Process

Algorithm 1 Retriever-based adversarial training

Require: Mathlib declarations $\mathcal{D}$, context-rich query data $\mathcal{D}_{\text{long}}$, SFT anchor data $\mathcal{D}_{\text{SFT}}$, base generator π_{θ_0}, number of rounds T

$R_0 \leftarrow \textsc{TrainRetriever}(\mathcal{D}_{\text{long}})$
for $t = 0, \ldots, T-1$ **do**
  $\theta_{t+1} \leftarrow \textsc{HybridObjective}(\theta_t, R_t, \mathcal{D}, \mathcal{D}_{\text{SFT}})$
  $\mathcal{D}_{t+1}^{\text{short}} \leftarrow \emptyset$
  for all $d \in \mathcal{D}$ **do**
    $q \sim \pi_{\theta_{t+1}}(\cdot \mid d)$
    $\mathcal{D}_{t+1}^{\text{short}} \leftarrow \mathcal{D}_{t+1}^{\text{short}} \cup \{(q, d)\}$
  end for
  $\mathcal{D}_{\text{short}} \leftarrow \mathcal{D}_{\text{short}} \cup \mathcal{D}_{t+1}^{\text{short}}$
  $R_{t+1} \leftarrow \textsc{TrainRetriever}(\mathcal{D}_{\text{long}} \cup \mathcal{D}_{\text{short}})$
end for
return R_T

G Model Scale and Backbone Choice

Table 13 compares the parameter counts of the retrieval systems we evaluated. Lean Finder V2 uses a single 8B retriever, while the reranked LeanSearch V2 configuration combines an 8B retriever with an 8B reranker.

Table 13: Retrieval-model scale for the systems in Table 1. For a two-stage system, the values before and after + give the retriever and reranker sizes, respectively.

System	Retrieval configuration	Parameters
LeanExplore	Retriever	109M
LeanSearch V2	Retriever only	8B
LeanSearch V2	Retriever + reranker	8B + 8B
TheoremGraph	Retriever	8B
Lean Finder	Retriever	7B
Lean Finder V2	Retriever	8B

Lean Finder V2 and LeanSearch V2 rely on the same Qwen3-Embedding-8B base checkpoint (Gao et al., 2026; Zhang et al., 2025). LeanSearch V2 applies this checkpoint without task-specific retriever fine-tuning, whereas Lean Finder V2 fine-tunes it for Mathlib retrieval. The shared initialization shows that our gains come from adapting the same backbone to Mathlib search rather than increasing retriever scale.

Our method is agnostic to the base model architecture and pretrained checkpoint. Although we can ablate other base models, our goal is to deliver a strong Mathlib search engine for the AI4Math and Lean communities so starting from a strong base model is necessary.

H Reward Hyperparameter Justifications

H.1 Length-penalty thresholds

The length penalty guards against unnecessarily long generations rather than forcing queries toward a fixed target length. In the deployment data summarized in Figure 1(a), the 90th-percentile lengths of distinct human and coding-agent queries are 37 and 28 tokens, respectively, while their 95th-percentile lengths are 45 and 34 tokens. We set $L_{\max} = 40$, above the 90th percentile for both sources, so that queries in the ordinary deployment range remain unpenalized. We use $L_{\text{buf}} = 10$ so that the penalty grows smoothly from 40 to 50 tokens and reaches its maximum only beyond the 95th percentile of both sources. Finally, $\lambda_{\text{len}} = 0.15$ caps the length deduction at 0.15 on the unit-scale difficulty reward, keeping length as a secondary regularizer. These choices preserve natural variation in query length while discouraging the generator from adding unnecessary text to lower the target declaration's rank.

H.2 Rank regions for the difficulty reward

We calibrate the rank regions to the retrieval depths used in deployment rather than to downstream benchmark performance. Our logs cover queries from human users and coding agents to deployed Lean search engines, each with a valid requested retrieval depth. Among these requests, 99.9% of human requests and 98.7% of coding-agent requests return at least five results. A target within the top five is therefore almost always exposed to the downstream user or agent. We treat such queries as already solved by the retriever and assign no difficulty reward for $k \leq 5$, directing generation toward queries that reveal unresolved retrieval gaps.

We set $k_{\text{sat}} = 30$ because 100% of human requests and 99.9% of coding-agent requests return at most 30 results. The logarithmic ramp from ranks 6 to 30 rewards increasingly challenging queries, while saturation beyond rank 30 treats all queries outside the effective deployment range equally. This plateau rewards the generator for exposing a meaningful retrieval gap without incentivizing it to push the target arbitrarily farther down the ranking by removing target information.

We set $k_{\text{cut}} = 100$ as a complementary retrieval-side safeguard. Queries that place the source declaration beyond rank 100 tend to omit target-identifying content so we assign zero difficulty reward for $k > 100$, keeping optimization focused on challenging queries that remain aligned to their target declarations.

Since each boundary changes the on-policy generator and consequently the retriever's training data, testing another setting requires retraining the complete generator-retriever pipeline. We therefore do not select these thresholds through a benchmark-driven hyperparameter sweep, as the sweep would cost significant GPU resources. The synthetic data produced by the generator trained with this fixed setting are already useful and show improvements across complementary model-side training (Table 1 and Table 2) and input-side prompt optimization (Table 4), without even performing careful tuning.

I Alignment-Gate Audit

We audit alignment-gate reliability on 100 query-declaration pairs picked from generator outputs across the two training rounds to reach a balanced negative and positive pair distribution for effective auditing (40 pairs are misaligned and 60 pairs are aligned). GPT-OSS-120B agrees with the human labels on 98/100 pairs and with Claude Opus 5 on 97/100 pairs. It incorrectly accepts 1/40 human-labeled misaligned pairs, corresponding to a 2.5% false-positive rate, and rejects 1/60 human-labeled aligned pairs, giving a 1.7% false-negative rate. These results demonstrate that false alignment approvals are rare while the gate preserves valid training queries.

J Query Rewriting Case Study

We consider a query whose target declaration is

`CategoryTheory.Adjunction.homEquiv_naturality_left`.

The original query states the desired equation but leaves the role of Φ implicit. Below are the complete query texts for one generation of each rewriting method.

Original (rank 7).

For $F \dashv G$, $\Phi(F.\text{map}\, f \gg g) = f \gg \Phi(g)$.

GEPA-optimized prompt rewriting (rank 1).

For an adjunction $F \dashv G$ and its hom-set equivalence Φ (Lean `Adjunction.homEquiv`), the equation $\Phi(F.\text{map}\, f \gg g) = f \gg \Phi(g)$ (left naturality of the adjunction hom-equivalence).

LeanSearch query rewriting (rank 8).

Adjunction Naturality (Left Triangle Identity): Given two functors F and G that form an adjunction with F left adjoint to G, and given the natural bijection between morphisms from F applied to an object to another object and morphisms from the original object to G applied to that other object, the natural bijection applied to the composition of F applied to a morphism followed by another morphism equals the composition of that first morphism followed by the natural bijection applied to the second morphism.

LeanDex query rewriting (rank 16).

The property $\Phi(F.\text{map}\, f \gg g) = f \gg \Phi(g)$ for an adjunction $F \dashv G$ relating natural transformations and functor maps within category theory.

Seed prompt query rewriting (rank 7)

Lemma that for an adjunction $F \dashv G$, $\Phi(F.\text{map}\, f \gg g) = f \gg \Phi(g)$.

Mathlib's theorem states precisely that the adjunction hom-set equivalence sends $F.\text{map}\, f \gg g$ to $f \gg \Phi(g)$. GEPA preserves this equation and its adjunction setting while supplying one useful clarification: Φ is `Adjunction.homEquiv`, and the requested relationship is its *left*

naturality. This follows the optimized prompt's principles to preserve mathematical intent, keep the exact relationship in focus, and use standard Lean terminology only when it directly denotes that relationship. GEPA introduces no additional assumptions or adjacent theorem.

The alternatives illustrate why more text is not necessarily better. LeanSearch augmentation replaces the exact equation with a lengthy paraphrase and incorrectly labels it a "Left Triangle Identity"; Mathlib's triangle identities instead concern the adjunction's unit and counit. LeanDex augmentation adds broad natural-transformation context without identifying the hom-set equivalence, shifting attention toward other adjunction declarations. The seed rewrite adds only "lemma that," leaving the missing context unresolved. GEPA's focused rewrite raises the target from rank 7 to rank 1, whereas these representative alternatives rank it 8, 16, and 7, respectively.

K PROMPTS

SEED PROMPT FOR QUERY REWRITING

```
Rewrite the user's search query for Mathlib4.

original query:  ...

OUTPUT FORMAT:
rewritten_query:  <the rewritten query>
```

OPTIMIZED PROMPT FOR QUERY REWRITING

```
You are an expert in mathematics, Lean 4, and theorem retrieval.

INSTRUCTION:
Rewrite the user's search query for mathematical/Lean theorem
retrieval.

REWRITING PRINCIPLES:
1.  Preserve the original mathematical intent, setting, assumptions,
and ambiguity.
2.  If the query already clearly names the object and relationship,
keep it nearly verbatim.
3.  Otherwise, expand terse terms only with standard synonyms or
equivalent formulations directly supported by the query, preferring
one concise clarification over stacked paraphrases or alternative
formulations.
...

original query:  ...

OUTPUT FORMAT:

rewritten_query:  <the rewritten query>
```

GEPA REFLECTION PROMPT

```
INPUT:
Ground-truth Lean statement:  ...
Description of the Lean statement:  ...


Original query:  ...
Original query ground-truth rank:  ...
```

```
Current rewritten query:  ...
Current rewritten query ground-truth rank:  ...


Seed-prompt rewritten query:  ...
Seed-prompt rewritten query ground truth rank:  ...


Optimization score:  ...

Retrieval outcome:  ...

...
```

OBJECTIVE:
Improve the reusable rewriting instruction ...

REFLECTION PRINCIPLES:
1. Use the provided examples to diagnose rewriting behavior, not to memorize their answers.

2. Compare the current, original, and seed outcomes when available. Consider both gains and regressions, distinguish observations from causal hypotheses, and propose a change only when its applicability follows from the query itself and is likely to transfer to unrelated topics.

3. Consider where a proposed change could hurt, and do not invent supporting evidence.

4. Remove inherited example-specific rules. Do not add topic or vocabulary lists, target names, copied formulas, worked examples, or query-to-target mappings.
...

QUERY GENERATOR PROMPT

```
You are a mathematician experienced in Lean4 helping stress-test
a semantic search engine over Mathlib4. Given the following
Mathlib4 statement, generate a single search query that is genuinely
relevant to the statement, while making the query adversarial to the
retriever so it struggles to rank this statement highly.
```

Guidelines:
- Relevance is mandatory: a mathematician reading the query and the statement must agree the statement is a correct answer.
- Make it something that a mathematician might actually write when searching for this statement.
- The query should not contain the formal name.
- The query should not be a near-verbatim rephrasing of the informal description.
- Keep it concise.

...

```
Lean4 Statement:  ...

Informal Description:  ...

Write your query wrapped in <search> and </search> tags.  Output
nothing else.
```