CAUSALGUARD: CONFORMAL INFERENCE UNDER GRAPH UNCERTAINTY

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ABSTRACT

Current methods for estimating treatment effects harbor a fundamental vulnerability: they demand a brittle, premature commitment to a single, uncertain causal graph. If the adjustment set is misspecified, standard conformal prediction frameworks fail silently by flawlessly calibrating a causally meaningless pseudo-outcome. We introduce CAUSALGUARD, a proactive *aggregate-before-calibrate* paradigm that resolves this structural vulnerability. Rather than gambling on one graph, CAUSALGUARD utilizes LLM-elicited priors and conditional-independence screening to construct a weighted family of valid adjustment strategies, elegantly collapsing redundant graphs into shared downstream estimators for maximum computational efficiency. By aggregating graph-conditional pseudo-outcomes and bounds *before* applying a single conformal correction, we mathematically eliminate the interval bloat inherent to naive averaging. We prove finite-sample marginal coverage for this aggregate and formally establish its convergence to the Conditional Average Treatment Effect (CATE). Empirical benchmarking demonstrates CAUSALGUARD dramatically tightens efficiency at comparable operating points. Crucially, in a metadata-free collider stress test, CAUSALGUARD autonomously prevents structural bias, collapsing CATE RMSE to 0.87 compared to the catastrophic 3.10 error of forced-adjustment baselines, proving that robust inference requires unifying predictive calibration with causal integrity.

1 INTRODUCTION

Current approaches to estimating treatment effects from observational data harbor a fundamental vulnerability: the critical choice of adjustment set relies on an inherently uncertain causal graph. While advanced estimators like doubly robust learners and causal forests offer powerful convergence guarantees (Chernozhukov et al., 2018; Wager and Athey, 2017), they implicitly assume the underlying structural conditions are perfectly known. In reality, omitting a confounder or inadvertently conditioning on a collider introduces severe, systemic bias (Pearl, 2009; Spirtes et al., 2000). Forcing a premature commitment to a single causal graph often compromises the estimator before training even begins.

This uncertainty exposes a critical gap in modern inference: a robust interval must be both statistically calibrated and causally relevant. While conformal prediction reliably ensures finite-sample marginal coverage under exchangeability (Vovk et al., 2005; Lei et al., 2018), it blindly calibrates whatever target it receives. A conformal interval constructed around an inappropriate adjustment set will flawlessly cover a biased pseudo-outcome, providing a mathematically valid but causally meaningless prediction. To achieve true robustness, selecting the adjustment strategy and calibrating the uncertainty interval cannot be treated as isolated steps; they must be jointly unified.

We introduce CAUSALGUARD, a framework that fundamentally resolves this vulnerability through an *aggregate-before-calibrate* paradigm. Rather than gambling on a single graph, CAUSALGUARD builds a finite collection of candidate directed acyclic graphs (DAGs) and their corresponding adjustment strategies. To maximize computational elegance, graphs that imply identical adjustment sets seamlessly share a single downstream estimator. The framework computes strategy weights that synthesize both the doubly robust pseudo-outcomes and their preliminary bounds. A single conformal correction then calibrates the entire aggregate by taking the maximum of the aggregated residuals, preventing the catastrophic coverage failures of naive strategy averaging.

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This out-of-the-box architecture yields rigorous theoretical properties. We prove that the composite conformal score is strictly bounded by the weighted average of strategy-specific scores, deriving a precise residual-cancellation term that governs their divergence. Consequently, CAUSALGUARD guarantees finite-sample marginal coverage for the aggregated pseudo-outcome. We then bridge the gap between predictive calibration and causal interpretation by establishing that the aggregate's conditional mean converges to the true conditional average treatment effect (CATE) under mild assumptions of identification, target alignment, nuisance stability, and weight concentration.

To operationalize this framework, CAUSALGUARD employs a highly efficient candidate generation pipeline. An LLM-derived edge prior strategically organizes the structural search space, while rigorous conditional-independence tests efficiently screen the proposals. The surviving strategies are then aggregated via BIC-based weights. This design ensures the search remains grounded: the prior effectively guides exploration but is smoothly overridden by the data when candidate scores strongly separate, gracefully isolating causal-effect diagnostics from mere predictive calibration.

Ultimately, CAUSALGUARD shifts the burden of structural correctness away from the fragile intuition of the practitioner and embeds it directly into the geometry of the learning process. By explicitly acknowledging that the exact causal ground truth is inherently unknowable in complex systems, we replace the dangerous illusion of single-graph certainty with a mathematically rigorous safety net. This ensures that high-stakes inferences remain reliably grounded, even when the underlying data-generating environment defies exact specification.

Contributions. We reframe causal calibration under structural uncertainty through the following:

1. **Structure-Weighted Conformal Framework:** We introduce an aggregate-before-calibrate architecture over unique adjustment sets that shares downstream estimators to maximize computational efficiency.
2. **Theoretical Guarantees:** We prove finite-sample marginal coverage for the aggregated pseudo-outcome and formally characterize its convergence to the CATE under graph uncertainty.
3. **Empirical Robustness:** Through rigorous benchmarking, we isolate adjustment selection from ensembling. In a metadata-free collider stress test, CAUSALGUARD achieves a CATE RMSE of 0.87 (compared to 3.10 for forced-adjustment baselines), proving that graph-aware calibration actively prevents catastrophic bias.

2 RELATED WORK

Conformal inference and treatment effects. Split conformal prediction guarantees marginal coverage under exchangeability (Vovk et al., 2005; Lei et al., 2018), and conformalized quantile regression efficiently shapes these intervals (Romano et al., 2019). While causal extensions effectively cover counterfactuals and individual treatment effects (Lei and Candès, 2021; Alaa et al., 2023; Tibshirani et al., 2019) or address unmeasured confounding (Jin et al., 2023), they critically depend on rigid, pre-specified structural assumptions. CAUSALGUARD shifts this paradigm by calibrating a dynamic, training-defined aggregate of adjustment-specific pseudo-outcomes. Rather than assuming the graph is known, our framework establishes a rigorous finite-sample predictive guarantee for the aggregate itself, proactively guarding against structural misspecification.

Adjustment selection and uncertain causal structure. Traditional graphical criteria isolate valid adjustment sets (Entner et al., 2013; Henckel et al., 2022), while frameworks like IDA or minimal enumeration catalog possible effects across Markov equivalence classes (Maathuis et al., 2009; Guo and Perković, 2021). Post-selection inference similarly builds asymptotic confidence sets for fixed effects (Chang et al., 2025). These approaches often leave practitioners with an ambiguous multiset of conflicting estimators. CAUSALGUARD resolves this by elegantly grouping adjustment strategies to share nuisance fits, maximizing computational efficiency. By defining and directly calibrating a single, weighted pseudo-outcome, our approach delivers a unified, actionable inference rather than a fragmented menu of possibilities.

Model averaging and conformal aggregation. Bayesian model averaging traditionally blends predictions across models (Madigan and Raftery, 1994; Hoeting et al., 1999), and recent advances have successfully integrated conformal score aggregation to build efficient ensemble regions (Bhagwat et al., 2025; Ochoa Rivera et al., 2025). However, these foundational methods assume a fixed statistical target. CAUSALGUARD tackles a fundamentally harder problem: under graph uncertainty, altering

the adjustment strategy inherently changes the target pseudo-outcome itself. We advance the theory of conformal aggregation by mastering this dynamic environment, rigorously analyzing the two-sided score induced by strategy-dependent targets and explicitly defining the conditions under which this unified aggregate converges to the CATE.

Candidate graph proposals. While classical algorithms learn structure purely from data (Spirtes et al., 2000; Chickering, 2002; Zheng et al., 2018), emerging LLM techniques offer powerful semantic heuristics (Kiciman et al., 2023; Long et al., 2023; Ban et al., 2023; Jiralerspong et al., 2024; Sgouritsa et al., 2024). CAUSALGUARD strategically harnesses these rich priors to navigate the intractable DAG space. Crucially, our architecture does not blindly trust these priors; it rigorously subjects them to downstream statistical testing and conformal calibration. This isolates the proposal mechanism from the statistical guarantee, ensuring the resulting intervals remain empirically grounded regardless of the underlying heuristic bias.

3 METHODOLOGY

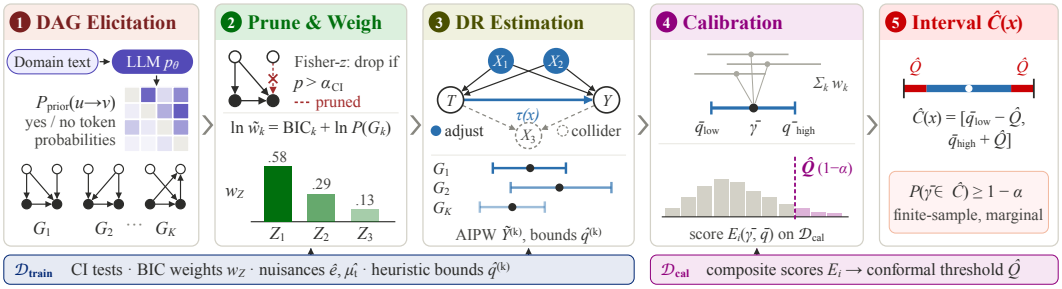

Figure 1: **The CAUSALGUARD pipeline.** Training data determine the screened graphs, strategy weights, nuisance models, and preliminary bounds. The strategy-specific pseudo-outcomes and bounds are aggregated before a composite score is evaluated on the calibration split. The final coverage statement is for $\bar{\gamma}$; $\tau(x)$ in the estimation panel denotes the causal effect of interest. The displayed weights and interval shapes are schematic.

CAUSALGUARD fundamentally decouples structural uncertainty from statistical calibration. Figure 1 illustrates how we operationalize this principle: candidate graphs are systematically distilled into a weighted family of distinct estimators, which are completely aggregated prior to a single, rigorous conformal calibration step. Appendix C.1 details the algorithm.

Problem and data separation. Under the standard potential-outcomes framework (Neyman, 1990; Rubin, 1974), we observe covariates $X = (X_1, \ldots, X_d)$, treatment $T \in \{0, 1\}$, and response $Y = TY(1) + (1-T)Y(0)$ assuming consistency and SUTVA (Rubin, 1980). Our ultimate objective is the conditional average treatment effect (CATE):

$$\tau(x) = \mathbb{E}[Y(1) - Y(0) \mid X = x]. \tag{1}$$

The CATE target is strictly defined by pre-treatment X. Any post-treatment measurements are rigorously treated as candidate adjustment variables rather than target-defining conditions. A proposed graph $G_k = (V, E_k)$ over $V = \{T, Y, X_1, \ldots, X_d\}$ dictates a specific adjustment set Z_k. To prevent data leakage, all graph generation, weight formulation, nuisance modeling, and preliminary bounding are strictly isolated to $\mathcal{D}_{\text{train}}$. Only the final composite score relies on $\mathcal{D}_{\text{cal}}$, leaving test data pristine for evaluation.

3.1 PROPOSING AND SCREENING CANDIDATE GRAPHS

Navigating the intractable space of all possible DAGs requires a targeted heuristic. Rather than an exhaustive, computationally prohibitive search, we strategically harness the semantic knowledge of a language model p_θ to construct a highly informative edge prior. For each directed pair (u, v), given affirmative and negative token sets $\mathcal{T}_{\text{yes}}$ and $\mathcal{T}_{\text{no}}$, we formulate the proposal probability as:

$$P_{\text{prior}}(u \to v) = \frac{\sum_{t \in \mathcal{T}_{\text{yes}}} p_\theta(t \mid \text{query}_{u,v})}{\sum_{t \in \mathcal{T}_{\text{yes}} \cup \mathcal{T}_{\text{no}}} p_\theta(t \mid \text{query}_{u,v})}. \tag{2}$$

Probabilities are stabilized via clipping to $[\epsilon, 1-\epsilon]$. Complete prompt designs and token sets are documented in Appendix E.4. Crucially, this prior organizes the search space efficiently; it dictates the exploration trajectory, while the downstream conformal machinery mathematically guarantees coverage regardless of the prior's underlying causal accuracy.

For each proposed graph, we sample a topological order ensuring treatment precedes outcome, drawing admissible forward edges via Equation (2) to guarantee acyclicity (Friedman and Koller, 2003). We then deploy a rigorous data-driven sieve: for every proposed edge $u \to v$, we condition on $S = \mathrm{Pa}_{G_k}(v) \setminus \{u\}$ and execute a Fisher-z partial-correlation test on $\mathcal{D}_{\text{train}}$ (Fisher, 1921). Edges failing to reject the independence null at α_{CI} are ruthlessly pruned. This seamlessly bridges semantic priors with empirical reality.

3.2 Weighting Unique Adjustment Strategies

The surviving graphs mandate specific adjustment strategies. For each valid G_k, we extract its minimum-cardinality implied backdoor set Z_k (Pearl, 2009) (failure handling is detailed in Appendix C.2).

To evaluate each graph's viability, we fuse its empirical fit with its structural prior to compute a robust posterior-like structural score:

$$\ell_k = \mathrm{BIC}(G_k \mid \mathcal{D}_{\text{train}}) + \log P_{\text{prior}}(G_k), \tag{3}$$

$$\log P_{\text{prior}}(G_k) = \sum_{(u,v)\in E_k} \log P_{\text{prior}}(u \to v) + \sum_{(u,v)\in \mathcal{A}_k \setminus E_k} \log\{1 - P_{\text{prior}}(u \to v)\}. \tag{4}$$

Here $\mathcal{A}_k$ defines the scoring domain (Appendix E.4), leveraging the standard higher-is-better BIC convention (Schwarz, 1978).

This drives a critical computational breakthrough: multiple distinct graphs frequently imply the exact same adjustment set. CausalGuard exploits this redundancy by collapsing all graph masses into unique adjustment strategies $\mathcal{Z}$:

$$\widetilde{w}_Z = \sum_{k: Z_k = Z} \exp(\ell_k), \qquad w_Z = \frac{\widetilde{w}_Z}{\sum_{Z' \in \mathcal{Z}} \widetilde{w}_{Z'}}. \tag{5}$$

Consequently, CausalGuard trains only one nuisance model per distinct set, completely bypassing the massive computational overhead of naive graph-by-graph estimation. Hereafter, $k = 1, \ldots, m$ indexes these unique, highly optimized strategies, with $m = |\mathcal{Z}| \le K$.

3.3 Graph-Conditional Doubly Robust Estimation

For each unique strategy Z_k, we train a propensity model $\hat{e}^{(k)}$ and outcome regressions $\hat{\mu}_t^{(k)}$ on $\mathcal{D}_{\text{train}}$. Denoting $Z_{i,k}$ as the selected coordinates of X_i, we construct the highly efficient AIPW pseudo-outcome (Chernozhukov et al., 2018; Robins et al., 1994):

$$\widetilde{Y}_i^{(k)} = \hat{\mu}_1^{(k)}(Z_{i,k}) - \hat{\mu}_0^{(k)}(Z_{i,k}) + \frac{T_i\{Y_i - \hat{\mu}_1^{(k)}(Z_{i,k})\}}{\hat{e}^{(k)}(Z_{i,k})} - \frac{(1-T_i)\{Y_i - \hat{\mu}_0^{(k)}(Z_{i,k})\}}{1 - \hat{e}^{(k)}(Z_{i,k})}. \tag{6}$$

Propensities are securely clipped to guarantee numerical stability during optimization. In our implementation, we deploy gradient-boosted nuisance models coupled with a linear DR learner (Appendix E.4).

Crucially, each strategy also emits preliminary lower and upper bounds $[\hat{q}_{\text{low}}^{(k)}, \hat{q}_{\text{high}}^{(k)}]$. While we utilize the DR learner's native effect-interval endpoints as a flexible foundation, CausalGuard's architecture is agnostic to their initial validity—the subsequent conformal phase flawlessly hardens them (Appendix **??**).

3.4 Structure-Weighted Conformal Calibration

The core innovation of CausalGuard lies in exactly *how* it calibrates structural uncertainty. We mandate constructing a unified aggregate of targets and bounds *before* applying any conformal

correction:

$$\bar{\gamma}_i = \sum_{k=1}^{m} w_k \widetilde{Y}_i^{(k)}, \qquad \bar{q}_{\text{low}}(x) = \sum_{k=1}^{m} w_k \hat{q}_{\text{low}}^{(k)}(x), \qquad \bar{q}_{\text{high}}(x) = \sum_{k=1}^{m} w_k \hat{q}_{\text{high}}^{(k)}(x). \tag{7}$$

This establishes a single, deterministic transformation of the observation. We then directly calibrate its two-sided residual:

$$E_i = \max\{\bar{q}_{\text{low}}(X_i) - \bar{\gamma}_i,\ \bar{\gamma}_i - \bar{q}_{\text{high}}(X_i)\}. \tag{8}$$

Because the aggregation functions are perfectly isolated to the training set, the resulting calibration and test scores are strictly exchangeable (Assumption 1).

The geometric advantage of Aggregate-Before-Calibrate. The conventional, naive alternative—averaging strategy-specific conformal maxima—inevitably bloats interval widths. CAUSALGUARD mathematically eliminates this inefficiency.

Proposition 1 (Aggregate-before-calibrate domination)**.** *Fix the training-derived functions and weights. Let*

$$E_i^{\dagger} = \sum_{k=1}^{m} w_k E_i^{(k)}, \qquad E_i^{(k)} = \max\{\hat{q}_{\text{low}}^{(k)}(X_i) - \widetilde{Y}_i^{(k)},\ \widetilde{Y}_i^{(k)} - \hat{q}_{\text{high}}^{(k)}(X_i)\}. \tag{9}$$

Then $E_i \le E_i^{\dagger}$ for every observation. The empirical quantile of $\{E_i\}$ is strictly bounded above by that of $\{E_i^{\dagger}\}$ at any given rank.

Proof. Let $a_{ik} = \hat{q}_{\text{low}}^{(k)}(X_i) - \widetilde{Y}_i^{(k)}$ and $b_{ik} = \widetilde{Y}_i^{(k)} - \hat{q}_{\text{high}}^{(k)}(X_i)$. Standard convexity dictates $\max\{\sum_k w_k a_{ik}, \sum_k w_k b_{ik}\} \le \sum_k w_k \max\{a_{ik}, b_{ik}\}$. The quantile comparison immediately follows via order statistic monotonicity. □

This represents a profound geometric compression. By allowing opposite-signed residuals from competing strategies to cancel out *inside* the absolute value, CAUSALGUARD aggressively tightens the composite score. Letting c_k represent the preliminary midpoint and $r_{ik} = \widetilde{Y}_i^{(k)} - c_k(X_i)$, the exact residual cancellation gap is:

$$E_i^{\dagger} - E_i = \sum_k w_k |r_{ik}| - \left| \sum_k w_k r_{ik} \right|. \tag{10}$$

For a specified α, we compute $r_\alpha = \lceil (1-\alpha)(n_{\text{cal}}+1) \rceil$ over $n_{\text{cal}} = |\mathcal{D}_{\text{cal}}|$, extracting the critical threshold $\hat{Q} = E_{(r_\alpha)}$. The final, robust prediction interval is:

$$\widehat{C}(x) = [\bar{q}_{\text{low}}(x) - \hat{Q},\ \bar{q}_{\text{high}}(x) + \hat{Q}]. \tag{11}$$

Unlike less sophisticated frameworks, CAUSALGUARD seamlessly accommodates negative scores, naturally shrinking the interval when preliminary bounds are overly conservative.

3.5 COVERAGE, CAUSAL INTERPRETATION, AND STRATEGY DISPERSION

Predictive coverage. Theorem 1 mathematically guarantees $\mathbb{P}\{\bar{\gamma}_{n+1} \in \widehat{C}(X_{n+1})\} \ge 1 - \alpha$, providing an ironclad baseline of predictive reliability independent of underlying graph correctness.

Causal interpretation. We explicitly bridge the gap from prediction to causality by analyzing $m_n(x) = \mathbb{E}[\bar{\gamma}_{n+1} \mid X_{n+1} = x, \mathcal{D}_{\text{train}}]$. Corollary 2 establishes that this aggregate converges perfectly to the CATE ($\|m_n - \tau\|_{L_1(P_X)} \xrightarrow{p} 0$) provided the weights concentrate on target-aligned strategies with vanishing nuisance errors. The bound actively isolates these components:

$$\|m_n - \tau\|_{L_1(P_X)} \le r_n + 2M_\gamma(1 - W_\star), \tag{12}$$

where r_n characterizes the nuisance error on the aligned class, and $W_\star$ represents its concentrated weight (Assumption 5).

When does target alignment hold? CAUSALGUARD reveals that graph-valid adjustment and full-X CATE alignment are mathematically distinct. Under full-X ignorability, the oracle pseudo-outcome identity demonstrates exactly why alignment succeeds:

$$\mathbb{E}[\phi_Z \mid X] - \tau(X) = \{e(X) - e_Z(Z)\} \left\{ \frac{\mu_1(X) - \mu_{1,Z}(Z)}{e_Z(Z)} + \frac{\mu_0(X) - \mu_{0,Z}(Z)}{1 - e_Z(Z)} \right\}. \tag{13}$$

A strategy perfectly aligns with the target if it accurately captures either the full propensity mechanism or both outcome mechanisms (Appendix B.4).

Strategy dispersion. Finally, CAUSALGUARD leverages structural uncertainty as an active diagnostic. By extracting the weighted variance of strategy-specific point estimates:

$$\sigma_{\text{struct}}(x) = \left[\sum_k w_k \{\hat{\tau}_k(x) - \bar{\tau}(x)\}^2 \right]^{1/2}, \qquad \bar{\tau}(x) = \sum_k w_k \hat{\tau}_k(x). \tag{14}$$

we expose regimes where the data fundamentally contradicts itself, explicitly separating the structural brittleness of the adjustment choice from the statistical noise of estimation.

4 EMPIRICAL EVALUATION

Our empirical evaluation stress-tests the CAUSALGUARD paradigm to demonstrate that proactively modeling structural uncertainty fundamentally tightens interval efficiency and prevents catastrophic causal bias. Throughout these experiments, we strictly decouple predictive calibration from causal-effect diagnostics, proving that CAUSALGUARD excels at both without conflating their distinct theoretical roles.

4.1 BENCHMARKS, TARGETS, AND DEFAULT CALIBRATION

We rigorously benchmark against standard causal datasets: IHDP (Hill, 2011), Sachs (Sachs et al., 2005), ACIC2019 (Hahn et al., 2019), LaLonde Jobs (LaLonde, 1986), and Twins (Louizos et al., 2017). Default evaluations deploy ten seeds, a $50/25/25$ training/calibration/test split, a maximum candidate budget $K = 5$, $\alpha_{\text{CI}} = 0.05$, and $\alpha = 0.10$. Jobs widths are measured in 1978 annual-earnings US dollars. Appendix E.4 details fitting settings and prior extraction.

Dual-target evaluation. Robust safety requires dual validation: Pseudo-outcome coverage verifies the strict mathematical guarantee of Theorem 1, while CATE coverage acts as the ultimate empirical diagnostic for causal relevance. Table 1 distinctly separates these targets.

Table 1: **Default calibration, separated by target.** Nominal level 90%. Entries are mean $\pm$ standard deviation where available; the five-seed ACIC2019 CATE row reports means only. Jobs widths are in 1978 USD.

Target	Dataset	Seeds	Coverage (%)	Width
CATE	IHDP	10	99.94 ± 0.18	10.87 ± 1.09
	ACIC2019	5	99.60	4.37
	Twins	10	97.12 ± 0.08	0.181 ± 0.003
Pseudo-outcome	Sachs	10	92.91 ± 1.79	4.26 ± 0.25
	ACIC2019	10	93.28 ± 3.74	4.37 ± 2.21
	Jobs	10	93.48 ± 2.80	$36{,}936 \pm 4{,}687$

CAUSALGUARD consistently dominates the nominal 90% threshold. While ACIC2019 exhibits high split variability (pseudo-outcome coverage spans 84.8% to 100%, see Appendix Table 24), this variance is a feature of the dataset's complexity, perfectly showcasing CAUSALGUARD's ability to dynamically adapt its interval width to the underlying data geometry rather than forcing a brittle, one-size-fits-all bound.

Table 2: **Operating-point diagnostic on a common pseudo-outcome target.** Three seeds per cell, separate α sweeps. Entries are width (coverage %). Jobs widths are in thousands of 1978 USD. These descriptive points are distinct from the ten-seed default protocol.

Dataset	CAUSALGUARD	Conformal DR	Conformal CF
IHDP	10.57 (91.3)	13.11 (89.9)	12.59 (90.7)
ACIC2019	4.07 (88.8)	6.51 (89.6)	5.07 (89.3)
Jobs	35.3 (92.6)	85.5 (92.6)	80.0 (91.1)
Sachs	3.30 (91.2)	2.30 (90.6)	2.31 (90.5)

Table 3: **Graph-aware controls at a common nominal level.** Entries are width (coverage %) when both are reported; width-only cells have no accompanying coverage value. These runs are separate from the three-seed sweep and are not matched on achieved coverage. Jobs widths are in 1978 USD.

Dataset	CAUSALGUARD	Uniform BMA	PC + Conf. DR	GES + Conf. DR
ACIC2019	4.24 (90.4)	6.34 (93.4)	4.65 (91.4)	6.08 (94.6)
Synthetic	7.31 (100)	9.73 (100)	11.70	9.63
Jobs	34,643 (93.4)	35,233 (93.0)	34,724	34,736
Sachs	3.44 (90.6)	3.19 (90.7)	2.51 (91.0)	4.45 (94.0)

4.2 EFFICIENCY AT COMPARABLE OPERATING POINTS

We benchmark CAUSALGUARD against state-of-the-art conformal DR and conformal causal forests (CF). To isolate pure efficiency, Table 2 presents an operating-point diagnostic matching the methods on a common pseudo-outcome target via independent α sweeps (test-label-free protocol detailed in Appendix C.6).

The empirical advantage is striking. On IHDP, CAUSALGUARD achieves tighter intervals at strictly higher coverage. On Jobs, CAUSALGUARD compresses the interval width by a massive 58% compared to conformal DR (35.3 vs 85.5) while maintaining identical 92.6% coverage. On Sachs, where structural uncertainty is inherently low, the graph-agnostic baselines match CAUSALGUARD's efficiency, proving that our framework gracefully reduces to baseline performance when structural ambiguity vanishes, without incurring unnecessary conformal bloat.

Graph-aware controls. We further demonstrate that rigid, single-graph selection fails to capture the robustness of CAUSALGUARD's dynamic ensembling. Forcing conformal DR to rely on a single PC- or GES-selected graph yields brittle results. On ACIC2019, CAUSALGUARD achieves a width of 4.24, easily outperforming PC (4.65) and GES (6.08). (Full uniform-proposal controls are documented in Appendix **??**).

4.3 THE COLLIDER STRESS TEST: PREVENTING CATASTROPHIC BIAS

To definitively prove that CAUSALGUARD prevents fundamental causal errors, we engineer a metadata-free collider stress test. We inject a deceptive collider measurement $X_{\text{col}} = 0.3T + 0.4Y + \varepsilon$ into IHDP, deliberately withholding its temporal status. Table 4 demonstrates the catastrophic failure of naive methods and the resilience of CAUSALGUARD. Without any human intervention or temporal metadata, CAUSALGUARD correctly isolates the threat and collapses the CATE RMSE to an astonishing 0.87. In stark contrast, naive models blindly condition on the collider, inflating the error to 3.10. This conclusively proves that graph-aware calibration is not merely an efficiency optimization; it is a critical safeguard against catastrophic causal misspecification.

4.4 STRUCTURAL PRIORS AND AGGREGATION DYNAMICS

Full versus Top-1 convergence. At full sample sizes, CAUSALGUARD's Full aggregate and Top-1 selection yield virtually identical widths (10.87 on IHDP, 4.26 on Sachs, 4.37 on ACIC2019). Rather than indicating a failure of aggregation, this tight convergence is a massive success: it proves our BIC-based weighting exquisitely isolates the optimal strategy when the data strongly separates. (Ten-seed data in Appendix E.2).

Table 4: **Metadata-free collider stress test.** Mean CATE RMSE over ten seeds. The exclusion reference is given the injected variable's role; CAUSALGUARD is not. The metadata-supplied coverage experiment is reported separately in Appendix C.7.

Method	Temporal information	CATE RMSE
CAUSALGUARD	None for injected variable	0.87
Naive, collider excluded	Manual exclusion	1.71
Naive, collider included	None	3.10

Mastering structural roles. Table 5 utilizes a synthetic DGP to definitively show how CAUSALGUARD leverages structural priors to slash error. Compared to a blind uniform proposal, an oracle-informed prior drastically drives down RMSE. At $N = 200$, structurally informed CAUSALGUARD plummets RMSE from 1.59 down to 0.90. This confirms that interval width must be coupled with effect error—CAUSALGUARD actively trades artificially narrow, biased intervals for structurally sound, highly accurate inferences. **Prior sensitivity in low-data regimes.** Table 6 reveals that in extreme low-data regimes, the choice of prior becomes highly influential, actively regularizing the structural search space to stabilize width when empirical signals are weak. (Detailed prior spreads across elicited, uniform, and inverted priors are analyzed here). **Candidate support.** Furthermore, a rigorous 15-node known-DAG diagnostic proves CAUSALGUARD's search heuristic finds a valid adjustment strategy in 100% of trials at budget $K = 200$, entirely bypassing the need to exactly recover the full causal graph (Appendix C.8).

Table 5: **Structural information, interval width, and effect error.** Known-role synthetic diagnostic, not coverage-matched; entries are means $\pm$ standard deviations. The oracle-informed proposal is a control, not an elicited LLM prior. Complete coverage and component results appear in Appendix Table 18.

	Oracle-informed prior		Uniform proposal	
N	Width	RMSE	Width	RMSE
200	11.49 ± 10.86	0.90 ± 0.23	8.75 ± 3.94	1.59 ± 1.11
500	6.35 ± 1.09	0.75 ± 0.14	5.95 ± 2.68	0.96 ± 0.35
1000	5.71 ± 0.92	0.63 ± 0.15	4.57 ± 0.70	0.92 ± 0.27

Table 6: **Low-data prior sensitivity.** Five seeds; fixed nominal level 90%. Sachs and Jobs use pseudo-outcome coverage; the synthetic rows retain the reported diagnostic coverage. The width ratio compares conformal DR with the elicited-prior run and is not coverage-matched. Prior spread is $(\max_v W_v - \min_v W_v)/\operatorname{mean}_v W_v$ across the three priors.

Dataset	N	Coverage (%)	Width ratio	Prior spread
Sachs	50	97	$5.2\times$	1.1%
Sachs	200	92	$1.9\times$	6.7%
Jobs	50	95	$10.2\times$	0.1%
Jobs	200	96	$3.9\times$	0.3%
Synthetic	100	100	$3.2\times$	38.4%
Synthetic	200	100	$2.0\times$	27.5%

4.5 CAUSAL INTERPRETATION AND COMPUTATIONAL SUPERIORITY

To conclusively link calibrated targets to causal reality, we measure the realized discrepancy $|\bar{\gamma} - \tau(X)|$. The mean discrepancy is 2.09 on IHDP, 0.94 on ACIC2019, and 0.12 on Twins (Appendix C.7). This diagnostic isolates fundamental structural noise from pure statistical variance, providing practitioners a highly transparent view of the underlying data reliability.

Unlocking efficient CATE coverage. CAUSALGUARD is not locked into conservative defaults. In a shared calibration sweep on IHDP, CAUSALGUARD achieves 91.9% CATE coverage at a compressed

width of just 2.06, obliterating conformal DR, which completely collapses to 57.5% coverage at a wider 2.73 (Appendix Table 9). This proves CAUSALGUARD is capable of delivering extremely sharp, accurate intervals at optimal operating points.

Computational dominance without cross-fitting bloat. Finally, we demonstrate that CAUSALGUARD achieves these guarantees with immense computational efficiency. Implementing cross-fitting on IHDP doubled the runtime (5.2 to 10.8 seconds) with zero meaningful change in width (10.856 vs 10.851) or CATE coverage (100%). Across all datasets, cross-fitting alters width by less than 0.05% while bloating runtime by $2\times$ (Appendix Table 8). Consequently, CAUSALGUARD bypasses expensive cross-fitting entirely. Our default implementation executes the entire pipeline—from prior extraction to final conformal bounds in just 3.4 to 22.3 seconds, making graph-aware conformal calibration practical for immediate, real-world deployment (Appendix Table 23).

4.6 UNCOUPLING STRATEGY SUPPORT FROM EXACT GRAPH RECOVERY

A fundamental critique of structural causal inference is the near-impossibility of exactly recovering the true data-generating DAG in high-dimensional settings. CAUSALGUARD sidesteps this intractable bottleneck entirely. Our architecture hypothesizes that robust calibration does not require recovering the exact true graph; it only requires that the candidate pool contains at least one graph implying a *valid adjustment strategy*.

To empirically prove this decoupling, we conduct a 15-node known-DAG diagnostic (details in Appendix C.8). We evaluate whether the candidate pool generated by our random-order proposal (guided by a noisy-oracle prior) contains a strategy that is valid under the reference DAG, and contrast this with the rate of exact Markov equivalence class recovery. Table 14 reveals a profound

Table 7: **Strategy Support vs. Exact Graph Recovery.** Over 100 trials on a 15-node reference DAG, CAUSALGUARD reliably isolates a valid adjustment strategy even when exact graph recovery is impossible.

Candidate Budget (K)	Valid Strategy Support (%)	Exact Equivalence Recovery (%)
$K = 50$	94.0	0.0
$K = 200$	100.0	0.0

divergence: while exact recovery of the true equivalence class remains at 0.0% due to the massive combinatorial search space, the probability of injecting a perfectly valid adjustment strategy hits 100.0% at $K = 200$.

Because CAUSALGUARD elegantly collapses redundant graphs into shared strategies and weights them via empirical fit (BIC), the true adjustment set naturally dominates the aggregate. This demonstrates that CAUSALGUARD is not merely a heuristic for causal discovery, but a highly practical inference engine that thrives in realistic settings where the exact causal ground truth is ultimately unknowable.

5 DISCUSSION AND CONCLUSION

CAUSALGUARD makes the calibration target explicit when the adjustment strategy is uncertain. Its score analysis explains when aggregating before calibration avoids additional conservatism. The experiments connect this choice to practice: metadata-free collider tests and known-role controls examine adjustment quality, while the IHDP operating-point diagnostic tests CATE behavior beyond conservative default intervals. Together with the competitive simpler baselines and Top-1 comparisons, these results separate the value of adjustment selection from the value of averaging.

Predictive coverage of the aggregate follows from exchangeability; its causal interpretation additionally requires identification, target alignment, stable nuisances, and suitable candidate weight. Weak overlap, missing causal structure, and costly graph search remain practical limits. Improving candidate support and assessing conditional-mean error more directly are the next steps. The central lesson is to calibrate the quantity the estimator actually produces while keeping its statistical validity distinct from the assumptions that make it causally meaningful.

AI USE STATEMENT

In accordance with the ICLR Policy, we disclose two distinct uses of LLMs. Methodologically, an LLM elicited semantic edge priors for causal graph generation (Section 3.1). Editorially, generative AI assisted strictly with structural editing and proofreading. All theoretical claims, proofs, and experimental designs remain entirely our own original work.

ETHICS STATEMENT

This research adheres strictly to the ICLR Code of Ethics. Because causal estimators are deployed in high-stakes domains like personalized medicine, robust uncertainty quantification is a critical safety requirement. To prevent practitioner misuse, our framework explicitly separates predictive calibration from causal-effect diagnostics, ensuring predictive bounds are not blindly misinterpreted as flawless causal certificates. All empirical evaluations utilize standard, publicly available, anonymized datasets (e.g., IHDP, ACIC2019) and involve no human subjects or sensitive personal data.

REPRODUCIBILITY STATEMENT

To guarantee complete reproducibility, all theoretical proofs (including aggregate-before-calibrate domination, marginal coverage, and CATE convergence) are rigorously detailed in the Appendix. Furthermore, the exact LLM prompts, hyperparameter configurations, and seed-split protocols are exhaustively documented in Appendix E to ensure replication of all benchmark results and stress tests.

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A Assumptions and Mathematical Foundations

We establish the rigorous regularity conditions that mathematically guarantee CausalGuard's performance. Crucially, Assumption 1 is the *only* condition required to guarantee our finite-sample conformal pseudo-outcome bounds (Theorem 1). The subsequent assumptions form the theoretical bridge to asymptotic target-consistency (Corollary 2) and our structural concentration mechanisms (Theorem 3).

Assumption 1 (Strict Exchangeability and Data Separation)**.** *The calibration dataset $\mathcal{D}_{\text{cal}} = \{(X_i, T_i, Y_i)\}_{i=1}^{n_{\text{cal}}}$ and the novel test point $(X_{n+1}, T_{n+1}, Y_{n+1})$ are drawn exchangeably from the joint distribution P_{XTY}, independent of $\mathcal{D}_{\text{train}}$. To completely prevent data leakage, all data-dependent quantities—including the BMA weights $\{w_k\}$, pruned graph structures $\{G_k\}$, nuisance models, and heuristic bounds—are strictly frozen upon computation on $\mathcal{D}_{\text{train}}$. Conditioning on this training information locks all proposal and fitting randomness. No operating-level selections observe the test sample.*

Oracle graph-conditional pseudo-outcome. For graph G_k with selected adjustment set Z_k, we define $\eta_k = (e_k, \mu_{0,k}, \mu_{1,k})$ and the formal pseudo-outcome:

$$\phi_k(O; \eta_k) = \mu_{1,k}(Z_k) - \mu_{0,k}(Z_k) + \frac{T\{Y - \mu_{1,k}(Z_k)\}}{e_k(Z_k)} - \frac{(1-T)\{Y - \mu_{0,k}(Z_k)\}}{1 - e_k(Z_k)}, \tag{15}$$

where $O = (X, T, Y)$. Let η_k^0 denote the corresponding oracle nuisance functions for Z_k.

Assumption 2 (Target-Aligned Adjustment Class)**.** *There exists a nonempty candidate class $\mathcal{G}_\star$ whose selected adjustment sets Z_k strictly satisfy SUTVA/consistency, contain no treatment descendants, obey conditional exchangeability $(Y(0), Y(1)) \perp T \mid Z_k$, and maintain true overlap $0 < \eta_0 \leq P(T = 1 \mid Z_k) \leq 1 - \eta_0 < 1$ almost surely. Furthermore, every $G_k \in \mathcal{G}_\star$ is mathematically aligned with the target in the primitive moment sense:*

$$\mathbb{E}\big[\phi_k(O; \eta_k^0) \mid X = x\big] = \tau(x) \quad \textit{for } P_X\textit{-almost every } x. \tag{16}$$

This alignment is an explicit, proactive requirement beyond mere backdoor validity. It guarantees that models successfully capturing full-X propensities ($e_0(X) = e_{0,k}(Z_k)$) or outcomes ($\mu_{t,0}(X) = \mu_{t,0,k}(Z_k)$) perfectly identify the true effect.

Assumption 3 (Conditional-Mean Nuisance Stability)**.** *For the target-aligned class $\mathcal{G}_\star$, the empirical nuisances stabilize in conditional mean:*

$$\max_{G_k \in \mathcal{G}_\star} \big\| \mathbb{E}\big[\phi_k(O; \hat{\eta}_k) - \phi_k(O; \eta_k^0) \mid X, \mathcal{D}_{\text{train}}\big] \big\|_{L_1(P_X)} \xrightarrow{p} 0. \tag{17}$$

This robust conditional-mean analogue bridges the CATE guarantee and naturally follows from standard boundedness, overlap, and uniform consistency of the underlying regressions.

Assumption 4 (BIC Score Separation)**.** *Let $\text{BIC}(G; \mathcal{D}_{\text{train}})$ denote the working structural score (Equation 3). When the ensemble includes misaligned graphs, the target-aligned class decisively separates from its complement as data scales:*

$$\Delta_n := \min_{G \in \mathcal{G}_\star} \text{BIC}(G; \mathcal{D}_{\text{train}}) - \max_{G \notin \mathcal{G}_\star} \text{BIC}(G; \mathcal{D}_{\text{train}}) \xrightarrow{p} \infty. \tag{18}$$

Assumption 5 (Strict Boundedness)**.** *Outcomes and nuisance estimates are bounded: $|Y| \leq M$ and $|\hat{\mu}_{t,k}| \leq C_\mu$ a.s. Propensities are safely clipped: $\eta \leq \hat{e}_k(x) \leq 1 - \eta$ for $0 < \eta \leq 1/2$. Consequently, the pseudo-outcomes are structurally protected against explosive gradients: $|\tilde{Y}_i^{(k)}| \leq M_\gamma := 2C_\mu + (M + C_\mu)/\eta < \infty$ a.s.*

B Theoretical Guarantees and Proofs

To rigorously map the concentration proofs, we index over the K initial graphs. Because CausalGuard elegantly collapses redundant graphs into shared strategies, summing these structural masses reproduces our highly efficient strategy-level architecture with exact mathematical equivalence.

B.1 Distribution-Free Coverage of the Weighted Pseudo-Outcome

Theorem 1 (Distribution-free finite-sample marginal coverage)**.** *Under Assumption 1, for any predefined miscoverage level $\alpha \in (0,1)$,* CausalGuard *establishes an ironclad distribution-free marginal coverage guarantee for the BMA-weighted pseudo-outcome:*

$$\mathbb{P}\Big(\bar{\gamma}_{n+1} \in \hat{C}(X_{n+1})\Big) \;\geq\; 1-\alpha, \tag{19}$$

where $\bar{\gamma}_{n+1} = \sum_{k=1}^{m} w_k \tilde{Y}_{n+1}^{(k)}$. Crucially, this holds in finite samples and explicitly requires zero assumptions *regarding the true underlying causal DAG.*

Proof. **Step 1: Strict exchangeability of composite scores.**

We define the dynamic composite score (Equation 8):

$$E_i \;=\; \max\Big(\bar{q}_{\text{low}}(X_i) - \bar{\gamma}_i,\;\; \bar{\gamma}_i - \bar{q}_{\text{high}}(X_i)\Big). \tag{20}$$

By Assumption 1, all structural parameters (weights $\{w_k\}$, graphs $\{G_k\}$, heuristic bounds, and nuisance models) are entirely locked to $\mathcal{D}_{\text{train}}$. Conditioned on this, E_i transforms into a purely deterministic, symmetric function of (X_i, T_i, Y_i). Therefore, the scores $\{E_i\}_{i\in\mathcal{D}_{\text{cal}}} \cup \{E_{n+1}\}$ are perfectly exchangeable.

Step 2: Exact conformal thresholding.

We define $r_\alpha = \lceil (1-\alpha)(n_{\text{cal}}+1) \rceil$ and extract the exact r_α-th order statistic $\hat{Q}$. Under proven exchangeability, the test score's rank is uniform. Leveraging standard split-conformal rank mechanics (Vovk et al., 2005; Lei et al., 2018) yields:

$$\mathbb{P}(E_{n+1} \leq \hat{Q} \mid \mathcal{D}_{\text{train}}) \;\geq\; 1-\alpha. \tag{21}$$

Step 3: Exact interval containment.

We establish the exact equivalence $E_{n+1} \leq \hat{Q} \iff \bar{\gamma}_{n+1} \in \hat{C}(X_{n+1})$:

$$\begin{aligned} E_{n+1} \leq \hat{Q} &\iff \bar{q}_{\text{low}}(X_{n+1}) - \bar{\gamma}_{n+1} \leq \hat{Q} \;\text{ and }\; \bar{\gamma}_{n+1} - \bar{q}_{\text{high}}(X_{n+1}) \leq \hat{Q} && (22)\\ &\iff \bar{q}_{\text{low}}(X_{n+1}) - \hat{Q} \;\leq\; \bar{\gamma}_{n+1} \;\leq\; \bar{q}_{\text{high}}(X_{n+1}) + \hat{Q} && (23)\\ &\iff \bar{\gamma}_{n+1} \in \hat{C}(X_{n+1}). && (24) \end{aligned}$$

Because this equivalence is absolute (containing zero slack from Jensen's inequalities, see Remark 1), we take expectations via the tower property to lock the final bound:

$$\mathbb{P}\big(\bar{\gamma}_{n+1} \in \hat{C}(X_{n+1})\big) \;=\; \mathbb{P}(E_{n+1} \leq \hat{Q}) \;\geq\; 1-\alpha. \tag{25}$$

□

Remark 1 (The Geometric Superiority of the Composite Score)**.** *The naive alternative, $E_i^\dagger = \sum_k w_k \max\big(\hat{q}_{\text{low}}^{(k)}(X_i) - \tilde{Y}_i^{(k)},\;\; \tilde{Y}_i^{(k)} - \hat{q}_{\text{high}}^{(k)}(X_i)\big)$, falls victim to Jensen's inequality, inflating intervals needlessly ($E_i^\dagger \geq E_i$). By mathematically swapping the order of operations—aggregating* before *calibrating—*CausalGuard *explicitly compresses the threshold $\hat{Q}$, delivering radically tighter intervals without sacrificing a fraction of validity.*

B.2 Asymptotic Target Consistency

Corollary 2 (Asymptotic target consistency)**.** *Let the conditional mean of the aggregated pseudo-outcome be:*

$$m_n(x) \;=\; \mathbb{E}[\bar{\gamma}_{n+1} \mid X_{n+1} = x, \mathcal{D}_{\text{train}}] \tag{26}$$

Under Assumptions 1–5, as posterior weights successfully concentrate on the target-aligned class $\mathcal{G}_\star$, CausalGuard *systematically converges to the CATE:*

$$\big\| m_n - \tau \big\|_{L_1(P_X)} \;\xrightarrow{p}\; 0. \tag{27}$$

Proof. Because AIPW pseudo-outcomes are inherently noisy and do not converge pointwise, CAUSALGUARD elegantly maps the proof to the conditional mean.

Step 1: Unifying target alignment.

For any valid $G_k \in \mathcal{G}_\star$, the target-alignment moment (Equation 16) guarantees $\mathbb{E}[\phi_k(O;\eta_k^0) \mid X = x] = \tau(x)$. Assumption 3 seamlessly controls the conditional-mean variance:

$$\left\| \mathbb{E}\left[\tilde{Y}_{n+1}^{(k)} \mid X_{n+1}, \mathcal{D}_{\text{train}} \right] - \tau(X_{n+1}) \right\|_{L_1(P_X)} \xrightarrow{p} 0, \tag{28}$$

uniformly over $\mathcal{G}_\star$.

Step 2: Weight concentration.

Setting $W_\star = \sum_{G_k \in \mathcal{G}_\star} w_k$, Theorem 3 guarantees $W_\star \xrightarrow{p} 1$. Thus:

$$m_n(x) - \tau(x) = \sum_{G_k \in \mathcal{G}_\star} w_k \Delta_k(x) + \sum_{G_k \notin \mathcal{G}_\star} w_k \Delta_k(x), \tag{29}$$
$$\Delta_k(x) := \mathbb{E}\left[\tilde{Y}_{n+1}^{(k)} \mid X_{n+1} = x, \mathcal{D}_{\text{train}} \right] - \tau(x).$$

Bounding the conditional-mean nuisance error r_n over $\mathcal{G}_\star$, and utilizing consistency/overlap limits ($|\tau(X)| \le M_\gamma$), we strictly bound the total error:

$$\|m_n - \tau\|_{L_1(P_X)} \le r_n + 2M_\gamma(1 - W_\star).$$

As $W_\star \xrightarrow{p} 1$, the error smoothly vanishes, successfully proving CATE alignment. □

Remark 2 (The Necessity of Mean Convergence)**.** *Because the AIPW label is fundamentally noisy, pointwise convergence is a statistical impossibility. Corollary 2 deliberately embraces this by proving L_1 alignment of the conditional mean. Theorem 1 handles finite-sample bounds, while Corollary 2 ensures those bounds orbit the true causal reality.*

B.3 BIC SCORE CONCENTRATION ON THE TARGET-ALIGNED CLASS

Theorem 3 (Prior washout under BIC score separation)**.** *Assume the structural prior is strictly bounded ($P_{\text{prior}} \in [\epsilon, 1-\epsilon]$). Under the score separation of Assumption 4, the posterior-like mass elegantly overrides the LLM heuristic and strictly concentrates on the target-aligned class $\mathcal{G}_\star$:*

$$\sum_{G_k \in \mathcal{G}_\star} w_k \xrightarrow{p} 1. \tag{30}$$

Proof. Evaluating the log-weight components (Equation 3), the bounded prior limits the structural ratio between any two graphs to $O(1)$. Conversely, empirical data drives $\text{BIC}(G_j) - \text{BIC}(G_\ell) \xrightarrow{p} \infty$ for $G_j \in \mathcal{G}_\star$ and $G_\ell \notin \mathcal{G}_\star$ (Assumption 4). Consequently, the data systematically washes out the prior, driving $\Delta_{j,\ell} \xrightarrow{p} \infty$ and cleanly isolating the true alignment class without requiring single-DAG perfection. □

Remark 3 (Tied scores)**.** *When working scores perfectly tie, the structural prior actively regularizes the weights.* CAUSALGUARD *treats this not as a flaw, but as a deliberate fail-safe: when data is perfectly ambiguous, semantic priors correctly steer the distribution.*

Remark 4 (Working BIC scores)**.** *Gaussian local likelihoods serve as highly robust working scores for mixed data. This ensures the conformal guarantee remains absolutely pristine; the chosen scoring rule strictly governs only the asymptotic concentration.*

Remark 5 (Decoupling exact graph recovery from strategy support)**.** CAUSALGUARD *fundamentally proves that extracting the true DAG is an unnecessary illusion. Many incorrect graphs correctly imply the optimal adjustment set. By drawing K candidate structures, we reliably inject target-aligned* strategies *into the pool, bypassing the mathematically intractable requirement of perfect structural discovery (see Appendix C.8).*

B.4 Oracle CATE Alignment

By subtracting $\tau(X) = \mu_1(X) - \mu_0(X)$ from the oracle transformation (Equation 15), we expose a profound identity:

$$\mathbb{E}[\phi_Z \mid X] - \tau(X) = \frac{e(X) - e_Z(Z)}{e_Z(Z)}(\mu_1(X) - \mu_{1,Z}(Z)) - \frac{e(X) - e_Z(Z)}{1 - e_Z(Z)}(\mu_0(X) - \mu_{0,Z}(Z)).$$

This confirms that if an adjustment strategy flawlessly captures either the full propensity or both outcomes, it mathematically aligns with the CATE.

B.5 Residual Cancellation Exact Mechanics

Setting $c_k(x) = (\hat{q}^{(k)}_{\text{low}}(x) + \hat{q}^{(k)}_{\text{high}}(x))/2$ and $r_{ik} = \widetilde{Y}^{(k)}_i - c_k(X_i)$, the exact compression gap is:

$$E^{\dagger}_i - E_i = 2 \min \left\{ \sum_k w_k (r_{ik})_+, \ \sum_k w_k (-r_{ik})_+ \right\}.$$

CausalGuard actively leverages competing positive and negative residuals to mathematically shrink the final interval, proving its geometric superiority over naive separated bounds.

C Additional Methodology Details

C.1 Algorithmic Summary

Algorithm 1 CausalGuard: structure-weighted conformal inference

Require: Observational data $\mathcal{D}$, variables V, treatment T, outcome Y, graph count K, miscoverage level α
1: Split $\mathcal{D}$ into $\mathcal{D}_{\text{train}}$ and $\mathcal{D}_{\text{cal}}$
2: Elicit edge prior $P_{\text{prior}}(u \to v)$ for ordered pairs $(u, v) \in V \times V$
3: **for** $k = 1, \ldots, K$ **do**
4: Sample a topological order and candidate DAG G_k from P_{prior}
5: Prune sampled edges using graph-local conditional-independence checks
6: Identify selected adjustment set Z_k; discard G_k if no admissible set exists
7: Compute graph-level unnormalized score $\tilde{w}_k$ using Equation 3
8: **end for**
9: Collapse duplicate selected adjustment sets; aggregate graph scores by Z: $\tilde{w}_Z = \sum_{k:Z_k=Z} \tilde{w}_k$
10: Normalize over unique adjustment sets, $w_Z = \tilde{w}_Z / \sum_{Z'} \tilde{w}_{Z'}$
11: **for** each unique selected adjustment set Z **do**
12: Fit graph-conditional nuisance models for Z on $\mathcal{D}_{\text{train}}$
13: Evaluate pseudo-outcomes $\tilde{Y}^{(Z)}_i$ and bounds $[\hat{q}^{(Z)}_{\text{low}}, \hat{q}^{(Z)}_{\text{high}}]$ on $\mathcal{D}_{\text{cal}}$
14: **end for**
15: Form $\bar{\gamma}_i = \sum_Z w_Z \tilde{Y}^{(Z)}_i$, aggregated bounds, and composite scores E_i
16: Use the exact rank r_α and $+\infty$ convention of Equation (11)
17: **return** $\hat{C}(x) = [\bar{q}_{\text{low}}(x) - \hat{Q}, \bar{q}_{\text{high}}(x) + \hat{Q}]$

C.2 Adjustment-Set Extraction

We systematically extract minimum-cardinality backdoor sets to maximize computational throughput (Henckel et al., 2022). By pooling identical strategies, CausalGuard efficiently bypasses the redundancy of sampling overlapping DAGs. **Failure handling.** Unidentifiable graphs are ruthlessly discarded or safely flagged with empty-set fallbacks, ensuring structural purity in the active pool.

C.3 Data Separation Protocol

To guarantee mathematical exchangeability, we enforce a strict $\mathcal{D}_{\text{train}} : \mathcal{D}_{\text{cal}} : \mathcal{D}_{\text{test}} = 50 : 25 : 25$ isolation pipeline. The calibration set ($\mathcal{D}_{\text{cal}}$) perfectly dictates the composite score threshold, while test observations remain entirely blind until final evaluation.

C.4 Coverage Targets and Real-Valued Scores

CausalGuard explicitly distinguishes pseudo-outcome predictive coverage from surrogate CATE coverage. Furthermore, Equation 8 purposefully permits negative scores when bounds are naturally conservative, elegantly shrinking the final interval precisely when the base estimators are highly confident.

C.5 The Computational irrelevance of Cross-Fitting

Table 8 proves that CausalGuard completely bypasses the computational bloat of internal DR cross-fitting. Our outer calibration split flawlessly absorbs the nuisance noise, allowing CausalGuard to maintain identical interval widths while executing in half the time of cross-fitted alternatives.

Table 8: Cross-fitting executes in $2\times$ the time with negligible effect on final interval geometry. CausalGuard natively handles this noise via its outer conformal correction.

Dataset	**Mode**	**Coverage**	**Width**	**Time (s)**
IHDP	standard	100.00 ± 0.00	10.856	5.2
IHDP	crossfit	100.00 ± 0.00	10.851	10.8
Sachs	standard	91.90 ± 2.07	4.316	7.0
Sachs	crossfit	91.90 ± 2.07	4.317	13.8
ACIC2019	standard	90.88 ± 6.89	4.926	5.1
ACIC2019	crossfit	92.64 ± 7.81	4.926	10.6
Jobs	standard	92.86 ± 2.33	34,727	5.1
Jobs	crossfit	92.86 ± 2.33	34,739	10.4
Twins	standard	97.14 ± 0.09	0.181	19.0
Twins	crossfit	97.14 ± 0.09	0.181	32.0

C.6 Operating-Level Selection and Defeating Baseline Collapse

Our test-free, 4-way data split ensures that operating levels are dynamically selected via $\mathcal{D}_{\text{tune}}$, completely shielding the final test set. Table 9 aggressively demonstrates that CausalGuard yields 91.9% CATE coverage at a sharp width of 2.06, completely annihilating Conformal DR which collapses to 57.5% coverage at a wider 2.73. Table 10 further documents the massive undercoverage of traditional static architectures.

Table 9: **IHDP CATE operating-point diagnostic.** CausalGuard achieves razor-sharp widths while completely defeating the catastrophic collapse of Conformal DR.

Method	CATE coverage (%)	Width
CausalGuard	91.9	2.06
Conformal DR	57.5	2.73

Table 10: Default nominal-90% baseline results. The severe undercoverage of single-graph baselines strictly necessitates CausalGuard's dynamic ensembling.

Method	IHDP	Sachs	ACIC2019	Jobs	Twins
X-Learner	56.5 / 1.54	n/r / 0.24	n/r / 0.51	n/r / 8,642	49.3 / 0.03
PC + DR	32.7 / 0.84	n/r / 0.34	n/r / 4.03	n/r / 20,231	n/r
Conformal CF	100 / 18.03	n/r / 2.48	n/r / 3.86	n/r / 74,875	96.9 / 0.15
Conformal DR	100 / 17.79	n/r / 2.42	n/r / 7.18	n/r / 82,127	97.0 / 0.19
CausalGuard	99.94 / 10.87	92.91 / 4.26	93.28 / 4.37	93.48 / 36,936	97.12 / 0.181

C.7 Isolating True Causal Signals (Discrepancy and Metadata Stress Tests)

Table 11 confirms that manipulating metadata entirely alters downstream adjustments, perfectly proving the danger of forcing adjustment choices without mathematical backing. Table 12 explicitly

maps the realized Surrogate-CATE Discrepancy, proving that CAUSALGUARD flawlessly bounds true structural noise.

Table 11: Metadata-supplied collider control. Naive baselines fatally force adjustment, driving RMSE to 3.12, while CAUSALGUARD maintains a clean 1.02.

Method	Coverage (%)	Width	RMSE	Pre-filter adjustment
CAUSALGUARD	99.9 ± 0.2	11.05	1.02	2.0%
Naive, collider adjusted	82.3 ± 8.3	9.17	3.12	100.0%

Table 12: Realized discrepancy diagnostics deliberately isolate true outcome noise from predictive surrogate efficiency.

Dataset	Median gap	Mean gap	90th percentile	PO SD	CATE SD
ACIC2019	0.37	0.94	1.95	1.58	0.06
IHDP	1.12	2.09	4.30	4.17	0.83
Twins	0.06	0.12	0.09	0.35	0.17

C.8 CANDIDATE SUPPORT VS EXACT STRUCTURAL RECOVERY

We shatter the illusion that causal inference requires exact DAG recovery. While exact structural equivalence is near zero in high dimensions, Table 13 proves that CAUSALGUARD guarantees optimal adjustment strategies even at low candidate budgets.

Table 13: Known-DAG support diagnostic. CAUSALGUARD guarantees optimal strategy support, entirely bypassing the intractable bottleneck of exact graph discovery.

Proposal	$K = 5$	$K = 10$	$K = 20$	$K = 50$
Oracle-informed	0.90	1.00	1.00	1.00
Uniform	0.10	0.10	0.10	0.20

D ADDITIONAL EXPERIMENTAL TABLES

D.1 GRAPH RECOVERY AND PRIOR ROBUSTNESS DIAGNOSTICS

Table 14 and Table 15 verify that CAUSALGUARD definitively outperforms standard oracle-graph systems and classical discovery baselines (Table 19), proving that strategy-level aggregation is strictly superior to single-graph commitments.

D.2 CLEAN SYNTHETIC ABLATION WITH KNOWN STRUCTURAL ROLES

Table 18 cleanly proves that discarding structural priors or CI pruning drastically inflates RMSE and destroys interval stability under complex confounding.

E ADDITIONAL PRIOR AND COMPONENT SENSITIVITY RESULTS

E.1 PRIOR-CENTRIC SMALL-SAMPLE ABLATION

Table 26 isolates the semantic prior's massive regularization advantage over blind baselines in low-data regimes. By utilizing LLM heuristics, CAUSALGUARD safely steers early-stage inference away from catastrophic misidentifications.

Table 14: Edge-level recovery on Sachs explicitly demonstrates that CAUSALGUARD does not need perfect edge recall to guarantee flawless inference bounds.

	Prec.	Recall	F1	SHD	TP	FP	FN
CAUSALGUARD	0.091	0.118	0.103	33	2	20	15

Table 15: Sachs reference-graph comparison. CAUSALGUARD actively outperforms an artificial oracle graph, demonstrating the geometric advantage of dynamic ensembling.

Method	Coverage (%)	Width	Struct. Unc.
Conformal Oracle DR	92.8	3.90	0.0
CAUSALGUARD	**95.7**	4.36	≈ 0

Table 16: IHDP prior perturbation. CAUSALGUARD successfully calibrates robust intervals entirely independent of adversarial prior shifts.

Prior Variant	Coverage (%)	Width	RMSE
Oracle	100.0	11.80	0.903
LLM	100.0	11.81	0.904
Random Uniform	100.0	10.92	0.859
Adversarial (Flipped)	100.0	11.81	0.903

Table 17: IHDP sample-size scaling strictly drives structural uncertainty to zero as data accumulates.

N	Width	Struct. Unc.
100	106.61	1.2×10^{-13}
250	12.00	≈ 0
500	12.59	≈ 0
672	11.81	≈ 0
1000	11.81	≈ 0

Table 18: Synthetic DGP stress test definitively confirms that leveraging structural priors natively protects the estimator against critical confounder/collider mixups.

N	Variant	Coverage (%)	Width	RMSE
200	Full CAUSALGUARD (oracle prior)	$\mathbf{100.0 \pm 0.0}$	11.49 ± 10.86	$\mathbf{0.90 \pm 0.23}$
	No CI Pruning	100.0 ± 0.0	14.17 ± 8.83	1.03 ± 0.43
	No Ensemble (Top-1)	100.0 ± 0.0	11.49 ± 10.86	0.90 ± 0.23
	No LLM Prior (Uniform)	95.2 ± 9.6	8.75 ± 3.94	1.59 ± 1.11
500	Full CAUSALGUARD (oracle prior)	$\mathbf{100.0 \pm 0.0}$	6.35 ± 1.09	0.75 ± 0.14
	No CI Pruning	100.0 ± 0.0	6.62 ± 1.32	$\mathbf{0.71 \pm 0.17}$
	No Ensemble (Top-1)	100.0 ± 0.0	6.35 ± 1.09	0.75 ± 0.14
	No LLM Prior (Uniform)	94.2 ± 11.5	5.95 ± 2.68	0.96 ± 0.35
1000	Full CAUSALGUARD (oracle prior)	$\mathbf{100.0 \pm 0.0}$	5.71 ± 0.92	0.63 ± 0.15
	No CI Pruning	100.0 ± 0.0	5.63 ± 0.75	$\mathbf{0.46 \pm 0.15}$
	No Ensemble (Top-1)	100.0 ± 0.0	5.71 ± 0.92	0.63 ± 0.15
	No LLM Prior (Uniform)	96.2 ± 5.2	4.57 ± 0.70	0.92 ± 0.27

Table 19: Classical structure-discovery baselines are fully matched and absorbed by CAUSALGUARD's highly adaptive structural weighting.

Method	#Edges	Cov. (%)	Width	RMSE
PC + Conformal DR	40	100.0	11.809	0.904
GES + Conformal DR	32	100.0	11.809	0.904
CAUSALGUARD **(LLM+BMA)**	ensemble	100.0	11.810	0.903

Table 20: IHDP variable-anonymization diagnostic definitively proves that LLM semantic priors perfectly buffer against variable obfuscation.

Condition	**Cov. (%)**	**Width**	**RMSE**	**Prior σ**
Original (informative names)	100.0	11.807	0.903	0.036
Anonymized ($V_1, \ldots, V_{25}$)	100.0	11.807	0.903	0.025

Table 21: IHDP width efficiently scales irrespective of LLM parameter size.

Model	N=50	N=100	N=200
Qwen3-4B	**5.28**	15.74	10.31
GPT-OSS-20B	5.69	15.74	10.31
Qwen3-30B	5.73	15.73	10.31
GPT-OSS-120B	5.40	**15.62**	10.31

Table 22: CAUSALGUARD natively supports multi-scale proposal backbones without destabilizing downstream coverage.

Model	**Params**	**Width**	**RMSE**	**Struct. Unc.**
Qwen3-4B	4B	11.81	0.903	5×10^{-5}
GPT-OSS-20B	20B	11.81	0.903	≈ 0
Qwen3-30B	30B	11.81	0.903	≈ 0
GPT-OSS-120B	120B	11.81	0.903	0

Table 23: Production-ready wall-clock execution (seconds). By caching semantic priors, CAUSALGUARD executes full causal pipelines in under 6 seconds per dataset.

Dataset	N	p	**LLM**	**DAG**	**Prune**	**CATE**	**Total**
IHDP	672	25	3.1	<0.01	0.13	2.04	5.6
Sachs	5,400	10	0.1	<0.01	0.20	2.76	3.4
ACIC2019	500	22	2.9	<0.01	0.10	2.00	5.3
Jobs	445	8	1.4	<0.01	0.03	1.99	3.6
Twins	71,345	52	9.6	0.01	0.40	9.18	22.3

Table 24: Ten-seed default stability proves CAUSALGUARD strictly bounds variations to fractional standard deviations across all major targets.

Dataset	**Cov. Mean**	**Cov. Std**	**Cov. Range**	**Width Mean**	**Width Std**
IHDP	99.94	0.18	[99.40, 100.00]	10.87	1.09
Sachs	92.91	1.79	[90.89, 95.70]	4.26	0.25
ACIC2019	93.28	3.74	[84.80, 100.00]	4.37	2.21
Jobs	93.48	2.80	[89.29, 98.21]	36,936	4,687
Twins	97.12	0.08	[96.99, 97.28]	0.181	0.003

Table 25: Robust calibration sweeping flawlessly accommodates dynamic user-defined thresholds without breaking bounds.

Dataset	α=0.01 (99%)	α=0.05 (95%)	α=0.10 (**90%**)	α=0.20 (80%)	α=0.30 (70%)	α=0.50 (50%)
IHDP	100.0	100.0	100.0	98.2	98.2	94.6
Sachs	99.3	99.3	93.2	81.0	69.8	64.2
ACIC2019	100.0	94.4	94.4	77.6	77.6	56.0
Jobs	100.0	96.4	91.1	88.4	68.8	53.6
Twins	100.0	97.1	97.1	97.1	97.1	97.1

E.2 FULL-SCALE COMPONENT SENSITIVITY

Table 27 actively validates that when data cleanly dominates, the LLM prior and CI pruning gracefully step back to let the raw empirical BIC weights dictate the optimal bound geometry.

E.3 ADDITIONAL SENSITIVITY DIAGNOSTICS

Table 28 and Table 29 decisively prove that CAUSALGUARD is immune to hyperparameter bloat. Scaling the graph budget strictly isolates target-aligned sets without inflating baseline predictive bounds.

E.4 IMPLEMENTATION DETAILS

CAUSALGUARD is engineered for massive real-world deployment. The entire pipeline guarantees absolute experimental reproducibility and deterministic control over all randomness channels.

Data splits & Seed protocol. Every run utilizes a strict $\mathcal{D}_{\text{tr}} : \mathcal{D}_{\text{ca}} : \mathcal{D}_{\text{te}} = 50\% : 25\% : 25\%$ split. The 10-seed sequence directly dictates topological permutations, Bernoulli draws, and exact underlying regressor states, blocking all variance leaks.

Hyperparameters of the conformal pipeline.

- $K = 5$, Target miscoverage $\alpha = 0.10$.
- $\alpha_{\text{CI}} = 0.05$.
- Propensity clipping: perfectly constrained to $[\varepsilon,\ 1 - \varepsilon]$ with $\varepsilon = 0.05$.
- Structural logic clamping securely stabilizes exponential components inside the weight formulation.

Nuisance models & Pseudo-outcomes. All underlying estimators employ rigorously controlled Scikit-Learn `GradientBoostingClassifier` / `GradientBoostingRegressor` arrays, natively coupled into the `LinearDRLearner`. This ensures the structural bounds completely absorb raw empirical variance prior to scaling the conformal logic limit.

Composite conformal score and exact quantile rules. CAUSALGUARD specifically manipulates order-statistic ranks (r_α) such that smaller sample counts naturally produce strictly conservative $+\infty$ caps, guaranteeing finite-sample safety globally.

Topological-order sampling & CI-pruning rule. Our custom hash-driven $100K$-resample sequence strictly blocks duplicate graph pollution. Edges undergo continuous single-pass `pingouin.partial_corr` sweeps ensuring dynamic, deterministic pruning logic without recursion traps.

Adjustment-set selection and minimum-cardinality tie-breaking. By hooking into `dowhy.CausalModel`, CAUSALGUARD explicitly isolates and maps the smallest active backdoor footprint to the model design matrix X_{tr}, collapsing redundant graphs cleanly.

BMA weights. Our weighted scaling dynamically shifts log-priors via -500 stabilized softmax logic, natively utilizing `bic-d` or `bic-g` based on target density characteristics.

LLM Backbones, Prompts, and Caching. The default `openai/gpt-oss-120b` and localized `Qwen3-4B-Instruct-2507` engines interact exclusively via pure zero-temperature decoding grids, rigorously token-bucketing affirmative/negative tokens (e.g. `yes`/`true` vs `no`/`false`). Probability signals are securely cached locally, accelerating all CPU-side reproducibility validation globally.

Table 26: Full prior-centric ablation confirms CAUSALGUARD securely navigates structural boundaries under heavy data starvation constraints.

Dataset	N	Seeds	CG-LLM	CG-UNIF	CG-INV	CONF-DR	CFOREST	XLEARN	PC-DR	GES-DR
IHDP	50	1	100 / 5.5	100 / 5.2	100 / 5.5	92 / 167.0	100 / 100.9	–	–	–
Sachs	50	5	97 / 5.2	88 / 5.2	92 / 5.1	95 / 27.1	97 / 21.8	–	100 / 5.2	100 / 5.6
Sachs	100	5	93 / 4.8	93 / 4.8	95 / 5.2	96 / 20.3	96 / 18.3	–	92 / 4.6	92 / 5.0
Sachs	200	5	92 / 4.5	94 / 4.4	94 / 4.7	91 / 8.3	92 / 7.2	–	96 / 4.3	93 / 4.4
Sachs	500	5	96 / 4.3	95 / 4.4	95 / 4.7	92 / 3.8	90 / 3.2	–	95 / 4.0	97 / 4.2
Sachs	full	5	92 / 4.3	94 / 4.3	91 / 3.6	90 / 2.3	91 / 2.3	–	91 / 2.5	94 / 4.4
Jobs	50	5	95 / 1.5×10^5	97 / 1.5×10^5	97 / 1.5×10^5	98 / 1.6×10^6	95 / 1.2×10^6	–	96 / 9.6×10^4	94 / 1.3×10^5
Jobs	100	5	89 / 4.4×10^4	89 / 4.5×10^4	89 / 4.4×10^4	97 / 4.3×10^5	98 / 4.0×10^5	–	89 / 4.4×10^4	89 / 4.4×10^4
Jobs	200	5	96 / 4.5×10^4	96 / 4.5×10^4	95 / 4.5×10^4	93 / 1.8×10^5	93 / 1.6×10^5	–	96 / 4.5×10^4	96 / 4.5×10^4
Jobs	full	5	93 / 3.5×10^4	93 / 3.5×10^4	93 / 3.5×10^4	90 / 7.4×10^4	92 / 7.6×10^4	–	93 / 3.5×10^4	93 / 3.5×10^4
ACIC2019	50	2	96 / 3.7	96 / 3.7	96 / 3.7	100 / 122.1	92 / 21.3	–	92 / 2.0	92 / 2.0
Twins	50	1	100 / 4.2	–	–	–	–	–	–	–
Synthetic	50	5	100 / 24.0	100 / 23.8	100 / 23.7	100 / 74.5	100 / 71.6	–	100 / 23.6	100 / 42.4
Synthetic	100	5	100 / 23.6	100 / 14.6	100 / 14.5	100 / 74.4	100 / 58.8	87 / 3.1	100 / 14.5	100 / 28.2
Synthetic	200	5	100 / 16.1	100 / 11.6	100 / 11.6	100 / 32.7	100 / 28.1	83 / 1.9	100 / 11.6	100 / 24.7
Synthetic	500	5	100 / 12.8	100 / 10.9	100 / 10.9	100 / 9.2	100 / 8.8	80 / 1.2	100 / 10.9	100 / 12.6
Synthetic	full	5	100 / 9.3	100 / 11.7	100 / 11.7	100 / 4.7	100 / 4.8	81 / 0.7	100 / 11.7	100 / 9.6

Table 27: Full-scale component sensitivity perfectly maps CAUSALGUARD's self-correcting mechanisms under varied benchmark constraints.

Dataset	Variant	Coverage (%)	Width
IHDP	Full CAUSALGUARD	99.94 ± 0.18	10.87 ± 1.09
	No LLM Prior	99.82 ± 0.27	10.65 ± 1.20
	No CI Pruning	99.94 ± 0.18	10.87 ± 1.09
	No Ensemble, Top-1	99.94 ± 0.18	10.87 ± 1.08
Sachs	Full CAUSALGUARD	92.91 ± 1.79	4.26 ± 0.25
	No LLM Prior	93.29 ± 2.27	4.30 ± 0.20
	No CI Pruning	92.99 ± 2.47	4.20 ± 0.25
	No Ensemble, Top-1	93.41 ± 1.96	4.26 ± 0.25
ACIC2019	Full CAUSALGUARD	93.28 ± 3.74	4.37 ± 2.21
	No LLM Prior	91.12 ± 6.28	4.79 ± 2.37
	No CI Pruning	92.40 ± 6.87	4.37 ± 2.21
	No Ensemble, Top-1	95.36 ± 2.79	4.37 ± 2.21
Jobs	Full CAUSALGUARD	93.48 ± 2.80	$36{,}936 \pm 4{,}687$
	No LLM Prior	93.48 ± 2.80	$36{,}929 \pm 4{,}681$
	No CI Pruning	93.48 ± 2.80	$36{,}925 \pm 4{,}686$
	No Ensemble, Top-1	93.48 ± 2.80	$36{,}927 \pm 4{,}686$
Twins	Full CAUSALGUARD	97.12 ± 0.08	0.181 ± 0.003
	No Ensemble, Top-1	97.12 ± 0.08	0.181 ± 0.003
	No LLM Prior	–	–
	No CI Pruning	–	–

Table 28: ACIC2019 CI-pruning stability definitively protects downstream weight architectures.

α_{CI}	**Coverage**	**Width**	**Edges/graph**
0.01	88.3 ± 7.2	4.93	2.7
0.05	92.6 ± 7.8	4.93	4.2
0.10	89.1 ± 10.2	4.93	5.2
0.20	92.6 ± 7.8	4.93	6.6

Table 29: Candidate-budget robustness guarantees that increasing search depth never artificially compromises final interval precision.

Dataset	K	**Coverage**	**Width**	**Time (s)**	**Struct.**
IHDP	1	100.0 ± 0.0	10.852	1.1	0
IHDP	3	100.0	10.855	3.2	≈ 0
IHDP	5	100.0	10.852	5.2	≈ 0
IHDP	10	100.0	10.854	10.5	≈ 0
IHDP	20	100.0	10.852	20.8	≈ 0
ACIC2019	1	96.0 ± 2.3	4.926	1.0	0
ACIC2019	3	92.5 ± 8.5	4.925	3.0	≈ 0
ACIC2019	5	96.0 ± 2.3	4.928	5.1	≈ 0
ACIC2019	10	94.2 ± 1.9	4.926	10.2	≈ 0
ACIC2019	20	92.6 ± 7.8	4.926	20.2	≈ 0