DEEP DIVIDE-AND-REDUCE IN SYMBOLIC REGRESSION

Anonymous authors
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ABSTRACT

Symbolic regression (SR) aims to discover underlying mathematical expressions from data while preserving interpretability. Most existing learning-based SR methods primarily optimize expressions from observations without explicitly exploiting their structural mathematical properties. AI Feynman introduced a complementary paradigm that leverages such properties to recursively decompose complex expressions, but its decomposition criteria cover only restricted structural forms and its treatment of nested composition can require brute-force search over candidate sub-expressions. Building on this paradigm, we propose Deep Divide-and-Reduce in Symbolic Regression (DDRSR), a mathematically grounded framework that systematically generalizes expression decomposition and variable reduction. DDRSR extends translational symmetry to coefficient- and exponent-interfered forms, enables variable separation under overlapping variables and additive constant offsets, and generalizes the identification of nested compositional structures. We further characterize an intrinsic non-identifiability limitation of decomposition when no effective variable separation is induced. Experiments across multiple symbolic regression algorithms and benchmark datasets show that DDRSR identifies a broader range of decomposable structures than AI Feynman and overall improves downstream regression accuracy and exact-expression recovery.

1 INTRODUCTION

Symbolic regression (SR) aims to discover interpretable analytical expressions from observational data and has become an important tool for scientific discovery. Existing SR methods are broadly dominated by search-based approaches, including genetic programming, reinforcement learning, and related heuristic search methods (Petersen et al., 2019; Sun et al., 2022; Mundhenk et al., 2021; Li et al., 2025; Liu et al., 2025; Dong & Zhong, 2025; Al-Helali et al., 2024; Xu et al., 2024), and pre-training-based approaches that formulate SR as sequence generation from large synthetic equation corpora (Biggio et al., 2021; Kamienny et al., 2022; Vastl et al., 2024; Meidani et al., 2023; Wu et al., 2023; Li et al., 2023a; Shojaee et al., 2024). Recent studies further combine learned priors with symbolic search to improve search efficiency and solution quality (Landajuela et al., 2022; Xu et al., 2024; Holt et al., 2023; Liu et al., 2023; Li et al., 2025). Despite their strong empirical performance, these approaches primarily optimize expressions from data and generally do not explicitly exploit the structural mathematical properties of the target function.

In contrast, AI Feynman (Udrescu & Tegmark, 2020; Udrescu et al., 2020) introduces a mathematically grounded paradigm that exploits translational symmetry, variable separability, and nested composition to recursively decompose complex expressions into simpler sub-problems. However, its decomposition criteria cover only relatively restricted structural forms. Its translational-symmetry rules are sensitive to coefficient or exponent interference, its separability criteria do not naturally handle overlapping variables or additive offsets, and its nested-composition mechanism may require searching over candidate sub-expressions. These restrictions substantially limit its applicability to complex, high-dimensional expressions. Concurrent work, NestyNet-SR (Xu et al., 2026), also extends the AI Feynman paradigm, with an emphasis on structure discovery through analytic neural surrogates within an integrated SR pipeline.

To address these limitations, we propose Deep Divide-and-Reduce in Symbolic Regression (DDRSR), which systematically generalizes the decomposition principles of AI Feynman through mathematical

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proofs and derivations for broader variable composition and expression separation. Our primary innovations and contributions are summarized as follows:

- We extend translational symmetry, variable separability, and nested composition to substantially broader structural forms, including coefficient- and exponent-interfered variable compositions, overlapping-variable separability, additive constant offsets, and top-down nested separation.
- We derive practical mathematical criteria for identifying these structures and establish a non-identifiability result showing the intrinsic limitation of decomposition without effective variable separation.
- Across multiple SR solvers and benchmark datasets, DDRSR identifies more decomposable structures than AI Feynman and overall improves downstream regression and exact-expression recovery, with additional evaluations on runtime and robustness to noise.

2 Related Work

AI Feynman (Udrescu & Tegmark, 2020; Udrescu et al., 2020) exploits translational symmetry, variable separability, and nested composition to recursively reduce high-dimensional SR problems. However, its criteria mainly cover idealized separable structures: translational symmetry is sensitive to coefficient or exponent interference, conventional separability assumes disjoint variable groups and does not handle additive offsets, and nested-composition detection either requires unknown inner functions or candidate sub-expression search. These restrictions limit its applicability to overlapping-variable and structurally complex expressions. DDRSR generalizes these three decomposition mechanisms through analytical criteria that directly identify broader compositional structures. Detailed definitions of the AI Feynman criteria are provided in Appendix A.

Concurrent work, NestyNet-SR(Xu et al., 2026), also extends the AI Feynman decomposition paradigm and uses derivative information from neural surrogates to identify richer structural patterns. Its main focus is on reliable structure discovery from sampled data within an integrated symbolic regression pipeline. In contrast, DDRSR focuses on the mathematical principles of decomposition itself, deriving explicit criteria and reconstruction mechanisms for generalized symmetry, separability, and nested structures, together with theoretical results on decomposition identifiability. Moreover, DDRSR serves as a model-agnostic decomposition framework compatible with different downstream SR methods.

3 Methodology

This section presents the primary definitions, criteria, and theorems utilized in our framework; detailed mathematical derivations and proofs are provided in Appendix D. Following the criterion-based reasoning adopted in AI Feynman, we derive characteristic analytical conditions that are necessarily satisfied by the corresponding structural forms and use them as practical criteria for structural identification. Unless explicitly stated as a theorem, these derivations are intended to provide analytical signatures for screening candidate structures rather than to establish complete necessary-and-sufficient characterizations.

In the tree representation of a mathematical expression, leaf nodes denote variables, whereas internal nodes represent mathematical operators. Within our methodology, the objective of translational symmetry is to consolidate multiple leaf nodes into a single equivalent leaf node, a process we define as bottom-up variable composition. Conversely, variable separability and nested composition attempt to partition the expression into two distinct sub-modules starting from the root node, a process we define as top-down expression separation.

Let $\Omega \subseteq \mathbb{R}^n$ be a connected open domain and $f : \Omega \to \mathbb{R}$. Unless otherwise specified, we assume that $f \in C^3(\Omega)$, and that all denominators and transformed expressions involved in the criteria are well defined on the considered subdomain.

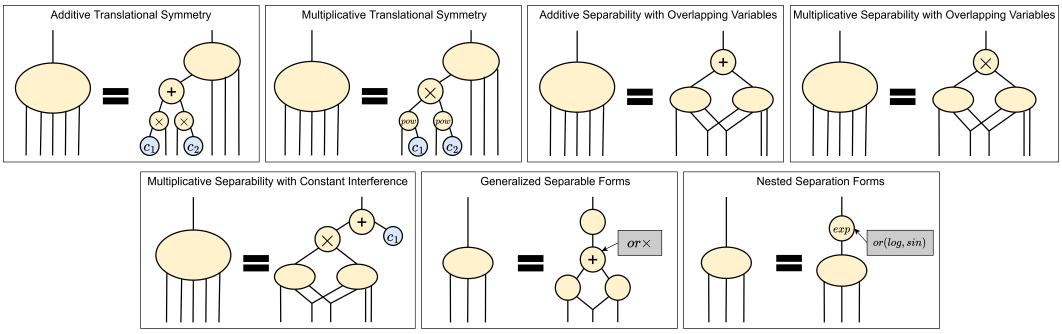


Figure 1: Examples of graph modularity that DDRSR can auto-discover. Lines denote real-valued variables and ovals denote functions.

3.1 Translational Symmetry

3.1.1 Additive Translational Symmetry

Definition: For an n-dimensional function $f(X)$, if it can be formulated as $f(x_1, x_2, ...) = f(c_1 \times x_1 + c_2 \times x_2, ...)$, we state that $f(X)$ possesses additive translational symmetry with respect to the dimensions x_1 and x_2.

Criterion: If, for any arbitrary X, the following condition holds:

$$\frac{\partial f(X)}{\partial x_1} \div \frac{\partial f(X)}{\partial x_2} = c$$

where c is a constant, then $f(X)$ is deemed to possess additive translational symmetry across dimensions x_1 and x_2, yielding the relationship $\frac{c_1}{c_2} = c$.

3.1.2 Multiplicative Translational Symmetry

Definition: For an n-dimensional function $f(X)$, if it can be formulated as $f(x_1, x_2, ...) = f(x_1^{c_1} \times x_2^{c_2}, ...)$, we state that $f(X)$ possesses multiplicative translational symmetry with respect to the dimensions x_1 and x_2.

Criterion: If, for any arbitrary X, the following condition holds:

$$\frac{\partial f(X)}{\partial x_1} \div \frac{\partial f(X)}{\partial x_2} \times \frac{x_1}{x_2} = c$$

where c is a constant, then $f(X)$ is deemed to possess multiplicative translational symmetry across dimensions x_1 and x_2, yielding the relationship $\frac{c_1}{c_2} = c$.

3.2 Variable Separability

3.2.1 Additive Separability with Overlapping Variables

Definition: For an n-dimensional function $f(X)$, if it exhibits the form $f(SX_1, SX_2, SX_3) = g(SX_1, SX_2) + h(SX_2, SX_3)$, we state that $f(X)$ possesses additive separability with overlapping variables, specifically separable with respect to SX_1 and SX_3.

Criterion: If $f(X)$ satisfies the following structure for any X:

$$f(SX_1, SX_2, SX_3) = f(SX_1, SX_2, SC_3) + f(SC_1, SX_2, SX_3) - f(SC_1, SX_2, SC_3)$$

where SC_i denotes setting the corresponding dimensions of SX_i to constants, we determine that $f(X)$ possesses additive separability with respect to SX_1 and SX_3.

Furthermore, we can extract the sub-functions as follows:

$$g(SX_1, SX_2) = f(SX_1, SX_2, SC_3) - f(SC_1, SX_2, SC_3)$$
$$h(SX_2, SX_3) = f(SC_1, SX_2, SX_3)$$

3.2.2 MULTIPLICATIVE SEPARABILITY WITH OVERLAPPING VARIABLES

Definition: For an n-dimensional function $f(X)$, if it exhibits the form $f(SX_1, SX_2, SX_3) = g(SX_1, SX_2) \times h(SX_2, SX_3)$, we state that $f(X)$ possesses multiplicative separability with overlapping variables, specifically separable with respect to SX_1 and SX_3.

Criterion: If $f(X)$ satisfies the following structure for any X:

$$f(SX_1, SX_2, SX_3) = \frac{f(SX_1, SX_2, SC_3) \times f(SC_1, SX_2, SX_3)}{f(SC_1, SX_2, SC_3)}$$

we determine that $f(X)$ possesses multiplicative separability with respect to SX_1 and SX_3.

Furthermore, we can extract the sub-functions as follows:

$$g(SX_1, SX_2) = \frac{f(SX_1, SX_2, SC_3)}{f(SC_1, SX_2, SC_3)}$$
$$h(SX_2, SX_3) = f(SC_1, SX_2, SX_3)$$

3.2.3 COMPOSITIONAL FORM OF ADDITIVE SEPARABILITY

Theorem 1: For an n-dimensional function $f(X)$, if f is additively separable with respect to dimensions SX_1 and SX_3, and concurrently additively separable with respect to SX_2 and SX_3, then f can be rigorously expressed as:

$$f(SX_1, SX_2, SX_3) = g(SX_1, SX_2) + h(SX_3)$$

3.2.4 COMPOSITIONAL FORM OF MULTIPLICATIVE SEPARABILITY

Theorem 2: For an n-dimensional function $f(X)$, if f is multiplicatively separable with respect to dimensions SX_1 and SX_3, and concurrently multiplicatively separable with respect to SX_2 and SX_3, then f can be rigorously expressed as:

$$f(SX_1, SX_2, SX_3) = g(SX_1, SX_2) \times h(SX_3)$$

3.2.5 MULTIPLICATIVE SEPARABILITY WITH ADDITIVE CONSTANT OFFSET

This criterion is applied after standard additive and multiplicative separability have been excluded. In addition, we require $\frac{\partial^2 f(X)}{\partial x_i \partial x_j} \neq 0$ on the considered subdomain to avoid degenerate cases.

Definition: For an n-dimensional function $f(X)$, if it takes the form $f(X) = g(x_i, SX) \times h(x_j, SX) + c$, where SX denotes the remaining dimensions excluding x_i and x_j, we state that $f(X)$ possesses multiplicative separability with a constant term across dimensions x_i and x_j.

Criterion: Let $u_i(X) = \frac{\partial^2 f(X)}{\partial x_i \partial x_j} \div \frac{\partial f(X)}{\partial x_i}$ and $u_j(X) = \frac{\partial^2 f(X)}{\partial x_i \partial x_j} \div \frac{\partial f(X)}{\partial x_j}$. If $\frac{\partial u_i(X)}{\partial x_i} = 0$ and $\frac{\partial u_j(X)}{\partial x_j} = 0$ hold for any arbitrary X, we determine that $f(X)$ may possess multiplicative separability with a constant term across dimensions x_i and x_j. Furthermore, if the recovered constant c remains invariant for arbitrary values of the remaining variables, we confirm that $f(X)$ indeed possesses this property.

3.2.6 SEPARATION MECHANISM FOR MULTIPLICATIVE FORMS WITH ADDITIVE CONSTANT OFFSET

When $f(X)$ satisfies the proposed differential criterion for multiplicative separability with a constant term, we further recover and verify the residual term.

Theorem 3: Given that $f(X)$ exhibits the form $f(X) = g(x_i, SX) \times h(x_j, SX) + c(SX)$, where SX denotes the remaining dimensions excluding x_i and x_j and $c(SX)$ denotes a residual term that is independent of x_i and x_j. If two sample points are obtained for dimensions x_i and x_j respectively, denoted as (x_{i1}, x_{i2}) and (x_{j1}, x_{j2}), the constant term $c(SX)$ can be analytically formulated as:

$$c(SX) = f(x_{i2}, x_{j2}, SX) - \frac{(f(x_{i2}, x_{j2}, SX) - f(x_{i2}, x_{j1}, SX)) \times (f(x_{i2}, x_{j2}, SX) - f(x_{i1}, x_{j2}, SX))}{f(x_{i2}, x_{j2}, SX) - f(x_{i1}, x_{j2}, SX) - f(x_{i2}, x_{j1}, SX) + f(x_{i1}, x_{j1}, SX)}$$

3.2.7 GENERALIZED SEPARABLE FORMS

Definition: For an n-dimensional function $f(X)$, if it takes the form $f(X) = U(g(x_i, SX) + h(x_j, SX))$ or $f(X) = U(g(x_i, SX) \times h(x_j, SX))$, where U represents an arbitrary operator and SX denotes the remaining dimensions excluding x_i and x_j, we state that $f(X)$ possesses a generalized separable form.

Criterion: If the ratio $\frac{\partial f/\partial x_i}{\partial f/\partial x_j}$ exhibits multiplicative separability with respect to x_i and x_j for any arbitrary X, we determine that $f(X)$ possesses a generalized separable form across dimensions x_i and x_j.

However, under this criterion, it cannot be directly ascertained whether the intrinsic structure is additive or multiplicative.

3.3 NESTED COMPOSITION

3.3.1 NESTED SEPARATION FORM OF STANDARD LOGARITHMIC FUNCTIONS

Definition: For an n-dimensional function $f(X)$, if it exhibits the form $f(X) = \log(g(X))$, where $g(X)$ can undergo further separation, we state that $f(X)$ possesses a nested separation form of standard logarithmic functions.

Criterion: Let $f_{new}(X) = \exp(f(X))$. If $f_{new}(X)$ exhibits separability, we verify that $f(X)$ possesses a nested separation form of standard logarithmic functions.

Consequently, we reduce the problem to the separation of the non-nested function $f_{new}(X)$. Notably, functions formulated as $f(X) = \log(g(x_i) \times h(x_j))$ can be analytically rewritten as $f(X) = \log(g(x_i)) + \log(h(x_j))$. This structure intrinsically exhibits additive separability and will thus be correctly processed during the additive separation phase.

3.3.2 NESTED SEPARATION FORM OF STANDARD EXPONENTIAL FUNCTIONS

Definition: For an n-dimensional function $f(X)$, if it exhibits the form $f(X) = \exp(g(X))$, where $g(X)$ can undergo further separation, we state that $f(X)$ possesses a nested separation form of standard exponential functions.

Criterion: Let $f_{new}(X) = \log(f(X))$. If $f_{new}(X)$ exhibits separability, we verify that $f(X)$ possesses a nested separation form of standard exponential functions.

Consequently, we reduce the problem to the separation of the non-nested function $f_{new}(X)$. Notably, functions formulated as $f(X) = \exp(g(x_i) + h(x_j))$ can be analytically rewritten as $f(X) = \exp(g(x_i)) \times \exp(h(x_j))$. This structure intrinsically exhibits multiplicative separability and will thus be correctly processed during the multiplicative separation phase.

3.3.3 NESTED SEPARATION OF LOGARITHMIC FUNCTIONS WITH CONSTANT TERMS

Functions exhibiting the form $f(X) = c_1 \times \log(g(x_i, SX) \times h(x_j, SX)) + c_2$ can be mathematically rewritten as $f(X) = c_1 \times \log(g(x_i, SX)) + c_1 \times \log(h(x_j, SX)) + c_2$, allowing them to be systematically reduced to additive separation.

For functions exhibiting the form $f(X) = \log(g(x_i, SX) + h(x_j, SX)) + c$, applying an inverse function mapping yields $\exp(f(X)) = \exp(c) \times g(x_i, SX) + \exp(c) \times h(x_j, SX)$. Therefore, the original function can be reduced to the standard logarithmic function (log) separation.

Functions exhibiting the form $f(X) = c \times \log(g(x_i, SX) + h(x_j, SX))$ are not subjected to standalone separation criteria.

3.3.4 NESTED SEPARATION OF EXPONENTIAL FUNCTIONS WITH CONSTANT TERMS

Functions exhibiting the form $f(X) = c_1 \times \exp(g(x_i, SX) + h(x_j, SX)) + c_2$ can be mathematically rewritten as $f(X) = c_1 \times \exp(g(x_i, SX)) \times \exp(h(x_j, SX)) + c_2$, allowing them to be reduced to multiplicative separation containing a constant term.

For functions exhibiting the form $f(X) = c \times \exp(g(x_i, SX) \times h(x_j, SX))$, applying an inverse function mapping yields $\log(f(X)) = \log(c) + g(x_i, SX) \times h(x_j, SX)$. Therefore, the original function can be reduced to the standard exponential function (exp) separation.

Functions exhibiting the form $f(X) = \exp(g(x_i, SX) \times h(x_j, SX)) + c$ are not subjected to standalone separation criteria.

3.3.5 Nested Separation Form of Standard Trigonometric Functions

Definition: For an n-dimensional function $f(X)$, if it takes the form $f(X) = \sin(\text{or } \cos)(g(x_i, SX) + h(x_j, SX) + c)$ or $f(X) = \sin(\text{or } \cos)(g(x_i, SX) \times h(x_j, SX) + c)$, where SX denotes the remaining dimensions excluding x_i and x_j, we state that $f(X)$ possesses a nested separation form of standard trigonometric functions.

Criterion: Let $T(X) = \frac{\partial f(X)}{\partial x_i} \times \frac{\partial f(X)}{\partial x_j} \div (1 - f^2(X))$. If, for any arbitrary X, $T(X)$ exhibits multiplicative separability with respect to dimensions x_i and x_j, $f(X)$ is considered a candidate for a standard trigonometric nested separation form.

3.3.6 Non-identifiability of Decomposition Without Variable Separability

During the process of expression decomposition, if an expression is partitioned into two components where the variables involved in the second component are merely a subset of those in the first component—meaning no distinct variable separation has occurred—such a separation strategy is fundamentally non-identifiable without additional constraints.

Theorem 4: For an n-dimensional function $f(X)$, if $f(X)$ can be formulated as $f(SX_1, SX_2) = g(SX_1, SX_2) \oplus h(SX_2)$, where $\oplus$ denotes an associative and commutative binary operation with an identity element and an inverse operation $\ominus$, there exist infinitely many valid functional configurations for g and h.

Consequently, it is analytically intractable to achieve identifiable expression decomposition under similar unconstrained conditions.

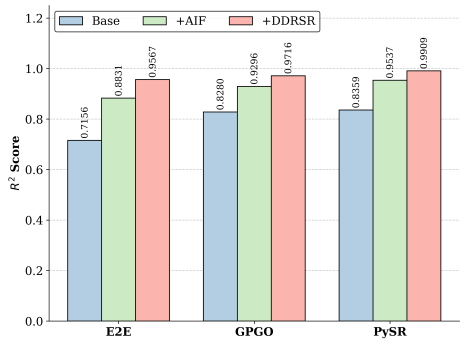
4 Experiments

4.1 Experimental Setup and Procedure

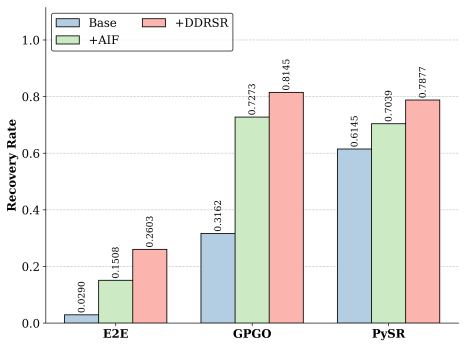
To evaluate the correctness and effectiveness of DDRSR, we apply the proposed decomposition principles to target expressions and subsequently solve the resulting sub-expressions using five symbolic regression algorithms: E2E (Kamienny et al., 2022), GP-GOMEA (Virgolin et al., 2021), NGGP (Mundhenk et al., 2021), PySR (Cranmer, 2023), and MetaSymNet (Li et al., 2023b). We compare DDRSR with both the original SR methods without decomposition and the decomposition principles of AI Feynman, and additionally investigate robustness under noisy observations.

We use R^2 in the ablation and noise experiments, while the comprehensive experiments additionally report the recovery rate, defined as the fraction of runs whose predicted expressions are symbolically or mathematically equivalent to the ground truth. Each experiment is repeated five times, and the reported R^2 is averaged over the five runs. We further introduce a Relative Separation Score to quantify decomposition completeness: the score is 0 when no valid decomposition is identified, 0.5 when a valid but less complete decomposition is obtained than by another method, and 1 when the maximum number of correct decomposition patterns is identified.

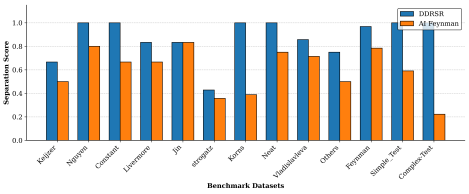
To isolate the effect of the underlying decomposition principles, all brute-force-search-related components are disabled, since exhaustive enumeration may solve otherwise non-decomposable sub-expressions and thereby confound the comparison. In ablation experiments, function values and required derivatives are evaluated directly from the analytical ground-truth functions to isolate and verify the proposed decomposition principles, while in comprehensive experiments, derivatives are approximated using finite differences from actual sampled values to evaluate the applicability of numerical estimation. To further assess DDRSR under non-ideal conditions, we explicitly introduce perturbations into the sampled observations, approximate derivatives using finite differences, and employ a relative-loss criterion with a numerical threshold to accommodate sampling and numerical



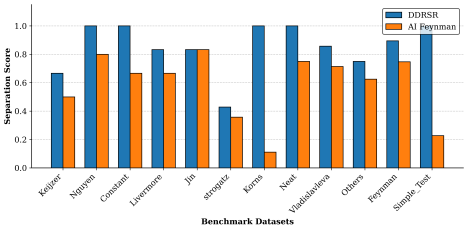
(a) R^2 of Comprehensive Regression



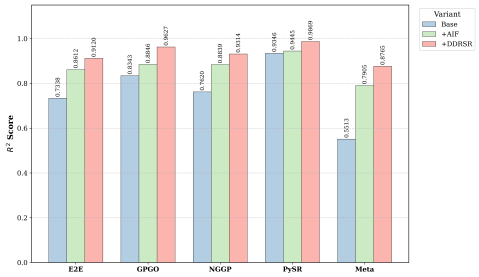
(b) Recovery Rate of Comprehensive Regression



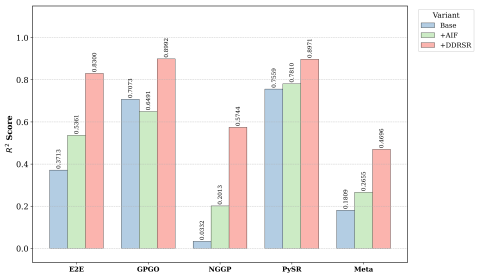
(c) Relative Separation Score of Comprehensive Experiments



(d) Relative Separation Score of Separation Ablation Experiments



(e) R^2 of Separation



(f) R^2 of Translation Symmetry

Figure 2: Comprehensive and ablation evaluation of DDRSR against the undecomposed SR baseline and AI Feynman decomposition. GPGO refers to GP-GOMEA. (a) Mean R^2, (b) exact-expression recovery rate, and (c) Relative Separation Score in the comprehensive experiments. (d) Relative Separation Score and (e) mean R^2 for the top-down expression-separation ablation (variable separability and nested composition). (f) Mean R^2 for the bottom-up variable-composition ablation based on translational symmetry on complex-test.

errors. The resulting effects on decomposition reliability and downstream symbolic regression performance are systematically evaluated in the noise robustness experiments. Equality-based mathematical criteria are evaluated using a relative threshold of 0.01; the exact normalized loss definition, numerical stabilization term, and related implementation details are provided in Appendix B.

For each unknown expression, DDRSR sequentially evaluates the proposed principles and recursively applies every successfully identified decomposition to the resulting sub-expressions until no branch can be further decomposed. Each terminal sub-expression is then processed once by the downstream SR algorithm, and the resulting expressions are recombined in reverse according to the decomposition structure to reconstruct the complete expression. Invalid numerical evaluations involving NaN or Inf are handled through finite resampling. Details of experimental procedure are in Appendix B. Dataset formulas, SR hyperparameters, and hardware configurations are also provided in the Appendix.

4.2 Comprehensive Symbolic Regression Experiments

We evaluate the complete DDRSR framework, including variable separation, nested composition, and translational symmetry, on public benchmarks and custom-constructed test expressions. For these experiments, we select the three best-performing baseline SR methods, E2E, GP-GOMEA, and PySR, with the results summarized in Figure2a – 2c.

DDRSR matches or exceeds AI Feynman in Relative Separation Score across all evaluated dataset groups (Figure 2c), confirming that the proposed principles can identify and mathematically parse a broader range of structural cases. In terms of R^2 (Figure 2a), decomposition using either DDRSR or AI Feynman generally improves over the corresponding undecomposed SR baselines, demonstrating the benefit of reducing complex expressions before regression; dimension-wise statistics in Appendix further show that this advantage becomes increasingly pronounced as expression dimensionality and complexity increase. Recovery-rate results (Figure 2b), together with the R^2 results, show that DDRSR provides better overall performance on the evaluated benchmarks because it can identify and simplify structural cases that remain inaccessible to AI Feynman.

4.3 Ablation Studies

Top-down expression separation.

We independently evaluate top-down decomposition, consisting of variable separation and nested composition, on all datasets except complex-test (Figure 2d and 2e). DDRSR overall achieves higher Relative Separation Scores than AI Feynman, demonstrating its ability to recognize a broader range of separable structures. Both decomposition-based variants outperform the corresponding undecomposed SR methods in R^2, confirming that expression separation alone effectively reduces the difficulty of the target problem; DDRSR further outperforms AI Feynman because its generalized principles successfully separate a wider variety of structures.

Bottom-up variable composition.

We isolate the variable-composition mechanism, that based on translational symmetry, on the custom complex-test dataset (Figure 2f). DDRSR achieves the best R^2 performance among the evaluated methods and obtains a Relative Separation Score of 0.7777, compared with 0.0555 for AI Feynman, demonstrating that the proposed variable-composition principles can identify and combine a substantially broader range of mathematical structures.

4.4 Runtime Analysis

We separately evaluate the running time of DDRSR, AI Feynman, and the baseline method without expression decomposition. The symbolic regression algorithms considered include E2E, PySR, and GP-GOMEA. The evaluation metric is the average running time of each expression over the entire test set, and the results are presented in Table 1.

Table 1: Average running time (s).

SR Method	Baseline	AI Feynman	DDRSR
E2E	4.31	17.84	30.06
PySR	143.52	351.84	439.27
GP-GOMEA	657.98	1246.43	1552.01

DDRSR introduces additional computational overhead, which is expected. This is because DDRSR can achieve a more thorough decomposition of expressions, and a larger number of sub-expressions requires more frequent calls to symbolic regression algorithms, thereby resulting in increased running time.

4.5 Robustness to Noise

Strict mathematical principles are accompanied by exact analytical conditions, which are theoretically correct and reasonable under ideal experimental conditions. However, for non-ideal scenarios where

data contain noise, the effectiveness of DDRSR requires further investigation. We add uniform random noise($\xi \sim \mathcal{U}(-\eta, \eta)$) to the data and count the number of expressions that fail to be successfully decomposed. The results are shown in Table 2.

Table 2: Number of expressions that fail to be decomposed under different noise levels.

	η=0	η=10^{-7}	η=10^{-6}	η=10^{-5}
DDRSR	23	37	66	113
AI Feynman	41	52	81	141

Theoretically, the decomposition principles of DDRSR involve more gradient computations, and gradients are more sensitive to noise. Therefore, noise is expected to have a greater impact on DDRSR than on AI Feynman. However, the experimental results show that, when using relative loss and thresholds for decision-making, DDRSR does not exhibit a substantially larger increase in decomposition failures than AI Feynman in the reported experiments. In addition, under noisy conditions, we apply AI Feynman and DDRSR to all expressions that remain decomposable, and perform symbolic regression on the resulting decomposed sub-expressions. We compare the experimental results under noisy conditions with those obtained without noise interference. Since AI Feynman and DDRSR retain different subsets of successfully decomposable expressions under noise, the PySR baseline in each row is evaluated on the corresponding method-specific subset. The results are shown in Table 3.

Table 3: Regression performance under noise conditions.

	PySR_R2	Divide_PySR_R2	Divide_PySR_R2_Noise
AI Feynman	0.8272	0.9814	0.9796
DDRSR	0.7730	0.9880	0.9861

Table 3 shows that once an expression is correctly decomposed by either DDRSR or AI Feynman, the improvement in symbolic regression performance brought by expression decomposition is almost identical to that under noise-free conditions.

Even when decomposition decisions fail due to noise interference, DDRSR falls back to applying the symbolic regression algorithm directly to the original expression. This fallback mechanism provides a practical safeguard against performance degradation caused by unsuccessful decomposition. Moreover, since the decomposition criteria require the corresponding mathematical properties to hold overall across the sampled domain, while noise is inherently random, noise-induced false-positive decompositions are unlikely to occur. Consequently, DDRSR is generally expected to perform no worse than direct symbolic regression in practice, while successful decomposition can provide additional performance gains, as also supported by our experimental results.

5 Discussion and Conclusion

To overcome the limited applicability of the decomposition principles in AI Feynman, we adopt a "Divide-and-Reduce" philosophy and systematically extend variable separation, nested composition, and translational symmetry, culminating in the proposed DDRSR. By identifying and mathematically decomposing a substantially broader range of structural patterns, DDRSR transforms complex symbolic regression problems into simpler sub-problems and thereby reduces the difficulty of downstream regression.

An important feature of this paradigm is its broad compatibility with existing symbolic regression algorithms: after decomposition, the resulting sub-expressions can be directly processed by different downstream SR methods and subsequently recombined to reconstruct the complete expression. Extensive experiments demonstrate that such decomposition overall improves downstream symbolic regression performance, with DDRSR achieving stronger predictive performance and recovery accuracy by recognizing and simplifying more complex structures than existing decomposition principles. Nevertheless, DDRSR and related mathematically grounded decomposition methods have inherent limitations, particularly in practical scenarios involving imperfect data and numerical estimation; these limitations and the corresponding directions for future development are discussed in Appendix C.

AI USE STATEMENT

In this work, we used generative AI tools solely for English translation and language polishing of text originally written by the authors. Generative AI tools were not used for research ideation, methodology development, algorithm design, code generation, experimental design or implementation, data analysis, result interpretation, or the formulation of scientific claims and conclusions. All AI-assisted translations and language edits were carefully reviewed and verified by the authors to ensure that they accurately reflected the original content and intended meaning. We take full responsibility for the final content of this work, including all text, claims, and results.

ETHICS STATEMENT

This work does not involve human subjects, personal or sensitive data, or other activities that raise specific ethical concerns. To the best of our knowledge, this study does not introduce any notable ethical risks.

REPRODUCIBILITY STATEMENT

The principles and procedures underlying our method are precisely defined, enabling the proposed approach to be reproduced from the descriptions provided in this paper. The symbolic regression methods employed in our experiments are well-established and effective; although they involve stochastic optimization, repeated runs are expected to yield comparable overall results. The source code and implementation details will be made publicly available upon acceptance of this paper.

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A Detailed definitions of the AI Feynman

A.1 Translational Symmetry

For an n-dimensional function $f(X)$, where $X = (x_1, ..., x_n)$ denotes the n-dimensional variables, if f satisfies $f(x_i, x_j, ...) = f(x_i + x_j, ...)$ for any given x_i, x_j, then $f(X)$ is said to possess additive

translational symmetry with respect to dimensions x_i and x_j. Note that the left-hand side of the equation, $f(x_i, x_j, ...)$, involves (n) variables, whereas the right-hand side, $f(x_i + x_j, ...)$, involves only (n-1) variables. This means that x_i and x_j can be combined into a single new variable due to translational symmetry. Similar statements appearing later should be interpreted in the same manner. In the AI Feynman framework, for any $f(x_i, x_j, ...)$, if the condition $f(x_i, x_j, ...) = f(x_i + c, x_j - c, ...)$ holds for an arbitrary constant c, the function $f(X)$ is considered to exhibit additive translational symmetry. Analogous definitions apply to subtraction, multiplication, and division.

However, the AI Feynman method fails to effectively identify these symmetries in the presence of interference of constant coefficients or exponents, such as in the cases of $f(x_1 + 2x_2)$ or $f(x_1 x_2^2)$.

A.2 Variable Separability

Consider an n-dimensional function $f(SX_1, SX_2)$, where SX denotes a set consisting of an arbitrary number of variables x_i ($i \in \{1, ..., n\}$), and all such disjoint sets SX span the n-dimensional space. If f satisfies $f(SX_1, SX_2) = g(SX_1) + h(SX_2)$ for any X, then $f(X)$ is termed additively separable with respect to SX_1 and SX_2. In AI Feynman, for any $f(SX_1, SX_2)$ and a constant c, if the relation $f(SX_1, SX_2) = f(SX_1, c) + f(c, SX_2) - f(c, c)$ is satisfied, $f(X)$ is deemed decomposable into $f(SX_1, SX_2) = g(SX_1) + h(SX_2)$. Multiplicative variable separability is determined via a similar criterion.

Nevertheless, this approach is incapable of effectively identifying separability in scenarios involving overlapping variables or interference of additive constant offset, such as $f(X) = g(x_1, x_2) + h(x_2, x_3)$ or $f(X) = g(x_1)h(x_2) + c$.

A.3 Nested Composition

Compositionality: An n-dimensional function $f(X)$ is said to possess compositionality if it can be formulated as $f(X) = g(h(X))$. AI Feynman postulates that if the gradient $\nabla f(X)$ aligns with the direction of $\nabla h(X)$ for all X, then $f(X)$ can be represented as $f(X) = g(h(X))$.

Generalized Symmetry: An n-dimensional function $f(SX_1, SX_2)$ exhibits generalized symmetry if it takes the form $f(X) = g(h(SX_1), SX_2)$. In AI Feynman, if the direction of the partial gradient $\nabla_{SX_1} f(SX_1, SX_2)$ is independent of SX_2 for all X, $f(X)$ is assumed to be representable as $f(X) = g(h(SX_1), SX_2)$.

Generalized Additivity: A 2-dimensional function $f(x_1, x_2)$ is characterized by generalized additivity if it can be expressed as $f(X) = F(g(x_1) + h(x_2))$. AI Feynman determines this property by checking whether the ratio of partial derivatives $\frac{\partial f/\partial x_1}{\partial f/\partial x_2}$ is multiplicatively separable with respect to x_1 and x_2.

The condition for compositionality requires evaluating $\nabla h(X)$, despite the analytical form of $h(X)$ being intrinsically unknown. The criterion for generalized additivity is not exclusively sensitive to additive structures and is prone to false positives when encountering multiplicative structures (a detailed proof is provided in 3.2.7 of Methodology and D.9 of Appendix). Even if generalized symmetry or generalized additivity is successfully identified, these conditions do not yield the explicit forms of the inner functions g and h.

B Numerical Decision Criteria

During the experiments, for the decision criteria derived from mathematical principles that theoretically require exact equality to zero, we set the threshold to 0.01. Specifically, we define the relative loss as

$$\text{loss}_{\text{relative}} = \frac{|\text{loss}_{\text{math}}|}{\max\limits_{x \in X_{\text{all}}} |x| + \epsilon}, \tag{1}$$

where $\text{loss}_{\text{math}}$ denotes the numerical residual, approximated using the sum of n(10 in experiments) randomly sampled points, of the corresponding expression that is theoretically required to be equal to zero, and X_{all} denotes the set of all computational terms involved in the evaluation of $\text{loss}_{\text{math}}$.

$\epsilon = 10^{-10}$ is introduced to avoid numerical instability when the denominator approaches zero. When $\text{loss}_{\text{relative}}$ is smaller than the predefined threshold, the corresponding criterion is considered satisfied.

For an unknown expression, we sequentially examine it according to the order of the proposed principles. If none of the principles is satisfied, or if valid values cannot be obtained during numerical sampling verification, the corresponding branch is terminated and returned. If a decomposition principle is successfully identified, the next-step computation is performed according to the formula of that principle, and the same decomposition procedure is recursively applied to the resulting sub-expressions until all branches can no longer be decomposed. This constitutes the expression decomposition process.

In the decomposition-based regression experiments, each terminal sub-expression at the end of every decomposition branch is processed by a symbolic regression algorithm once. The symbolic regression results of multiple sub-expressions are then recombined in reverse according to the decomposition structure to reconstruct the original complete expression.

It is worth noting that, for cases where NaN or Inf values occur during numerical evaluation, we perform finite resampling attempts. If valid sampled values cannot be obtained after these attempts, the corresponding criterion is considered invalid.

C Limitation and Future Work

Methods of this kind, such as DDRSR and AI Feynman, are inherently grounded in rigorous mathematical principles. The implementation of these principles typically involves strict numerical criteria and therefore requires access to sufficiently accurate data, which constitutes an inherent limitation of such methods.

In practical applications, we usually have access only to an initial set of randomly sampled data points, whereas rigorous mathematical criteria often require information at specific critical coordinates. This discrepancy restricts the implementation and applicability of such algorithms. AI Feynman alleviates this issue by employing a surrogate model, namely a neural network, to approximate the underlying ground-truth mapping. However, this approach requires a large number of initial samples to adequately capture the fine-grained characteristics of the target domain. Alternatively, when an active sampling environment is available, such as a directly queryable physical field, accurate numerical values and gradient information can be efficiently obtained by combining direct sampling with numerical differentiation, like DDRSR.

Moreover, in the presence of noise, sampling errors, or other perturbations, all sampled data may contain deviations. Consequently, quantities that are theoretically required to be exactly zero may take random nonzero values because of these deviations. This can interfere with the evaluation of the mathematical principles underlying methods such as DDRSR, thereby degrading their decomposition performance.

Nevertheless, when expression decomposition fails because of approximation errors, sampling biases, or other perturbations, the symbolic regression algorithm can still be applied directly to the undecomposed expression, which is equivalent to performing symbolic regression without expression decomposition. Conversely, once a candidate decomposition is successfully identified and verified, the expression can be simplified according to the corresponding decomposition structure. Therefore, decomposition algorithms based on rigorous mathematical principles are designed to avoid performance degradation when decomposition fails, while successful decomposition can provide additional performance gains.

These limitations also suggest two main directions for future development. First, additional mathematical principles with broader applicability, or principles tailored to specific application scenarios, may be incorporated to further extend the theoretical scope of the framework. Second, further improvements can focus on the practical implementation of these principles, including mitigating the effects of noise and sampling errors and developing more accurate surrogate models for reliable estimation of function values and derivative information.

D MATHEMATICAL DERIVATIONS AND PROOFS

D.1 ADDITIVE TRANSLATIONAL SYMMETRY

Derivation: For an n-dimensional function $f(X)$, if $f(X)$ possesses additive translational symmetry with respect to dimensions x_i and x_j, then $f(X)$ can be expressed as:

$$f(x_i, x_j, SX) = f(c_1 \times x_i + c_2 \times x_j, SX)$$

Here, SX denotes the remaining dimensions excluding x_i and x_j. By introducing a new variable $x_{ij} = c_1 \times x_i + c_2 \times x_j$, we obtain:

$$\frac{\partial f(X)}{\partial x_i} = \frac{\partial f(X)}{\partial x_{ij}} \times \frac{\partial x_{ij}}{\partial x_i} = c_1 \times \frac{\partial f(X)}{\partial x_{ij}}$$
$$\frac{\partial f(X)}{\partial x_j} = \frac{\partial f(X)}{\partial x_{ij}} \times \frac{\partial x_{ij}}{\partial x_j} = c_2 \times \frac{\partial f(X)}{\partial x_{ij}}$$

Consequently, the ratio of these partial derivatives is given by:

$$\frac{\partial f(X)}{\partial x_i} \div \frac{\partial f(X)}{\partial x_j} = \frac{c_1}{c_2} = \text{constant}$$

If the above equation holds for any arbitrary x_i and x_j, we determine that $f(X)$ possesses additive translational symmetry along the x_i and x_j dimensions, prompting the introduction of $x_{new} = x_i + \frac{c_2}{c_1} \times x_j$.

D.2 MULTIPLICATIVE TRANSLATIONAL SYMMETRY

Derivation: For an n-dimensional function $f(X)$, if $f(X)$ possesses multiplicative translational symmetry with respect to dimensions x_i and x_j, then $f(X)$ can be formulated as:

$$f(x_i, x_j, SX) = f(x_i^{c_1} \times x_j^{c_2}, SX)$$

Here, SX denotes the remaining dimensions excluding x_i and x_j. By introducing a new variable $x_{ij} = x_i^{c_1} \times x_j^{c_2}$, we obtain:

$$\frac{\partial f(X)}{\partial x_i} = \frac{\partial f(X)}{\partial x_{ij}} \times \frac{\partial x_{ij}}{\partial x_i} = c_1 \times \frac{\partial f(X)}{\partial x_{ij}} \times x_i^{c_1 - 1} \times x_j^{c_2}$$
$$\frac{\partial f(X)}{\partial x_j} = \frac{\partial f(X)}{\partial x_{ij}} \times \frac{\partial x_{ij}}{\partial x_j} = c_2 \times \frac{\partial f(X)}{\partial x_{ij}} \times x_i^{c_1} \times x_j^{c_2 - 1}$$

Consequently, dividing the two equations yields:

$$\frac{\partial f(X)}{\partial x_i} \div \frac{\partial f(X)}{\partial x_j} = \frac{c_1}{c_2} \times \frac{x_j}{x_i}$$
$$\frac{\partial f(X)}{\partial x_i} \div \frac{\partial f(X)}{\partial x_j} \times \frac{x_i}{x_j} = \frac{c_1}{c_2} = \text{constant}$$

If the above equation holds for any arbitrary x_i and x_j, we determine that $f(X)$ possesses multiplicative translational symmetry along the x_i and x_j dimensions, prompting the introduction of $X_{new} = x_i^{c_1} \times x_j^{c_2}$.

D.3 ADDITIVE SEPARABILITY WITH OVERLAPPING VARIABLES

Derivation: For an n-dimensional function $f(X)$, if $f(X)$ exhibits additive separability with overlapping variables with respect to dimensions SX_1 and SX_3, then $f(X)$ can be expressed as:

$$f(SX_1, SX_2, SX_3) = g(SX_1, SX_2) + h(SX_2, SX_3)$$

By manipulating the terms, we derive:

$$f(SX_1, SX_2, SX_3) = [g(SX_1, SX_2) + h(SX_2, SC_3)] + [g(SC_1, SX_2) + h(SX_2, SX_3)] - [g(SC_1, SX_2) + h(SX_2, SC_3)]$$
$$= f(SX_1, SX_2, SC_3) + f(SC_1, SX_2, SX_3) - f(SC_1, SX_2, SC_3)$$

If the above relationship holds for any SX_1 and SX_3 alongside any fixed constants SC, we conclude that $f(X)$ is additively separable across dimensions SX_1 and SX_3. To obtain the explicit function mappings for the sub-functions g and h to facilitate further separation, we define:

$$g(SX_1, SX_2) = f(SX_1, SX_2, SC_3) - f(SC_1, SX_2, SC_3)$$
$$h(SX_2, SX_3) = f(SC_1, SX_2, SX_3)$$

D.4 Multiplicative Separability with Overlapping Variables

Derivation: For an n-dimensional function $f(X)$, if $f(X)$ exhibits multiplicative separability with overlapping variables with respect to dimensions SX_1 and SX_3, then $f(X)$ can be expressed as:

$$f(SX_1, SX_2, SX_3) = g(SX_1, SX_2) \times h(SX_2, SX_3)$$

Through algebraic substitution, we establish:

$$f(SX_1, SX_2, SX_3) = \frac{[g(SX_1, SX_2) \times h(SX_2, SC_3)] \times [g(SC_1, SX_2) \times h(SX_2, SX_3)]}{[g(SC_1, SX_2) \times h(SX_2, SC_3)]}$$

$$= \frac{f(SX_1, SX_2, SC_3) \times f(SC_1, SX_2, SX_3)}{f(SC_1, SX_2, SC_3)}$$

If the above relationship holds for any SX_1 and SX_3 alongside any fixed constants SC, we conclude that $f(X)$ is multiplicatively separable across dimensions SX_1 and SX_3. To obtain the function mappings for sub-functions g and h to enable further decomposition, we define:

$$g(SX_1, SX_2) = \frac{f(SX_1, SX_2, SC_3)}{f(SC_1, SX_2, SC_3)}$$

$$h(SX_2, SX_3) = f(SC_1, SX_2, SX_3)$$

D.5 Compositional Form of Additive Separability

Proof: Assume the function f is additively separable with respect to dimensions SX_1 and SX_3, and concurrently additively separable with respect to SX_2 and SX_3. The function f can therefore be represented as:

$$f(SX_1, SX_2, SX_3) = g_1(SX_1, SX_2) + h_1(SX_2, SX_3) = g_2(SX_1, SX_2) + h_2(SX_1, SX_3)$$

Taking the partial derivative with respect to SX_3 yields:

$$\frac{\partial f(SX_1, SX_2, SX_3)}{\partial SX_3} = \frac{\partial h_1(SX_2, SX_3)}{\partial SX_3} = \frac{\partial h_2(SX_1, SX_3)}{\partial SX_3}$$

Let $T(X) = h_1(SX_2, SX_3) - h_2(SX_1, SX_3)$. It follows that:

$$\frac{\partial T(X)}{\partial SX_3} = \frac{\partial h_1(SX_2, SX_3)}{\partial SX_3} - \frac{\partial h_2(SX_1, SX_3)}{\partial SX_3} = 0$$

This indicates that $T(X)$ is independent of SX_3 and can be strictly expressed in the form $T(X) = T(SX_1, SX_2)$. Simultaneously, we have:

$$\frac{\partial \frac{\partial T(SX_1, SX_2)}{\partial SX_1}}{\partial SX_2} = \frac{\partial \frac{\partial h_2(SX_1, SX_3)}{\partial SX_1}}{\partial SX_2} = 0, \quad \frac{\partial \frac{\partial T(SX_1, SX_2)}{\partial SX_2}}{\partial SX_1} = \frac{\partial \frac{\partial h_1(SX_2, SX_3)}{\partial SX_2}}{\partial SX_1} = 0$$

Thus, $T(X)$ can be further decomposed into the form $T(X) = A(SX_1) + B(SX_2)$. Furthermore:

$$h_1(SX_2, SX_3) - B(SX_2) = h_2(SX_1, SX_3) + A(SX_1)$$

Given that the left side is a function exclusively of SX_2 and SX_3, and the right side is a function exclusively of SX_1 and SX_3, both sides must equal a function solely dependent on SX_3. We therefore deduce:

$$h_1(SX_2, SX_3) = C(SX_3) + B(SX_2), \quad h_2(SX_1, SX_3) = C(SX_3) - A(SX_1)$$

Substituting this back yields:

$$f(SX_1, SX_2, SX_3) = g_1(SX_1, SX_2) + h_1(SX_2, SX_3) = [g_1(SX_1, SX_2) + B(SX_2)] + C(SX_3)$$

This rigorously proves that f admits a representation of the form $f(SX_1, SX_2, SX_3) = g(SX_1, SX_2) + h(SX_3)$.

D.6 Compositional Form of Multiplicative Separability

Proof: Assume that the function f is multiplicatively separable with respect to dimensions SX_1 and SX_3, and concurrently multiplicatively separable with respect to SX_2 and SX_3. The function f can therefore be represented as:

$$\begin{aligned} f(SX_1, SX_2, SX_3) &= g_1(SX_1, SX_2) \times h_1(SX_2, SX_3) \\ &= g_2(SX_1, SX_2) \times h_2(SX_1, SX_3). \end{aligned} \tag{2}$$

The following derivation is conducted on the considered subdomain where the relevant factors and denominators are well defined. We fix SX_1 to an arbitrary valid constant SC_1. It follows that:

$$g_1(SC_1, SX_2) \times h_1(SX_2, SX_3) = g_2(SC_1, SX_2) \times h_2(SC_1, SX_3). \tag{3}$$

Rearranging the above equation yields:

$$h_1(SX_2, SX_3) = \frac{g_2(SC_1, SX_2)}{g_1(SC_1, SX_2)} \times h_2(SC_1, SX_3). \tag{4}$$

Let

$$q(SX_2) = \frac{g_2(SC_1, SX_2)}{g_1(SC_1, SX_2)}, \tag{5}$$

and

$$h(SX_3) = h_2(SC_1, SX_3). \tag{6}$$

We therefore obtain:

$$h_1(SX_2, SX_3) = q(SX_2) \times h(SX_3). \tag{7}$$

Substituting this relationship back into the original expression gives:

$$\begin{aligned} f(SX_1, SX_2, SX_3) &= g_1(SX_1, SX_2) \times h_1(SX_2, SX_3) \\ &= g_1(SX_1, SX_2) \times q(SX_2) \times h(SX_3). \end{aligned} \tag{8}$$

Defining

$$g(SX_1, SX_2) = g_1(SX_1, SX_2) \times q(SX_2), \tag{9}$$

we finally obtain:

$$f(SX_1, SX_2, SX_3) = g(SX_1, SX_2) \times h(SX_3). \tag{10}$$

This rigorously proves that f can be expressed in the form

$$f(SX_1, SX_2, SX_3) = g(SX_1, SX_2) \times h(SX_3). \tag{11}$$

D.7 Multiplicative Separability with Additive Constant Offset

This criterion is applied after standard additive and multiplicative separability have been excluded. In addition, we require $\frac{\partial^2 f(X)}{\partial x_i \partial x_j} \neq 0$ on the considered subdomain to avoid degenerate cases.

Derivation: For an n-dimensional function $f(X)$, if $f(X)$ exhibits multiplicative separability containing a constant term across dimensions SX, it can be expressed as:

$$f(X) = g(x_i, SX) \times h(x_j, SX) + c$$

Here, SX denotes the remaining dimensions excluding x_i and x_j. The partial derivatives are:

$$\frac{\partial f(X)}{\partial x_i} = h(x_j, SX) \times \frac{\partial g(x_i, SX)}{\partial x_i}, \quad \frac{\partial f(X)}{\partial x_j} = g(x_i, SX) \times \frac{\partial h(x_j, SX)}{\partial x_j}$$

$$\frac{\partial^2 f(X)}{\partial x_i \partial x_j} = \frac{\partial g(x_i, SX)}{\partial x_i} \times \frac{\partial h(x_j, SX)}{\partial x_j}$$

By computing the ratios, we obtain:

$$\frac{\partial^2 f(X)}{\partial x_i \partial x_j} \div \frac{\partial f(X)}{\partial x_i} = \frac{\partial h(x_j, SX)}{\partial x_j} \div h(x_j, SX), \quad \frac{\partial^2 f(X)}{\partial x_i \partial x_j} \div \frac{\partial f(X)}{\partial x_j} = \frac{\partial g(x_i, SX)}{\partial x_i} \div g(x_i, SX)$$

Let $u_i(X) = \frac{\partial^2 f(X)}{\partial x_i \partial x_j} \div \frac{\partial f(X)}{\partial x_i}$ and $u_j(X) = \frac{\partial^2 f(X)}{\partial x_i \partial x_j} \div \frac{\partial f(X)}{\partial x_j}$. If $f(X)$ exhibits multiplicative separability with a constant term, $u_i(X)$ is independent of x_i, and $u_j(X)$ is independent of x_j. Thus, $\frac{\partial u_i(X)}{\partial x_i} = 0$ and $\frac{\partial u_j(X)}{\partial x_j} = 0$.

Therefore, the above derivation establishes that multiplicative separability with a constant term necessarily satisfies the proposed differential criterion. In the practical decomposition procedure, this criterion is used as a candidate screening mechanism rather than a standalone proof of separability. When the criterion is satisfied, the corresponding constant term is further recovered and verified using the analytical procedure described in Appendix D.8. Only when the recovered constant term remains invariant under different values of the remaining variables is the structure accepted as a multiplicative separable form with a constant term.

D.8 Separation Mechanism for Multiplicative Forms with Additive Constant Offset

When $f(X)$ satisfies the proposed differential criterion for multiplicative separability with a constant term, we further recover and verify the constant term.

Derivation: Assuming that the candidate structure corresponds to $f(X) = g(x_i, SX) \times h(x_j, SX) + c(SX)$, where $c(SX)$ denotes a residual term that is independent of x_i and x_j, but may depend on the remaining variables S_X. First, we fix all dimensions excluding x_i and x_j to valid random values, establishing a 2-dimensional subspace spanned by x_i and x_j. We sample two valid points along the x_i and x_j dimensions, denoted as (x_{i1}, x_{i2}) and (x_{j1}, x_{j2}) respectively, forming a 2-dimensional bounding rectangle. Within this subspace (where SX acts as a constant, and is thus omitted from the subsequent notation for brevity), we evaluate the integrals:

$$f(x_{i2}, x_{j2}) = f(x_{i1}, x_{j2}) + \int_{x_{i1}}^{x_{i2}} \frac{\partial f(X)}{\partial x_i}\bigg|_{x_j = x_{j2}} = f(x_{i1}, x_{j2}) + h(x_{j2}) \times \int_{x_{i1}}^{x_{i2}} \frac{\partial g(x_i)}{\partial x_i}$$

$$f(x_{i2}, x_{j1}) = f(x_{i1}, x_{j1}) + h(x_{j1}) \times \int_{x_{i1}}^{x_{i2}} \frac{\partial g(x_i)}{\partial x_i}$$

Therefore, by computing the ratio, we derive:

$$\frac{h(x_{j2})}{h(x_{j1})} = \frac{f(x_{i2}, x_{j2}) - f(x_{i1}, x_{j2})}{f(x_{i2}, x_{j1}) - f(x_{i1}, x_{j1})}$$

By identical logic, we deduce:

$$\frac{g(x_{i2})}{g(x_{i1})} = \frac{f(x_{i2}, x_{j2}) - f(x_{i2}, x_{j1})}{f(x_{i1}, x_{j2}) - f(x_{i1}, x_{j1})}$$

Furthermore, we establish the difference:

$$f(x_{i2}, x_{j2}) - f(x_{i2}, x_{j1}) = (g(x_{i2}) \times h(x_{j2}) + c) - (g(x_{i2}) \times h(x_{j1}) + c)$$

Which yields:

$$h(x_{j2}) - h(x_{j1}) = \frac{f(x_{i2}, x_{j2}) - f(x_{i2}, x_{j1})}{g(x_{i2})}$$

Similarly:

$$g(x_{i2}) - g(x_{i1}) = \frac{f(x_{i2}, x_{j2}) - f(x_{i1}, x_{j2})}{h(x_{j2})}$$

By solving this system of simultaneous equations, we can analytically extract:

$$g(x_{i2}) \times h(x_{j2}) = \frac{(f(x_{i2}, x_{j2}) - f(x_{i2}, x_{j1})) \times (f(x_{i2}, x_{j2}) - f(x_{i1}, x_{j2}))}{f(x_{i2}, x_{j2}) - f(x_{i1}, x_{j2}) - f(x_{i2}, x_{j1}) + f(x_{i1}, x_{j1})}$$

Substituting this back into the original expression for $f(x_{i2}, x_{j2})$, the precise value of the constant c is computed as:

$$c(SX) = f(x_{i2}, x_{j2}) - \frac{(f(x_{i2}, x_{j2}) - f(x_{i2}, x_{j1})) \times (f(x_{i2}, x_{j2}) - f(x_{i1}, x_{j2}))}{f(x_{i2}, x_{j2}) - f(x_{i1}, x_{j2}) - f(x_{i2}, x_{j1}) + f(x_{i1}, x_{j1})}$$

It should be emphasized that the above derivation is performed within a two-dimensional subspace with fixed SX, and therefore the recovered residual is initially denoted as $c(SX)$. We repeat the recovery procedure for different values of SX. If

$$c(SX^{(1)}) = c(SX^{(2)}) = \cdots = c,$$

the residual is independent of all remaining variables and is therefore a global constant. Only in this case is the criterion for multiplicative separability with a constant term accepted. Once this invariance is verified, we denote the common value of $c(SX)$ by the global constant c.

Finally, by setting $f_{new}(X) = f(X) - c$, the problem is mathematically reduced to standard multiplicative separability, thereby enabling the execution of subsequent variable separation procedures.

D.9 Generalized Separable Forms

Derivation: For an n-dimensional function $f(X)$, if $f(X)$ exhibits a generalized separable form with respect to dimensions SX_1 and SX_3, it can be expressed as:

$$f(X) = U(g(x_i, SX) + h(x_j, SX)) \quad \text{or} \quad f(X) = U(g(x_i, SX) \times h(x_j, SX))$$

Here, SX denotes the remaining dimensions excluding x_i and x_j. If $f(X)$ assumes the additive internal form $f(X) = U(g(x_i, SX) + h(x_j, SX))$, applying the chain rule yields:

$$\frac{\partial f(X)}{\partial x_i} = \frac{\partial f(X)}{\partial (g(x_i, SX) + h(x_j, SX))} \times \frac{\partial g(x_i, SX)}{\partial x_i}$$

$$\frac{\partial f(X)}{\partial x_j} = \frac{\partial f(X)}{\partial (g(x_i, SX) + h(x_j, SX))} \times \frac{\partial h(x_j, SX)}{\partial x_j}$$

$$\frac{\partial f(X)}{\partial x_i} \div \frac{\partial f(X)}{\partial x_j} = \frac{\partial g(x_i, SX)}{\partial x_i} \div \frac{\partial h(x_j, SX)}{\partial x_j}$$

Let $v(X) = \frac{\partial f(X)}{\partial x_i} \div \frac{\partial f(X)}{\partial x_j}$. It is evident that $v(X)$ is multiplicatively separable across dimensions x_i and x_j. Similarly, if $f(X)$ assumes the multiplicative internal form $f(X) = U(g(x_i, SX) \times h(x_j, SX))$, we have:

$$v(X) = \frac{\partial f(X)}{\partial x_i} \div \frac{\partial f(X)}{\partial x_j} = \frac{\partial g(x_i, SX)}{\partial x_i} \times h(x_j, SX) \div \left(\frac{\partial h(x_j, SX)}{\partial x_j} \times g(x_i, SX) \right)$$

In this scenario, $v(X)$ is equally multiplicatively separable across dimensions x_i and x_j.

Therefore, for any unknown function $f(X)$, we construct the metric $v(X)$ according to the definitions above. If $v(X)$ satisfies multiplicative separability across dimensions x_i and x_j for any X, we identify $f(X)$ as possessing a generalized separable form.

D.10 Nested Separation of Standard Logarithmic and Exponential Functions

Derivation: For an n-dimensional function $f(X)$, if $f(X)$ possesses a nested separation form of a standard logarithmic function, it can be expressed as $f(X) = \log(g(X))$, where $g(X)$ can be subjected to further separation.

Let $f_{new}(X) = \exp(f(X)) = g(X)$. Under this mapping, f_{new} inherently maintains the capacity for further separation. Therefore, for any unknown target mapping $f(X)$, if $f_{new}(X)$ is proven to be separable, we confirm that the original function $f(X)$ possesses a nested separation form of a standard logarithmic function. The derivation and logic for exponential functions (exp) follow an identical paradigm.

It is worth noting that when evaluating the internal separability during this nested decomposition, we typically prohibit subsequent nested separation to prevent infinite recursive calls. Theoretically, however, guided by specific priors, the maximum nesting depth can be established as a tunable hyperparameter, thereby permitting a predefined number of nested hierarchical layers.

D.11 Nested Separation Form of Standard Trigonometric Functions

Derivation: For an n-dimensional function $f(X)$, if $f(X)$ possesses a nested separation form of standard trigonometric functions, it can be expressed as:

$$f(X) = \sin(\text{or } \cos)(g(x_i, SX)+h(x_j, SX)+c) \quad \text{or} \quad f(X) = \sin(\text{or } \cos)(g(x_i, SX)\times h(x_j, SX)+c)$$

Here, SX denotes the remaining dimensions excluding x_i and x_j. If $f(X)$ assumes the internal additive structure $f(X) = \sin(g(x_i, SX) + h(x_j, SX) + c)$, computing the partial derivatives yields:

$$\frac{\partial f(X)}{\partial x_i} = \frac{\partial f(X)}{\partial g(x_i, SX)} \times \frac{\partial g(x_i, SX)}{\partial x_i} = \cos(g(x_i, SX) + h(x_j, SX) + c) \times \frac{\partial g(x_i, SX)}{\partial x_i}$$

$$\frac{\partial f(X)}{\partial x_j} = \cos(g(x_i, SX) + h(x_j, SX) + c) \times \frac{\partial h(x_j, SX)}{\partial x_j}$$

Further multiplying these terms produces:

$$\frac{\partial f(X)}{\partial x_i} \times \frac{\partial f(X)}{\partial x_j} = \cos^2(g(x_i, SX) + h(x_j, SX) + c) \times \frac{\partial g(x_i, SX)}{\partial x_i} \times \frac{\partial h(x_j, SX)}{\partial x_j}$$

$$= (1 - f^2(X)) \times \frac{\partial g(x_i, SX)}{\partial x_i} \times \frac{\partial h(x_j, SX)}{\partial x_j}$$

Let $T(X) = \frac{\partial f(X)}{\partial x_i} \times \frac{\partial f(X)}{\partial x_j} \div (1 - f^2(X))$. If $f(X)$ conforms to the specified structure, $T(X) = \frac{\partial g(x_i, SX)}{\partial x_i} \times \frac{\partial h(x_j, SX)}{\partial x_j}$ is demonstrably multiplicatively separable with respect to dimensions x_i and x_j. Analogously, assuming the function f takes the internal multiplicative form $f(X) = \sin(g(x_i, SX) \times h(x_j, SX) + c)$, we derive:

$$\frac{\partial f(X)}{\partial x_i} = \frac{\partial f(X)}{\partial g(x_i, SX)} \times \frac{\partial g(x_i, SX)}{\partial x_i} = \cos(g(x_i, SX) \times h(x_j, SX) + c) \times h(x_j, SX) \times \frac{\partial g(x_i, SX)}{\partial x_i}$$

$$\frac{\partial f(X)}{\partial x_j} = \cos(g(x_i, SX) \times h(x_j, SX) + c) \times g(x_i, SX) \times \frac{\partial h(x_j, SX)}{\partial x_j}$$

Further multiplying these terms produces:

$$\frac{\partial f(X)}{\partial x_i} \times \frac{\partial f(X)}{\partial x_j} = \cos^2(g(x_i, SX) \times h(x_j, SX) + c) \times \left[g(x_i, SX) \times \frac{\partial g(x_i, SX)}{\partial x_i}\right] \times \left[h(x_j, SX) \times \frac{\partial h(x_j, SX)}{\partial x_j}\right]$$

$$= (1 - f^2(X)) \times \left[g(x_i, SX) \times \frac{\partial g(x_i, SX)}{\partial x_i}\right] \times \left[h(x_j, SX) \times \frac{\partial h(x_j, SX)}{\partial x_j}\right]$$

Let $T(X) = \frac{\partial f(X)}{\partial x_i} \times \frac{\partial f(X)}{\partial x_j} \div (1 - f^2(X))$. If $f(X)$ conforms to the specified structure, $T(X) = \left[g(x_i, SX) \times \frac{\partial g(x_i, SX)}{\partial x_i}\right] \times \left[h(x_j, SX) \times \frac{\partial h(x_j, SX)}{\partial x_j}\right]$ is multiplicatively separable across dimensions x_i and x_j.

The derivation for the cosine function ($\cos$) follows an identical mathematical progression. Therefore, for an unknown function $f(X)$, we construct the metric $T(X)$. When $T(X)$ is verified to be multiplicatively separable across any two arbitrary dimensions, the function $f(X)$ is considered to exhibit a standard trigonometric nested separation form.

It is critical to note that, due to the inherent periodicity of trigonometric operators and the uncertainty regarding the internal sub-functions, acquiring explicit deterministic mappings for the sub-functions is analytically intractable. Consequently, upon identifying this special nested form for $\sin$ (or $\cos$), the algorithm does not proceed with further variable separation along these nodes.

D.12 NON-IDENTIFIABILITY OF DECOMPOSITION WITHOUT VARIABLE SEPARABILITY

Proof: Assume a given function f structurally satisfies the following form:

$$f(SX_1, SX_2) = g(SX_1, SX_2) \oplus h(SX_2)$$

where $\oplus$ denotes an associative and commutative binary operation with an identity element and an inverse operation $\ominus$. Then for any arbitrary function $t(SX_2)$ defined over the subspace SX_2, the following identity holds:

$$f(SX_1, SX_2) = g(SX_1, SX_2) \oplus h(SX_2) \oplus t(SX_2) \ominus t(SX_2)$$

$$= (g(SX_1, SX_2) \ominus t(SX_2)) \oplus (h(SX_2) \oplus t(SX_2)) = g_t(SX_1, SX_2) \oplus h_t(SX_2)$$

This rigorously demonstrates that an infinite number of valid functional configurations exist for $g(SX_1, SX_2)$ and $h(SX_2)$, rendering identifiable decomposition fundamentally impossible under these unconstrained conditions.

E HYPERPARAMETER CONFIGURATIONS OF SYMBOLIC REGRESSION METHODS.

Table 4: Hyperparameter configurations of symbolic regression methods.

Method	Hyperparameter	Value
Hardware	GPU	four NVIDIA GV100(TITAN V, 12GB memory)
	CPU	Intel(R) Xeon(R) Gold 6138 CPU @ 2.00GHz(40 cores)
General	DDR sample variable number	20
	DDR sample constant number	10
	SR sample number	40
	SR test number	40
	variable sample range	(0,1)
	difference step size	1e-5
	Others	default
MetaSymNet	Function Set (S)	$+, *, /, \sin, \cos, \exp, \log, \sqrt{\cdot}$
	Outer Iterations (`ITE`)	5
	Others	default
PySR	`populations`	6
	`population_size`	40
	`ncycles_per_iteration`	400
	`niterations`	10,000
	`timeout_in_seconds`	300
	`maxsize`	50
	`maxdepth`	15
	`binary_operators`	$*, +, -, /$
	`unary_operators`	square, cube, exp, log, sin, cos, sqrt
	`early_stop_condition`	loss $< 1e-6$ && complexity < 30
	`constraints`	$/$: (-1, 20), square: 9, cube: 9, exp: 9, sqrt: 9
	`nested_constraints` (square/cube)	limits nested trig/exp operations
	`complexity_of_operators`	$/$: 2, exp: 3
	`complexity_of_constants`	2
	`turbo`	True
	`Others`	default

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Table 4 – continued from previous page

Method	Hyperparameter	Value
NGGP (DSO)	task.function_set	add, sub, mul, div, sin, cos, exp, log, sqrt, const
	task.metric	inv_nrmse
	metric_params	[1.0]
	task.threshold	1e-6
	task.protected	True
	training.n_samples	20,000
	training.batch_size	500
	training.epsilon	0.05
	training.alpha	0.5
	training.n_cores_batch	1
	controller.learning_rate	0.0005
	controller.entropy_weight	0.005
	controller.entropy_gamma	0.7
	controller.cell	lstm
	prior.length	min: 1, max: 50
	prior.repeat	tokens: [sin, cos, exp, log, sqrt], max: 1
	prior.inverse / trig / const / no_inputs	on: True
	prior.soft_length	loc: 10, scale: 5, on: True
	gp_meld.run_gp_meld	True
	gp_meld.population_size	100
	gp_meld.generations	20
	gp_meld.p_crossover	0.8
	gp_meld.p_mutate	0.05
	gp_meld.tourney_size	3
	Others	default
GP-GOMEA	gomea	True
	functions	$+$, $-$, $*$, $p/$, $\sqrt{\cdot}$, $plog$, exp, $sqrt$
	coeffmut	0.5_0.5_0.5_10
	time	120
	generations / evaluations	-1 (No limit)
	initmaxtreeheight	5
	ims (Interleaved multistart)	5_1
	popsize	64
	batchsize	256
	parallel	4
	linearscaling	False
	Others	default
E2E	max_input_points	200
	n_trees_to_refine	10
	batch_size_eval	1
	rescale	True
	eval_only	True
	replace_ops	add→+, mul→*, sub→-, pow→**, inv→1/
	Others	default

F EXPRESSIONS OF DATASETS

Table 5: Expressions of Datasets

Dataset	Variables	Formula
Keijzer-10	2	pow(x1,x2)
Keijzer-11	2	x1*x2+sin((x1-1)*(x2-1))
Keijzer-12	2	pow(x1,4)-pow(x1,3)+div(pow(x2,2),2)-x2
Keijzer-13	2	6*sin(x1)*cos(x2)
Keijzer-14	2	div(8,2+pow(x1,2)+pow(x2,2))
Keijzer-15	2	div(pow(x1,3),5)+div(pow(x2,3),2)-x2-x1
Nguyen-9	2	sin(x1)+sin(pow(x2,2))
Nguyen-10	2	2*sin(x1)*cos(x2)
Nguyen-11	2	pow(x1,x2)
Nguyen-12	2	pow(x1,4)-pow(x1,3)+div(pow(x2,2),2)-x2
Nguyen-10c	2	sin(1.5*x1)*cos(0.5*x2)
Constant-3	2	sin(1.5*x1)*cos(0.5*x2)
Constant-4	2	2.7*pow(x1,x2); 2.7exp(x2*ln(x1))
Constant-7	2	2*sin(1.3*x1)*cos(x2)
Livermore-5	2	pow(x1,4)-pow(x1,3)+pow(x2,2)-x2
Livermore-10	2	6*sin(x1)*cos(x2)
Livermore-11	2	div(pow(x1,2)*pow(x2,2),(x1+x2))
Livermore-12	2	div(pow(x1,5),pow(x2,3))
Livermore-14	2	pow(x1,3)+pow(x1,2)+x1+sin(x1)+sin(pow(x2,2))
Livermore-17	2	4*sin(x1)*cos(x2)
Jin-1	2	2.5*pow(x1,4)-1.3*pow(x1,3)+0.5*pow(x2,2)-1.7*x2
Jin-2	2	8.0*pow(x1,2)+8.0*pow(x2,3)-15.0
Jin-3	2	0.2*pow(x1,3)+0.5*pow(x2,3)-1.2*x2 -0.5*x1
Jin-4	2	1.5*exp(x1)+5.0*cos(x2)
Jin-5	2	6.0*sin(x1)*cos(x2)
Jin-6	2	1.35*x1*x2 +5.5*sin((x1-1.0)*(x2-1.0))
bacterial respiration 1	2	20-x1-(x1*x2/(1+0.5*pow(x1,2)))
bacterial respiration 3	2	10-(x1*x2/(1+0.5*pow(x1,2)))
bar magnets 1	2	0.5*sin(x1-x2)-sin(x1)
bar magnets 2	2	0.5*sin(x2-x1)-sin(x2)
glider 1	2	-0.05*pow(x1,2)-sin(x2)
glider 2	2	x1-cos(x2)/x1
Lotka-Volterra 1	2	3*x1-2*x1*x2-pow(x1,2)
Lotka-Volterra 2	2	2*x2-x1*x2-pow(x2,2)
predator-prey 1	2	x1*(4-x1-x2/(1+x1))
predator-prey 2	2	x2*(x1/(1+x1)-0.075*x2)
shear flow 1	2	cot(x2)*cos(x1)
shear flow 2	2	(pow(cos(x2),2)+0.1*pow(sin(x2),2))*sin(x1)
van der Pol 1	2	10*(x2-(1/3*(pow(x1,3)-x1)))
van der Pol 2	2	-1/10*x1
Korns-12	2	2 -2.1 *cos(9.8*x1)*sin(1.3*x2)
Korns-2	3	0.23 +14.2 *(x1+x3)/(3*x2)
Korns-8	3	6.87 +11*sqrt(abs(7.23*x1*x2*x3))
Korns-9	3	12*sqrt(abs(4.2*x1*x2*x3))
Korns-3	3	4.9 *(x3-x1+(x2/x4))/(3*x4)-5.41
Korns-10	4	0.81 +24.3 *(2*x1 +3*x2**2)/(4*x3**3 +5*x4**4)
Korns-13	4	32.0 -3.0 *(tan(x1)*tan(x3))/(tan(x2)*tan(x4))
Korns-14	4	22.0 -(4.2*cos(x1)-tan(x2))*tanh(x3)/sin(x4)
Korns-15	4	12.0 -6.0 *tan(x1)*(log(x3)-tan(x4))/exp(x2)
Neat-5	2	2 *sin(x1)*cos(x2)
Neat-7	2	2 -2.1 *cos(9.8*x1)*sin(1.3*x2)
Neat-8	2	exp(-x1**2)/(1.2 +(x2-2.5)**2)
Neat-9	2	1.0/(1+x1**(-4))+1.0/(1+x2**(-4))
Vladislavleva-1	2	exp(-(x1-1)**2)/(1.2+(x2-2.5)**2)
Vladislavleva-3	2	x1**3 *exp(-x1)*cos(x1)*sin(x1)*(sin(x1)**2 *cos(x1)-1)*(x2-5)
Vladislavleva-6	2	6 *sin(x1)*cos(x2)
Vladislavleva-7	2	2 -2.1*cos(9.8*x1)*sin(1.3*x2)
Vladislavleva-8	2	exp(-x1**2)/(1.2 +(x2-2.5)**2)

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Table 5 – continued from previous page

Dataset	Variables	Formula
Vladislavleva-5	3	`30.0 *(x1-1)*(x3-1)*x2**2 /(x1-10)`
Vladislavleva-4	5	`10.0/(5 +(x1-3)**2 +(x2-3)**2 +(x3-3)**2 +(x4-3)**2 +(x5-3)**2)`
Pagie-1	2	`1.0/(1+x1**(-4))+1.0/(1 +x2**(-4)))`
Meier-3	2	`x1**2 *x2**2 /(x1 +x2)`
Meier-4	2	`x1**5 /(x2**3)`
Poly-10	9	`x1*x2 +x3*x4 +x5*x6 +x1*x7*x8 +x3*x6*x9`
Feynman-I.6.2	2	`exp(-(x2/x1)**2/2)/(sqrt(2*pi)*x1)`
Feynman-I.12.1	2	`x1*x2`
Feynman-I.12.5	2	`x1*x2`
Feynman-I.14.4	2	`1/2*x1*pow(x2,2)`
Feynman-I.25.13	2	`x1/x2`
Feynman-I.29.4	2	`x1/x2`
Feynman-I.34.27	2	`(x2/(2*pi))*x1`
Feynman-I.39.1	2	`3/2*x1*x2`
Feynman-II.8.31	2	`x1*pow(x2,2)/2`
Feynman-II.11.28	2	`1+x1*x2/(1-(x1*x2/3))`
Feynman-II.27.18	2	`x1*pow(x2,2)`
Feynman-II.38.14	2	`x1/(2*(1+x2))`
Feynman-III.12.43	2	`x1*(x2/(2*pi))`
Feynman-I.6.20b	3	`exp(-(x1-x2)**2/(2*x3**2))/sqrt(2*pi*x3**2)`
Feynman-I.12.4	3	`x1/(4*pi*x2*x3**2)`
Feynman-I.14.3	3	`x1*x2*x3`
Feynman-I.16.6	3	`(x1+x2)/(1+x1*x2/(x3**2))`
Feynman-I.18.12	3	`x1*x2*sin(x3)`
Feynman-I.27.6	3	`1.0/(1.0/x1 +x3/x2)`
Feynman-I.30.3	3	`x1 *(sin(x2*x3/2.0))**2 /sin(x3/2.0)`
Feynman-I.34.10	3	`x1 /(1-x2/x3)`
Feynman-I.34.14	3	`(1+x1/x2)*x3 /sqrt(1-x1**2/x2**2)`
Feynman-I.37.4	3	`x1 +x2 +2*sqrt(x1*x2)*cos(x3)`
Feynman-I.39.11	3	`x1*x2/(1-x3)`
Feynman-I.43.31	3	`x1*x2*x3`
Feynman-I.48.20	3	`x1*x2**2/(sqrt(1-(x3/x2)**2))`
Feynman-II.4.23	3	`x1 /(4*pi*x2*x3)`
Feynman-II.8.7	3	`0.6 *x1**2 /(4*pi*x2*x3)`
Feynman-II.10.9	3	`x1 /(x2 *(1+x3))`
Feynman-II.15.4	3	`-x1*x2*cos(x3)`
Feynman-II.15.5	3	`-x1*x2*cos(x3)`
Feynman-II.24.17	3	`sqrt((x1/x2)**2 -(pi/x3)**2)`
Feynman-II.27.16	3	`x1*x2*x3**2`
Feynman-II.34.2a	3	`x1*x2/(2*pi*x3)`
Feynman-II.34.2	3	`x1*x2*x3/2`
Feynman-II.34.11	3	`x1*x2/(2*x3)`
Feynman-II.34.29a	3	`x1*x2/(4*pi*x3)`
Feynman-II.37.1	3	`x1*(1+x2)*x3`
Feynman-III.8.54	3	`sin((x1*x2/x3)**2)`
Feynman-III.15.12	3	`2*x1*(1-cos(x2*x3))`
Feynman-III.15.14	3	`x1**2 /(2*x2*x3**2)`
Feynman-III.15.27	3	`2*pi*x1/(x2*x3)`
Feynman-III.17.37	3	`x1*(1+x2*cos(x3))`
Feynman-I.8.14	4	`sqrt((x2-x1)**2 +(x4-x3)**2)`
Feynman-I.12.2	4	`x1*x2 /(4*pi*x3*x4**2)`
Feynman-I.13.4	4	`0.5 *x1 *(x2**2 +x3**2 +x4**2)`
Feynman-I.15.3x	4	`(x1 -x2*x3)/sqrt(1 -(x2/x4)**2)`
Feynman-I.15.3t	4	`(x1 -x2*x3/(x4**2))/sqrt(1 -(x2/x4)**2)`
Feynman-I.18.4	4	`(x1*x3 +x2*x4)/(x1+x2)`
Feynman-I.18.16	4	`x1 *x2 *x3 *sin(x4)`
Feynman-I.24.6	4	`0.25 *x1 *(x2**2 +x3**2)*x4**2`
Feynman-I.29.16	4	`sqrt(x1**2 +x2**2 -1*x1*x2*cos(x3-x4))`
Feynman-I.32.5	4	`x1**2 *x2**2 /(6*pi*x3*x4**3)`
Feynman-I.34.8	4	`x1*x2*x3/x4`

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Table 5 – continued from previous page

Dataset	Variables	Formula
Feynman-I.38.12	4	`4*pi*x1*x2**2/(x3*x4**2)`
Feynman-I.39.22	4	`x1*x2*x3/x4`
Feynman-I.43.16	4	`x1*x2*x3/x4`
Feynman-I.43.43	4	`x1*x2/((x3-1)*x4)`
Feynman-I.47.23	4	`sqrt(x1*x2*x3/x4)`
Feynman-I.50.26	4	`x1*abs(cos(x3*x4)+x2*(cos(x3*x4)**2))`
Feynman-II.6.11	4	`x1 *cos(x2)/(4*pi*x3*x4**2)`
Feynman-II.6.15b	4	`3 *x1 *cos(x2)*sin(x2)/(4*pi*x3*x4**3)`
Feynman-II.11.27	4	`x1*x2*x3*x4 /(1 -x1*x2/3)`
Feynman-II.13.17	4	`(2*x1)/(4*pi*x2*x3**2*x4)`
Feynman-II.13.23	4	`x1*x2 /sqrt(1 -(x3/x4)**2)`
Feynman-II.13.34	4	`x1*x2*x3 /sqrt(1 -(x3/x4)**2)`
Feynman-II.34.29b	4	`x1*x2*x3/x4`
Feynman-II.38.3	4	`x1*x2*x3/x4`
Feynman-III.4.32	4	`1.0/(exp(x1*x2/(x3*x4))-1)`
Feynman-III.4.33	4	`x1*x2/(exp(x1*x2/(x3*x4))-1)`
Feynman-III.7.38	4	`2*x1*x2*x3/x4`
Feynman-III.10.19	4	`x1*sqrt(x2**2+x3**2+x4**2)`
Feynman-III.13.18	4	`2*x1*x2**2*x3/x4`
Feynman-I.12.11	5	`x1*(x2+x3*x4*sin(x5))`
Feynman-I.13.12	5	`x1 *x2 *x3 *(1.0/x5 -1.0/x4)`
Feynman-I.41.16	5	`x1*x2**3/((pi)**2*x3**2*(exp(x1*x2/(x4*x5))-1))`
Feynman-I.44.4	5	`x1*x2*x3*log(x5/x4)`
Feynman-II.2.42	5	`x1*(x3-x2)*x4 /x5`
Feynman-II.11.3	5	`x1 *x2 /(x3*(x4**2-x5**2))`
Feynman-II.11.20	5	`x1*x2**2*x3 /(3*x4*x5)`
Feynman-II.21.32	5	`x1 /(4*pi*x2*x3*(1-x4/x5))`
Feynman-II.35.18	5	`x1 /(exp(x2*x3/(x4*x5))+exp(-x2*x3/(x4*x5)))`
Feynman-II.35.21	5	`x1*x2*tanh(x2*x3/(x4*x5))`
Feynman-III.14.14	5	`x1*(exp(x2*x3/(x4*x5))-1)`
Feynman-III.19.51	5	`x1*x2**4 /(2 *(4*pi*x3)**2 *x4**2 *x5**2)`
Feynman-III.21.20	5	`-x1*x2*x3*x4/x5`
Feynman-I.11.19	6	`x1*x4 +x2*x5 +x3*x6`
Feynman-I.32.17	6	`(0.5*x1*x2*x3**2)*(8*pi*x4**4/3.0)*(x5**4/(x5**2-x6**2)**2)`
Feynman-I.40.1	6	`x1 *exp(x2*x3*x4/(x5*x6))`
Feynman-II.6.15a	6	`3 *x1 *x2 *sqrt(x3**2 +x4**2)/(4*pi*x5*x6**5)`
Feynman-II.11.7	6	`x1 *(1 +(x2*x3*cos(x4)/(x5*x6)))`
Feynman-III.9.52	6	`x1*x2*x3*sin(((x4-x5)*x3/2)**2)/(x6*((x4-x5)*x3/2)**2)`
Feynman-II.36.28	8	`x1*x2/(x3*x4)+x1*x5*x6/(x7*x8**2*x3*x4)`
Feynman-I.9.18	9	`x9*x1*x2 /((x3-x4)**2 +(x5-x6)**2 +(x7-x8)**2)`
Feynman-I.9.18-variant	9	`x9*x1*x2 /sqrt(((x3-x4)**2 +(x5-x6)**2 +(x7-x8)**2))`
simple_test-1	3	`x1*x2+x2*x3`
simple_test-2	3	`sin(x1)*x2+x2*cos(x3)`
simple_test-3	3	`sin(x1)*exp(x2)+x2*cos(x3)`
simple_test-4	3	`(x1+x2)*(x2+x3)`
simple_test-5	3	`(sin(x1)+x2)*(x2+cos(x3))`
simple_test-6	3	`(sin(x1)+exp(x2))*(x2+cos(x3))`
simple_test-7	3	`exp(x1*x2*x3)`
simple_test-8	3	`log(x1+x2+x3)`
simple_test-9	3	`sin(x1*x2*x3)`
simple_test-10	3	`x1*x2*x3+1.2846`
simple_test-11	5	`x1*x2*x3+x3*x4*x5+x1*x3*x5`
complex-test1	5	`log(x1*x2*x3+x3*x4*x5+x1*x3*x5)`
complex-test2	5	`exp(x1*x2*x3+x3*x4*x5+x1*x3*x5)`
complex-test3	5	`x1*x2**2*(0.5*x3+1)/(2*(x4+1)*x5)+1.234`
complex-test4	5	`(x1+1.2*x2+1.4*x3)*(x3+1.2*x4+1.4*x5)`
complex-test5	5	`sin(x1*(x2+1)*(x3-1)+x4*x5**2)`
complex-test6	6	`sqrt(x1*x2**2+(x3+1.5*x4))/(x5**2*x6**3)+2.134`

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Table 5 – continued from previous page

Dataset	Variables	Formula
complex-test7	8	`(x1+0.5)*(x2-0.5)/(x3+2*x4)+(x1-0.5)*(x5-0.5*x6)/(x7*x8**2)`
complex-test8	9	`(x9+1.5)*(x1-1.2)*(x2-1.4)/((x3-2*x4)**2 +(x5-3*x6)**2 +(x7-4*x8)**2)`
complex-test9	9	`x1*x2*x3*x4*x5+x1*x3*x5*x7*x9+x5*x6*x7*x8*x9`

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Table 6: Separation Score of DDRSR and AI Feynman in comprehensive experiments.

Dataset	DDRSR	AI Feynman
Keijzer	0.6666	0.5000
Nguyen	1.0000	0.8000
Constant	1.0000	0.6666
Livermore	0.8333	0.6666
Jin	0.8333	0.8333
strogatz	0.4285	0.3571
Korns	1.0000	0.3888
Neat	1.0000	0.7500
Vladislavleva	0.8571	0.7142
Others	0.7500	0.5000
Feynman	0.9685	0.7842
Simple_Test	1.0000	0.5909
Complex-Test	1.0000	0.2222

Table 7: Performance (R^2 and Recovery Rate) across different number of variables in comprehensive experiments.

Method	Metric	Variant	Number of Variables				
			2	3	4	5	6+
GP-GOMEA	R^2	Orig.	0.9664	0.9441	0.8315	0.8190	-0.1853
		AIF.	0.9918	0.9410	0.9252	0.9853	0.5313
		DDRSR	0.9957	0.9563	0.9798	0.9946	0.8526
	Rec.	Orig.	0.3575	0.4772	0.2514	0.1400	0.0285
		AIF.	0.7393	0.8136	0.7714	0.6300	0.4285
		DDRSR	0.7969	0.8636	0.8285	0.8300	0.6857
E2E	R^2	Orig.	0.9386	0.8129	0.6418	0.3675	0.0405
		AIF.	0.9758	0.9449	0.7755	0.8188	0.6134
		DDRSR	0.9912	0.9659	0.9241	0.9518	0.8529
	Rec.	Orig.	0.0666	0.0181	0.0000	0.0000	0.0000
		AIF.	0.1666	0.2045	0.1257	0.1200	0.0142
		DDRSR	0.1909	0.3772	0.2457	0.2000	0.3428
PySR	R^2	Orig.	0.9974	0.9776	0.8466	0.7466	-0.2700
		AIF.	0.9933	0.9882	0.9246	0.9175	0.7835
		DDRSR	0.9997	0.9934	0.9779	0.9833	0.9843
	Rec.	Orig.	0.6818	0.7500	0.5428	0.5000	0.2142
		AIF.	0.7272	0.8181	0.6000	0.6500	0.5714
		DDRSR	0.8030	0.8863	0.6857	0.7500	0.7142

Table 8: Performance (R^2 and Recovery Rate) across different datasets in comprehensive experiments.

Dataset	Metric	Variant	E2E	GP-GOMEA	PySR	Dataset	Metric	Variant	E2E	GP-GOMEA	PySR
Keijzer	R^2	Orig.	0.9382	0.9896	0.9946	Nguyen	R^2	Orig.	0.9507	0.9936	0.9960
		AIF.	0.9830	0.9940	0.9971			AIF.	0.9864	0.9999	0.9981
		DDRSR	0.9876	0.9949	0.9995			DDRSR	0.9993	1.0000	0.9996
	Rec.	Orig.	0.0000	0.3000	0.6666		Rec.	Orig.	0.0000	0.2000	0.8000
		AIF.	0.0000	0.6666	0.1000			AIF.	0.0000	0.9600	0.5200
		DDRSR	0.0000	0.6666	0.2333			DDRSR	0.1200	1.0000	0.5600
Constant	R^2	Orig.	0.9633	0.9978	0.9212	Livermore	R^2	Orig.	0.9743	0.9673	0.9973
		AIF.	0.9578	1.0000	0.9994			AIF.	0.9967	1.0000	0.9983
		DDRSR	0.9977	1.0000	1.0000			DDRSR	0.9979	1.0000	0.9985
	Rec.	Orig.	0.0000	0.3333	0.3333		Rec.	Orig.	0.0000	0.3000	1.0000
		AIF.	0.0000	1.0000	0.0000			AIF.	0.0000	1.0000	0.4666
		DDRSR	0.0000	1.0000	1.0000			DDRSR	0.0000	1.0000	0.4666
Jin	R^2	Orig.	0.9826	0.9878	0.9998	strogatz	R^2	Orig.	0.9547	0.9984	0.9982
		AIF.	0.9983	0.9900	0.9998			AIF.	0.9892	0.9985	0.9990
		DDRSR	0.9987	0.9906	0.9997			DDRSR	0.9952	0.9993	0.9992
	Rec.	Orig.	0.0000	0.0333	0.8333		Rec.	Orig.	0.0714	0.4000	0.5000
		AIF.	0.1666	0.8000	0.3333			AIF.	0.0857	0.4285	0.3428
		DDRSR	0.1666	0.8333	0.3333			DDRSR	0.1285	0.5714	0.3428
Korns	R^2	Orig.	0.8455	0.5284	0.9866	Neat	R^2	Orig.	0.7414	0.9027	0.9938
		AIF.	0.7685	0.8918	0.9978			AIF.	0.9707	0.9307	0.9990
		DDRSR	0.9278	0.9757	0.9994			DDRSR	0.9961	0.9785	0.9987
	Rec.	Orig.	0.0000	0.0222	0.3333		Rec.	Orig.	0.0000	0.0000	0.5000
		AIF.	0.0000	0.5333	0.3111			AIF.	0.0000	0.5000	0.2000
		DDRSR	0.1111	0.7555	0.4000			DDRSR	0.0000	0.6500	0.3000
Vladislavleva	R^2	Orig.	0.9529	0.8762	0.9463	Others	R^2	Orig.	0.9258	0.8501	0.9917
		AIF.	0.9916	0.9731	0.9747			AIF.	0.9336	0.9172	0.9753
		DDRSR	0.9950	0.9776	0.9878			DDRSR	0.9966	0.9998	0.9975
	Rec.	Orig.	0.0000	0.0571	0.2857		Rec.	Orig.	0.0000	0.1500	0.7500
		AIF.	0.0000	0.6000	0.2571			AIF.	0.0000	0.5000	0.2500
		DDRSR	0.1428	0.7142	0.2571			DDRSR	0.2500	0.7500	0.3500
Feynman	R^2	Orig.	0.5745	0.7724	0.8923	Simple-Test	R^2	Orig.	0.9860	0.9963	0.9995
		AIF.	0.8442	0.9187	0.9882			AIF.	0.9970	0.9978	0.9998
		DDRSR	0.9360	0.9555	0.9986			DDRSR	0.9990	0.9998	0.9998
	Rec.	Orig.	0.0400	0.4105	0.6315		Rec.	Orig.	0.0363	0.4545	1.0000
		AIF.	0.2442	0.8421	0.6294			AIF.	0.1454	0.7272	0.7090
		DDRSR	0.3684	0.8715	0.6715			DDRSR	0.3090	0.8181	0.7090
Complex-Test	R^2	Orig.	0.3713	0.7073	0.7559						
		AIF.	0.6572	0.6700	0.8287						
		DDRSR	0.9141	0.9666	0.9856						
	Rec.	Orig.	0.0000	0.0000	0.2222						
		AIF.	0.0000	0.0666	0.1555						
		DDRSR	0.2000	0.6222	0.5333						

Table 9: Separation Score of DDRSR and AI Feynman in Ablation Experiments of Expression Separation.

Dataset	DDRSR	AI Feynman
Keijzer	0.6666	0.5000
Nguyen	1.0000	0.8000
Constant	1.0000	0.6666
Livermore	0.8333	0.6666
Jin	0.8333	0.8333
strogatz	0.4285	0.3571
Korns	1.0000	0.1111
Neat	1.0000	0.7500
Vladislavleva	0.8571	0.7142
Others	0.7500	0.6250
Feynman	0.8947	0.7473
Simple_Test	1.0000	0.2272

Table 10: Performance (R^2) across different number of variables in Ablation Experiments of Expression Separation.

Method	Variant	Variable Dimensionality				
		2	3	4	5	6+
E2E	Orig.	0.9386	0.8129	0.6418	0.2491	0.0839
	AIF.	0.9618	0.9038	0.8313	0.6166	0.4820
	DDRSR	0.9785	0.9415	0.8833	0.8211	0.5801
GP-GOMEA	Orig.	0.9664	0.9440	0.8314	0.7703	-0.4135
	AIF.	0.9729	0.9572	0.9023	0.8424	-0.0152
	DDRSR	0.9880	0.9747	0.9593	0.9615	0.7573
NGGP	Orig.	0.9754	0.9036	0.6042	0.2963	-0.0179
	AIF.	0.9883	0.9345	0.8432	0.6393	0.4866
	DDRSR	0.9988	0.9410	0.9102	0.7774	0.7500
PySR	Orig.	0.9889	0.9794	0.9463	0.7863	0.5602
	AIF.	0.9936	0.9872	0.9846	0.8605	0.4186
	DDRSR	0.9985	0.9989	0.9977	0.9762	0.8364
MetaSymNet	Orig.	0.8993	0.5268	0.2653	0.1255	0.0026
	AIF.	0.9638	0.7966	0.7423	0.4715	0.2261
	DDRSR	0.9819	0.8870	0.8428	0.6932	0.5285

Table 11: Performance (R^2) across different datasets in Ablation Experiments of Expression Separation.

Dataset	Variant	E2E	GP-GOMEA	NGGP	PySR	MetaSymNet
Keijzer	Orig.	0.9382	0.9896	0.9941	0.9946	0.8527
	AIF.	0.9737	0.9910	0.9959	0.9837	0.9256
	DDRSR	0.9889	0.9890	0.9998	0.9981	0.9360
Nguyen	Orig.	0.9507	0.9936	0.9935	0.9960	0.9822
	AIF.	0.9580	0.9998	0.9997	0.9952	0.9864
	DDRSR	0.9992	0.9999	0.9974	0.9988	0.9985
Constant	Orig.	0.9633	0.9978	0.9981	0.9212	0.9691
	AIF.	0.9750	0.9996	0.9999	0.9991	0.9951
	DDRSR	0.9979	1.0000	1.0000	0.9993	0.9991
Livermore	Orig.	0.9743	0.9673	0.9860	0.9973	0.7776
	AIF.	0.9941	0.9998	0.9832	0.9951	0.9836
	DDRSR	0.9951	1.0000	0.9976	0.9980	0.9865
Jin	Orig.	0.9826	0.9878	0.9937	0.9998	0.9667
	AIF.	0.9971	0.9917	0.9839	0.9993	0.9768
	DDRSR	0.9991	0.9906	0.9948	0.9995	0.9953
strogatz	Orig.	0.9547	0.9984	0.9991	0.9982	0.9714
	AIF.	0.9579	0.9974	0.9993	0.9922	0.9683
	DDRSR	0.9647	0.9990	0.9996	0.9991	0.9903
Korns	Orig.	0.8455	0.5284	0.7417	0.9866	0.1139
	AIF.	0.8561	0.6252	0.8042	0.9838	0.6085
	DDRSR	0.9632	0.9543	0.8790	0.9995	0.9474
Neat	Orig.	0.7414	0.9027	0.8873	0.9938	0.9391
	AIF.	0.9840	0.8817	0.9645	0.9992	0.9352
	DDRSR	0.9944	0.9027	1.0000	0.9967	0.9519
Vladislavleva	Orig.	0.9529	0.8762	0.8639	0.9463	0.7817
	AIF.	0.9482	0.9243	0.9579	0.9789	0.9073
	DDRSR	0.9941	0.9742	0.9601	0.9896	0.9457
Others	Orig.	0.9258	0.8501	0.7722	0.9917	0.4961
	AIF.	0.9612	0.9850	0.9570	0.9965	0.9166
	DDRSR	0.9946	0.9999	0.9990	0.9964	0.9838
Feynman	Orig.	0.5745	0.7723	0.6300	0.8923	0.3753
	AIF.	0.7842	0.8420	0.8207	0.9068	0.7047
	DDRSR	0.8534	0.9452	0.8925	0.9783	0.7985
Simple-Test	Orig.	0.9860	0.9963	0.9586	0.9995	0.8026
	AIF.	0.9888	0.9978	0.9855	0.9998	0.8556
	DDRSR	0.9991	0.9990	0.9993	0.9999	0.9913

H Detailed Experimental Results

Table 12: Detailed Separation of DDRSR and AI Feynman in comprehensive experiments.

Dataset	DDRSR Score	DDRSR Form	AIF Score	AIF Form
Keijzer-10	1.0	exp(f1(x1)*f2(x2))	0.0	f1(x1,x2)
Keijzer-11	0.0	f1(x1,x2)	0.0	f1(x1,x2)
Keijzer-12	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Keijzer-13	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Keijzer-14	0.0	f1(x1,x2)	0.0	f1(x1,x2)
Keijzer-15	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Nguyen-9	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Nguyen-10	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Nguyen-11	1.0	exp(f1(x1)*f2(x2))	0.0	f1(x1,x2)
Nguyen-12	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Nguyen-10c	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Constant-3	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Constant-4	1.0	exp(f1(x1,x2)+0.9933)	0.0	f1(x1,x2)
Constant-7	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Livermore-5	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Livermore-10	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Livermore-11	0.0	f1(x1,x2)	0.0	f1(x1,x2)
Livermore-12	1.0	f1((x1^(5)*x2^(-3)))	0.0	f1(x1,x2)
Livermore-14	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Livermore-17	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Jin-1	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Jin-2	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Jin-3	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Jin-4	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Jin-5	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Jin-6	0.0	f1(x1,x2)	0.0	f1(x1,x2)
bacterial respiration 1	0.0	f1(x1,x2)	0.0	f1(x1,x2)
bacterial respiration 3	1.0	(f1(x1))*(f2(x2))+10.0	0.0	f1(x1,x2)
bar magnets 1	0.0	f1(x1,x2)	0.0	f1(x1,x2)
bar magnets 2	0.0	f1(x1,x2)	0.0	f1(x1,x2)
glider 1	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
glider 2	0.0	f1(x1,x2)	0.0	f1(x1,x2)
Lotka-Volterra 1	0.0	f1(x1,x2)	0.0	f1(x1,x2)
Lotka-Volterra 2	0.0	f1(x1,x2)	0.0	f1(x1,x2)
predator-prey 1	0.0	f1(x1,x2)	0.0	f1(x1,x2)
predator-prey 2	0.0	f1(x1,x2)	0.0	f1(x1,x2)

Dataset	DDRSR Score	DDRSR Form	AIF Score	AIF Form
shear flow 1	1.0	`(f1(x1))*(f2(x2))`	1.0	`(f1(x1))*(f2(x2))`
shear flow 2	1.0	`(f1(x1))*(f2(x2))`	1.0	`(f1(x1))*(f2(x2))`
van der Pol 1	1.0	`f1(x1)+f2(x2)`	1.0	`f1(x1)+f2(x2)`
van der Pol 2	1.0	`f1(x1)+f2(x2)`	1.0	`f1(x1)+f2(x2)`
Korns-12	1.0	`(f1(x1))*(f2(x2))+2.0`	0.0	`f1(x1,x2)`
Korns-2	1.0	`f1(((x1+(1.0000)*x3)^(1)*x2^(-1)))`	1.0	`f1(((x1+x3)/x2))`
Korns-8	1.0	`f1((x1^(1)*x2^(1)*x3^(1)))`	1.0	`f1((x1*x2*x3))`
Korns-9	1.0	`f1((x1^(1)*x2^(1)*x3^(1)))`	1.0	`f1((x1*x2*x3))`
Korns-3	1.0	`(f1(((x1-x3)*x4^(-1)))+f2((x2*x4^(-2))))`	0.5	`f1((x1-x3),x2,x4)`
Korns-10	1.0	`(f1(x1))*(f2(x3,x4))+f3(x2,x3,x4)`	0.0	`f1(x1,x2,x3,x4)`
Korns-13	1.0	`(f1(x1)*f2(x4)*f3(x3)*f4(x2)+32.0)`	0.0	`f1(x1,x2,x3,x4)`
Korns-14	1.0	`(f1(x1))*(f2(x4))*(f3(x3))+(f4(x2))*(f5(x4))*(f6(x3))+22.00`	0.0	`f1(x1,x2,x3,x4)`
Korns-15	1.0	`(f1(x3))*(f2(x2))*(f3(x1))+(f4(x4))*(f5(x2))*(f6(x1))+12.00`	0.0	`f1(x1,x2,x3,x4)`
Neat-5	1.0	`(f1(x1))*(f2(x2))`	1.0	`(f1(x1))*(f2(x2))`
Neat-7	1.0	`(f1(x1))*(f2(x2))+2.0`	0.0	`f1(x1,x2)`
Neat-8	1.0	`(f1(x1))*(f2(x2))`	1.0	`(f1(x1))*(f2(x2))`
Neat-9	1.0	`f1(x1)+f2(x2)`	1.0	`f1(x1)+f2(x2)`
Vladislavleva-1	1.0	`(f1(x1))*(f2(x2))`	1.0	`(f1(x1))*(f2(x2))`
Vladislavleva-3	1.0	`(f1(x1))*(f2(x2))`	1.0	`(f1(x2))*(f2(x1))`
Vladislavleva-6	1.0	`(f1(x1))*(f2(x2))`	1.0	`(f1(x1))*(f2(x2))`
Vladislavleva-7	1.0	`(f1(x1))*(f2(x2))+2.0`	0.0	`f1(x1,x2)`
Vladislavleva-8	1.0	`(f1(x1))*(f2(x2))`	1.0	`(f1(x1))*(f2(x2))`
Vladislavleva-5	1.0	`(f1(x1))*(f2(x3))*(f3(x2))`	1.0	`(f1(x1))*((f2(x2))*(f3(x3)))`
Vladislavleva-4	0.0	`f1(x1,x2,x3,x4,x5)`	0.0	`f1(x1,x2,x3,x4,x5)`
Pagie-1	1.0	`(f1(x1)+f2(x2))`	1.0	`(f1(x1)+f2(x2))`
Meier-3	0.0	`f1(x1,x2)`	0.0	`f1(x1,x2)`
Meier-4	1.0	`f1((x1^(5)*x2^(-3)))`	0.5	`(f1(x1))*(f2(x2))`
Poly-10	1.0	`f1((x2^(1)*x1^(1)))+(f2((x7^(1)*x8^(1))))*(f3(x1))+f4((x4^(1)*x3^(1)))+f5((x9^(1)*x6^(1)*x3^(1)))+f6((x5^(1)*x6^(1)))`	0.5	`f1((x1*(x2+(x7*x8))))+f2(x3,x4,x5,x6,x9)`
Feynman I.6.2	0.0	`f1(x1,x2)`	0.0	`f1(x1,x2)`
Feynman I.12.1	1.0	`f1((x1^(1)*x2^(1)))`	1.0	`f1((x1*x2))`
Feynman I.12.5	1.0	`f1((x1^(1)*x2^(1)))`	1.0	`f1((x1*x2))`
Feynman I.14.4	1.0	`f1((x1^(1)*x2^(2)))`	0.5	`(f1(x1))*(f2(x2))`
Feynman I.25.13	1.0	`f1((x1^(1)*x2^(-1)))`	1.0	`f1((x1/x2))`
Feynman I.29.4	1.0	`f1((x1^(1)*x2^(-1)))`	1.0	`f1((x1/x2))`

Dataset	DDRSR Score	DDRSR Form	AIF Score	AIF Form
Feynman I.34.27	1.0	f1((x1^(1)*x2^(1)))	1.0	f1((x1*x2))
Feynman I.39.1	1.0	f1((x1^(1)*x2^(1)))	1.0	f1((x1*x2))
Feynman II.8.31	1.0	f1((x1^(1)*x2^(2)))	0.5	(f1(x1))*(f2(x2))
Feynman II.11.28	1.0	f1((x1^(1)*x2^(1)))	1.0	f1((x1*x2))
Feynman II.27.18	1.0	f1((x1^(1)*x2^(2)))	0.5	(f1(x1))*(f2(x2))
Feynman II.38.14	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Feynman III.12.43	1.0	f1((x1^(1)*x2^(1)))	1.0	f1((x1*x2))
Feynman I.6.20b	1.0	f1((x1+(-1.0000)*x2),x3)	1.0	f1((x1-x2),x3)
Feynman I.12.4	1.0	f1((x1^(1)*x2^(-1)*x3^(-2)))	0.5	(f1((x1/x2)))*(f2(x3))
Feynman I.14.3	1.0	f1((x1^(1)*x2^(1)*x3^(1)))	1.0	f1((x1*x2*x3))
Feynman-I.16.6	0.0	f1(x1,x2,x3)	0.0	f1(x1,x2,x3)
Feynman-I.18.12	1.0	(f1((x1^(1)*x2^(1))))*(f2(x3))	1.0	(f1((x1*x2)))*(f2(x3))
Feynman-I.27.6	1.0	f1(x1,(x2^(1)*x3^(-1)))	1.0	f1(x1,(x2/x3))
Feynman-I.30.3	1.0	(f1(x1))*(f2(x2,x3))	0.0	f1(x1,x2,x3)
Feynman-I.34.10	1.0	(f1(x1))*(f2((x2^(1)*x3^(-1))))	1.0	(f1(x1))*(f2((x2/x3)))
Feynman-I.34.14	1.0	(f1((x1^(1)*x2^(-1))))*(f2(x3))	0.5	(f1(x3))*(f2(x1,x2))
Feynman-I.37.4	0.0	f1(x1,x2,x3)	0.0	f1(x1,x2,x3)
Feynman-I.39.11	1.0	(f1((x1^(1)*x2^(1))))*(f2(x3))	1.0	(f1((x1*x2)))*(f2(x3))
Feynman-I.43.31	1.0	f1((x1^(1)*x2^(1)*x3^(1)))	1.0	f1((x1*x2*x3))
Feynman-I.48.20	1.0	(f1(x1))*(f2(x2,x3))	1.0	(f1(x1))*(f2(x2,x3))
Feynman-II.4.23	1.0	f1((x1^(1)*x2^(-1)*x3^(-1)))	1.0	f1((x1/x2/x3))
Feynman-II.8.7	1.0	f1((x1^(2)*x2^(-1)*x3^(-1)))	0.5	(f1(x1))*(f2((x2*x3)))
Feynman-II.10.9	1.0	(f1((x1^(1)*x2^(-1))))*(f2(x3))	1.0	(f1((x1/x2)))*(f2(x3))
Feynman-II.15.4	1.0	(f1((x1^(1)*x2^(1))))*(f2(x3))	1.0	(f1((x1*x2)))*(f2(x3))
Feynman-II.15.5	1.0	(f1((x1^(1)*x2^(1))))*(f2(x3))	1.0	(f1((x1*x2)))*(f2(x3))
Feynman-II.24.17	1.0	f1((x1^(1)*x2^(-1)),x3)	0.0	f1(x1,x2,x3)
Feynman-II.27.16	1.0	f1((x1^(1)*x2^(1)*x3^(2)))	0.5	(f1((x1*x2)))*(f2(x3))
Feynman-II.34.2a	1.0	f1((x1^(1)*x2^(1)*x3^(-1)))	1.0	f1((x1*x2/x3))
Feynman-II.34.2	1.0	f1((x1^(1)*x2^(1)*x3^(1)))	1.0	f1((x1*x2*x3))
Feynman-II.34.11	1.0	f1((x1^(1)*x2^(1)*x3^(-1)))	1.0	f1((x1*x2/x3))
Feynman-II.34.29a	1.0	f1((x1^(1)*x2^(1)*x3^(-1)))	1.0	f1((x1*x2/x3))
Feynman-II.37.1	1.0	(f1((x1^(1)*x3^(1))))*(f2(x2))	1.0	(f1((x1*x3)))*(f2(x2))
Feynman-III.8.54	1.0	f1((x1^(1)*x2^(1)*x3^(-1)))	1.0	f1(((x1*x2)/(x3)))
Feynman-III.15.12	1.0	(f1(x1))*(f2((x2^(1)*x3^(1))))	1.0	(f1((x2*x3)))*(f2(x1))
Feynman-III.15.14	1.0	f1((x1^(2)*x2^(-1)*x3^(-2)))	0.5	(f1((x1/x3)))*(f2(x2))
Feynman-III.15.27	1.0	f1((x1^(1)*x2^(-1)*x3^(-1)))	1.0	f1((x1/x2/x3))
Feynman-III.17.37	1.0	(f1(x1))*((f2(x2))*(f3(x3))+1.0000)	0.5	(f1(x1))*(f2(x2,x3))
Feynman-I.8.14	1.0	f1((x1+(-1.0000)*x2),(x3+(-1.0000)*x4))	1.0	f1((x1-x2),(x3-x4))

Dataset	DDRSR Score	DDRSR Form	AIF Score	AIF Form
Feynman-I.12.2	1.0	f1((x1^(1)*x2^(1)*x3^(-1)*x4^(-2)))	0.5	(f1((x1*x2/x3)))*(f2(x4))
Feynman-I.13.4	1.0	f1((x2^(2)*x1^(1)))+f2((x4^(2)*x1^(1)))+f3((x3^(2)*x1^(1)))	0.5	(f1(x1))*(f2(x2)+f3(x3)+f4(x4))
Feynman-I.15.3x	1.0	(f1(x1))*(f2((x2^(1)*x4^(-1))))+(f3(x3))*(f4(x2,x4))	0.0	f1(x1,x2,x3,x4)
Feynman-I.15.3t	1.0	(f1(x1))*(f2((x2^(1)*x4^(-1))))+(f3(x3))*(f4(x2,x4))	0.0	f1(x1,x2,x3,x4)
Feynman-I.18.4	1.0	(f1(x3))*(f2((x1^(1)*x2^(-1))))+(f3(x4))*(f4((x1^(1)*x2^(-1))))	0.5	f1((x1/x2),x3,x4)
Feynman-I.18.16	1.0	(f1((x1^(1)*x2^(1)*x3^(1))))*(f2(x4))	0.5	(f1((x1*x2*x3)))*(f2(x4))
Feynman-I.24.6	1.0	(f1(x2))*(f2((x1^(1)*x4^(2))))+(f3(x3))*(f4((x1^(1)*x4^(2))))	0.5	(f1(x1))*((f2(x4))*(f3(x2)+f4(x3)))
Feynman-I.29.16	1.0	f1(x1,x2,(x3+(-1.0000)*x4))	1.0	f1(x1,x2,(x3-x4))
Feynman-I.32.5	1.0	f1((x1^(2)*x2^(2)*x3^(-1)*x4^(-3)))	0.5	(f1((x1*x2)))*((f2(x3))*(f3(x4)))
Feynman-I.34.8	1.0	f1((x1^(1)*x2^(1)*x3^(1)*x4^(-1)))	1.0	f1((x1*x2*x3/x4))
Feynman-I.38.12	1.0	f1((x1^(1)*x2^(2)*x3^(-1)*x4^(-2)))	0.5	(f1((x1/x3)))*(f2((x2/x4)))
Feynman-I.39.22	1.0	f1((x1^(1)*x2^(1)*x3^(1)*x4^(-1)))	1.0	f1((x1*x2*x3/x4))
Feynman-I.43.16	1.0	f1((x1^(1)*x2^(1)*x3^(1)*x4^(-1)))	1.0	f1((x1*x2*x3/x4))
Feynman-I.43.43	1.0	(f1((x1^(1)*x2^(1)*x4^(-1))))*(f2(x3))	1.0	(f1((x1*x2/x4)))*(f2(x3))
Feynman-I.47.23	1.0	f1((x1^(1)*x2^(1)*x3^(1)*x4^(-1)))	1.0	f1((x1*x2*x3/x4))
Feynman-I.50.26	1.0	(f1(x1))*(f2(x2,(x3^(1)*x4^(1))))	1.0	(f1(x1))*(f2(x2,(x3*x4)))
Feynman-II.6.11	1.0	(f1((x1^(1)*x3^(-1)*x4^(-2))))*(f2(x2))	0.5	(f1((x1/x3)))*((f2(x2))*(f3(x4)))
Feynman-II.6.15b	1.0	(f1((x1^(1)*x3^(-1)*x4^(-3))))*(f2(x2))	0.5	(f1((x1/x3)))*((f2(x2))*(f3(x4)))
Feynman-II.11.27	1.0	(f1((x1^(1)*x2^(1))))*(f2((x3^(1)*x4^(1))))	1.0	(f1((x1*x2)))*(f2((x3*x4)))
Feynman-II.13.17	1.0	f1((x1^(1)*x2^(-1)*x3^(-2)*x4^(-1)))	0.5	(f1((x1/x2/x4)))*(f2(x3))
Feynman-II.13.23	1.0	(f1((x1^(1)*x2^(1))))*(f2((x3^(1)*x4^(-1))))	1.0	(f1((x1*x2)))*(f2(x3,x4))
Feynman-II.13.34	1.0	(f1((x1^(1)*x2^(1))))*(f2(x3,x4))	1.0	(f1((x1*x2)))*(f2(x3,x4))
Feynman-II.34.29b	1.0	f1((x1^(1)*x2^(1)*x3^(1)*x4^(-1)))	1.0	f1((x1*x2*x3/x4))
Feynman-II.38.3	1.0	f1((x1^(1)*x2^(1)*x3^(1)*x4^(-1)))	1.0	f1((x1*x2*x3/x4))
Feynman-III.4.32	1.0	f1((x1^(1)*x2^(1)*x3^(-1)*x4^(-1)))	1.0	f1((x1*x2/x3/x4))
Feynman-III.4.33	1.0	f1((x1^(1)*x2^(1)),(x3^(1)*x4^(1)))	1.0	f1((x1*x2),(x3*x4))
Feynman-III.7.38	1.0	f1((x1^(1)*x2^(1)*x3^(1)*x4^(-1)))	1.0	f1((x1*x2*x3/x4))
Feynman-III.10.19	1.0	(f1(x1))*(f2(x2,x3,x4))	1.0	(f1(x1))*(f2(x2,x3,x4))
Feynman-III.13.18	1.0	f1((x1^(1)*x2^(2)*x3^(1)*x4^(-1)))	0.5	(f1((x1*x3/x4)))*(f2(x2))
Feynman I.12.11	1.0	f1((x2^(1)*x1^(1)))+(f2((x3^(1)*x4^(1))))*(f3(x1))*(f4(x5))	1.0	(f1(x1))*(f2(x2)+(f3((x3*x4)))*(f4(x5)))

Dataset	DDRSR Score	DDRSR Form	AIF Score	AIF Form
Feynman I.13.12	1.0	`(f1(x4))*(f2((x1^(1)*x2^(1)*x3^(1))))+(f3(x5))*(f4((x1^(1)*x2^(1)*x3^(1))))`	1.0	`(f1((x1*x2*x3)))*(f2(x4)+f3(x5))`
Feynman I.41.16	1.0	`(f1(x1,x2,(x4^(1)*x5^(1))))*(f2(x3))`	1.0	`(f1(x3))*(f2(x1,x2,(x4*x5)))`
Feynman I.44.4	1.0	`(f1((x1^(1)*x2^(1)*x3^(1))))*(f2((x4^(1)*x5^(-1))))`	1.0	`(f1((x1*x2*x3)))*(f2((x4/x5)))`
Feynman II.2.42	1.0	`f1((x1^(1)*(x2+(-1.0000)*x3)^(1)*x4^(1)*x5^(-1)))`	1.0	`f1((x1*(x2-x3)*x4/x5))`
Feynman II.11.3	1.0	`(f1((x1^(1)*x2^(1)*x3^(-1))))*(f2(x4,x5))`	1.0	`(f1((x1*x2/x3)))*(f2(x4,x5))`
Feynman II.11.20	1.0	`f1((x1^(1)*x2^(2)*x3^(1)*x4^(-1)*x5^(-1)))`	0.5	`(f1((x1*x3/x4/x5)))*(f2(x2))`
Feynman II.21.32	1.0	`(f1((x1^(1)*x2^(-1)*x3^(-1))))*(f2((x4^(1)*x5^(-1))))`	1.0	`(f1((x1/x2/x3)))*(f2((x4/x5)))`
Feynman II.35.18	1.0	`(f1(x1))*(f2((x2^(1)*x3^(1)*x4^(-1)*x5^(-1))))`	1.0	`(f1(x1))*(f2((x2*x3/x4/x5)))`
Feynman II.35.21	1.0	`(f1(x1))*(f2(x2,(x3^(1)*x4^(-1)*x5^(-1))))`	1.0	`(f1(x1))*(f2(x2,(x3/x4/x5)))`
Feynman III.14.14	1.0	`(f1(x1)+f2((x2^(1)*x3^(1))))*(f3(x4,x5,(x2^(1)*x3^(1))))`	0.5	`(f1(x1))*(f2(x2,x3,x4,x5))`
Feynman III.19.51	1.0	`f1((x1^(1)*x2^(4)*x3^(-2)*x4^(-2)*x5^(-2)))`	0.5	`f1(x1,x2,(x3*x4*x5))`
Feynman III.21.20	1.0	`f1((x1^(1)*x2^(1)*x3^(1)*x4^(1)*x5^(-1)))`	1.0	`f1((x1*x2*x3*x4/x5))`
Feynman-I.11.19	1.0	`f1((x1^(1)*x4^(1)))+f2((x3^(1)*x6^(1)))+f3((x2^(1)*x5^(1)))`	1.0	`f1(((x1*x4)+(x2*x5)+(x3*x6)))`
Feynman-I.32.17	1.0	`(f1((x1^(1)*x2^(1)*x3^(2)*x4^(4))))*(f2((x5^(1)*x6^(-1))))`	0.5	`f1((x1*x2),x3,x4,(x5/x6))`
Feynman-I.40.1	1.0	`(f1(x1)+f2((x2^(1)*x3^(1)*x4^(1)*x5^(-1)))*(f3((x6^(1)),(x2^(1)*x3^(1)*x4^(1)*x5^(-1))))`	0.5	`(f1(x1))*(f2(x2,x3,x4,x5,x6))`
Feynman-II.6.15a	1.0	`(f1((x1^(1)*x2^(1)*x5^(-1)*x6^(-5))))*(f2(x3,x4))`	1.0	`(f1((x1*x2/x5)))*((f2(x6))*(f3(x3,x4)))`
Feynman-II.11.7	1.0	`(f1(x1))*((f2((x2^(1)*x3^(1)*x5^(-1)*x6^(-1))))*(f3(x4))+1.0000)`	0.5	`(f1(x1))*(f2((x2*x3/x5/x6),x4))`
Feynman-III.9.52	1.0	`(f1((x1^(1)*x2^(1)*x6^(-1))))*(f2(x3,x5))+f3(x4)+f4(x5)+f5(x3)`	1.0	`(f1((x1*x2/x6)))*(f2(x3,(x4-x5)))`
Feynman-II.36.28	1.0	`(f1(x2))*(f2((x1^(1)*x3^(-1)*x4^(-1))))+(f3((x5^(1)*x6^(1)*x7^(-1)*x8^(-2))))*(f4((x1^(1)*x3^(-1)*x4^(-1))))`	0.5	`(f1((x1/x3/x4)))*(f2(x2)+f3((x5*x6/x7),x8))`
Feynman-I.9.18	1.0	`(f1((x1^(1)*x2^(1)*x9^(1))))*(f2((x3+(-1.0000)*x4),(x5+(-1.0000)*x6),(x7+(-1.0000)*x8)))`	1.0	`(f1((x1*x2*x9)))*(f2((x3-x4),(x5-x6),(x7-x8)))`

Dataset	DDRSR Score	DDRSR Form	AIF Score	AIF Form
Feynman-I.9.18-variant	1.0	(f1((x1^(1)*x2^(1)*x9^(1))))*(f2((x3+(-1.0000)*x4),(x5+(-1.0000)*x6),(x7+(-1.0000)*x8)))	1.0	(f1((x1*x2*x9)))*(f2((x3-x4),(x5-x6),(x7-x8)))
simple_test-1	1.0	f1(((x1+(1.0000)*x3)^(1)*x2^(1)))	1.0	f1(((x1+x3)*x2))
simple_test-2	1.0	(f1(x1))*(f2(x2))+(f3(x3))*(f4(x2))	1.0	(f1(x2))*(f2(x1)+f3(x3))
simple_test-3	1.0	(f1(x1))*(f2(x2))+(f3(x3))*(f4(x2))	0.0	f1(x1,x2,x3)
simple_test-4	1.0	((f1(x1))*(f2(x2))+1.0000)*(f3(x3,x2))	0.0	f1(x1,x2,x3)
simple_test-5	1.0	(f1(x1,x2)+1.0000)*(f2(x3,x2))	0.0	f1(x1,x2,x3)
simple_test-6	1.0	((f1(x1))*(f2(x2))+1.0000)*(f3(x3,x2))	0.0	f1(x1,x2,x3)
simple_test-7	1.0	f1((x1^(1)*x2^(1)*x3^(1)))	1.0	f1((x1*x2*x3))
simple_test-8	1.0	f1((x1+(1.0000)*x2+(1.0000)*x3))	1.0	f1((x1+x2+x3))
simple_test-9	1.0	f1((x1^(1)*x2^(1)*x3^(1)))	1.0	f1((x1*x2*x3))
simple_test-10	1.0	f1((x1^(1)*x2^(1)*x3^(1)))	1.0	f1((x1*x2*x3))
simple_test-11	1.0	f1((x2^(1)*x1^(1)*x3^(1)))+f2((x5^(1)*x1^(1)*x3^(1)))+f3((x4^(1)*x3^(1)*x5^(1)))	0.5	(f1(x3))*(f2(x1,x2,x4,x5))
complex-test1	1.0	log((f1(x2))*(f2(x1))+(f3(x5))*(f4(x1))+f5((x4^(1)*x5^(1))))+f6(x3)	0.5	f1(x3)+f2(x1,x2,x4,x5)
complex-test2	1.0	(exp(f1(x2)*f2((x1^(1)*x3^(1)))))*(exp(f3(x5)*f4(x3)*f5(x1)))*(exp((f6(x4))*(f7(x5))+0.3333*f8(x3)))	0.0	f1(x1,x2,x3,x4,x5)
complex-test3	1.0	(f1((x1^(1)*x2^(2)*x5^(-1))))*(f2(x4))*(f3(x3))+1.2339	0.5	f1((x1/x5),x2,x3,x4)
complex-test4	1.0	(f1((x1+(1.2000)*x2),x3)+1.0001)*(f2((x4+(1.1667)*x5),x3))	0.0	f1(x1,x2,x3,x4,x5)
complex-test5	1.0	sin(f1((x1+(2.0000)*x2+(3.0000)*x3),(x4^(1)*x5^(2))))	0.0	f1(x1,x2,x3,x4,x5)
complex-test6	1.0	f1((x1^(1)*x2^(2)),(x3+(1.5000)*x4),(x5^(2)*x6^(3)))	0.0	f1(x1,x2,x3,x4,x5,x6)
complex-test7	1.0	((f1(x2))*(f2((x3+(2.0000)*x4)))+0.3847605586051941)*(f3(x1))+f4(((x5+(-0.5000)*x6)^(1)*x7^(-1)*x8^(-2)),x1)	0.0	f1(x1,x2,x3,x4,x5,x6,x7,x8)
complex-test8	1.0	(f1(x1))*(f2(x9))*(f3((x3+(-2.0000)*x4),(x5+(-3.0000)*x6),(x7+(-4.0000)*x8)))*(f4(x2))	0.5	(f1(x1))*((f2(x2))*((f3(x9))*(f4(x3,x4,x5,x6,x7,x8))))

Dataset	DDRSR Score	DDRSR Form	AIF Score	AIF Form
complex-test9	1.0	`(f1((x2^(1)*x4^(2))))*(f2((x1^(1)*x3^(1)*x5^(1))))+f3((x7^(2)*x9^(1)*x1^(1)*x3^(2)*x5^(1)))+(f4((x6^(2)*x8^(1))))*(f5((x5^(1)*x7^(1)*x9^(1))))`	0.5	`(f1(x5))*(f2(x1,x2,x3,x4,x6,x7,x8,x9))`

Table 13: Detailed Performance (R^2 and Recovery Rate) in comprehensive experiments.

Dataset	GP-GOMEA						E2E						PySR					
	Base		+AIF		+Div		Base		+AIF		+Div		Base		+AIF		+Div	
	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec
Keijzer-10	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.7149	0.00	0.9800	0.00	0.9973	0.00	0.9987	0.00	0.9990	0.00	1.0000	1.00
Keijzer-11	0.9848	0.00	0.9818	0.00	0.9828	0.00	0.9941	0.00	0.9923	0.00	0.9977	0.00	0.9904	0.00	1.0000	1.00	1.0000	1.00
Keijzer-12	0.9611	0.00	1.0000	1.00	1.0000	1.00	0.9816	0.00	0.9939	0.00	0.9987	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Keijzer-13	0.9992	0.00	1.0000	1.00	1.0000	1.00	0.9993	0.00	0.9981	0.00	0.9971	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Keijzer-14	0.9930	0.00	0.9823	0.00	0.9867	0.00	0.9458	0.00	0.9340	0.00	0.9358	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Keijzer-15	0.9999	0.80	1.0000	1.00	1.0000	1.00	0.9938	0.00	0.9997	0.00	0.9993	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Nguyen-9	0.9968	0.00	1.0000	1.00	1.0000	1.00	0.9891	0.00	0.9984	0.00	0.9994	0.40	1.0000	1.00	1.0000	1.00	1.0000	1.00
Nguyen-10	0.9988	0.00	0.9999	0.80	1.0000	1.00	0.9992	0.00	0.9988	0.00	0.9997	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Nguyen-11	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.8117	0.00	0.9414	0.00	0.9985	0.00	0.8932	0.00	0.9989	0.00	0.9992	0.00
Nguyen-12	0.9743	0.00	1.0000	1.00	1.0000	1.00	0.9562	0.00	0.9940	0.00	0.9997	0.20	1.0000	1.00	1.0000	1.00	1.0000	1.00
Nguyen-10c	0.9984	0.00	1.0000	1.00	1.0000	1.00	0.9974	0.00	0.9996	0.00	0.9996	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Constant-3	0.9956	0.00	1.0000	1.00	1.0000	1.00	0.9966	0.00	0.9996	0.00	0.9995	0.00	0.9999	0.00	0.9998	0.00	1.0000	1.00
Constant-4	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.8952	0.00	0.8751	0.00	0.9945	0.00	0.9960	0.00	0.9994	0.00	0.9998	0.00
Constant-7	0.9980	0.00	1.0000	1.00	1.0000	1.00	0.9982	0.00	0.9988	0.00	0.9992	0.00	1.0000	1.00	0.9998	0.00	1.0000	1.00
Livermore-5	0.9042	0.00	1.0000	1.00	1.0000	1.00	0.8973	0.00	0.9996	0.00	0.9991	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Livermore-10	0.9994	0.20	1.0000	1.00	1.0000	1.00	0.9987	0.00	0.9997	0.00	0.9992	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Livermore-11	0.9999	0.80	1.0000	1.00	1.0000	1.00	0.9903	0.00	0.9914	0.00	0.9920	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Livermore-12	0.9013	0.00	1.0000	1.00	1.0000	1.00	0.9642	0.00	0.9915	0.00	0.9991	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Livermore-14	0.9998	0.80	1.0000	1.00	1.0000	1.00	0.9974	0.00	0.9996	0.00	0.9996	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Livermore-17	0.9993	0.00	1.0000	1.00	1.0000	1.00	0.9983	0.00	0.9989	0.00	0.9986	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Jin-1	0.9875	0.00	1.0000	1.00	1.0000	1.00	0.9796	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Jin-2	0.9995	0.00	1.0000	1.00	1.0000	1.00	0.9852	0.00	0.9962	0.00	0.9980	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Jin-3	0.9960	0.00	1.0000	1.00	1.0000	1.00	0.9432	0.00	0.9982	0.00	0.9998	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Jin-4	0.9978	0.00	0.9999	0.80	1.0000	1.00	0.9984	0.00	0.9997	0.00	0.9997	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Jin-5	0.9994	0.20	1.0000	1.00	1.0000	1.00	0.9952	0.00	0.9995	0.00	0.9991	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Jin-6	0.9466	0.00	0.9406	0.00	0.9438	0.00	0.9940	0.00	0.9967	0.00	0.9956	0.00	0.9885	0.00	0.8761	0.00	0.9999	0.00
bacterial respiration 1	0.9999	0.80	0.9915	0.00	0.9975	0.00	0.9116	0.00	0.9917	0.00	0.9866	0.00	0.9885	0.00	1.0000	1.00	1.0000	1.00
bacterial respiration 3	0.9909	0.00	0.9967	0.00	1.0000	1.00	0.9982	0.00	0.9409	0.00	0.9995	0.00	0.9997	0.00	0.9999	0.00	0.9999	0.00
bar magnets 1	0.9980	0.00	0.9977	0.00	0.9977	0.00	0.9930	0.00	0.9985	0.00	0.9975	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
bar magnets 2	0.9968	0.00	0.9985	0.00	0.9984	0.00	0.9947	0.00	0.9941	0.00	0.9980	0.00	0.9998	0.00	1.0000	1.00	1.0000	1.00
glider 1	0.9999	0.80	1.0000	1.00	1.0000	1.00	0.9994	0.20	0.9996	0.20	0.9998	0.20	0.9997	0.00	1.0000	1.00	1.0000	1.00

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Table 13 – continued from previous page

Dataset	GP-GOMEA						E2E						PySR					
	Base		+AIF		+Div		Base		+AIF		+Div		Base		+AIF		+Div	
	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec
glider 2	0.9994	0.00	0.9991	0.40	0.9997	0.60	0.4977	0.00	0.9410	0.00	0.9660	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Lotka-Volterra 1	0.9985	0.00	0.9984	0.60	0.9994	0.00	0.9994	0.00	0.9993	0.00	0.9994	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Lotka-Volterra 2	0.9999	0.80	1.0000	0.00	1.0000	1.00	0.9992	0.00	0.9995	0.00	0.9995	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
predator-prey 1	0.9999	0.80	0.9998	0.00	0.9995	0.40	0.9993	0.00	0.9994	0.00	0.9999	0.00	0.9999	0.00	0.9999	0.00	0.9994	0.00
predator-prey 2	0.9972	0.00	0.9986	0.00	0.9989	0.00	0.9925	0.00	0.9910	0.00	0.9948	0.00	0.9986	0.00	0.9999	0.00	1.0000	1.00
shear flow 1	0.9986	0.00	1.0000	1.00	1.0000	1.00	0.9911	0.00	0.9974	0.00	0.9926	0.00	1.0000	1.00	0.9999	0.00	1.0000	1.00
shear flow 2	0.9999	0.60	1.0000	1.00	1.0000	1.00	0.9908	0.00	0.9977	0.20	0.9998	0.40	0.9996	0.00	1.0000	1.00	1.0000	1.00
van der Pol 1	0.9999	0.80	1.0000	1.00	1.0000	1.00	0.9996	0.00	0.9996	0.00	0.9998	0.20	1.0000	1.00	1.0000	1.00	1.0000	1.00
van der Pol 2	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9999	0.80	1.0000	0.80	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Korns-12	0.3539	0.00	1.0000	1.00	1.0000	1.00	0.5815	0.00	0.4608	0.00	0.9271	0.00	0.9999	0.00	0.8278	0.00	0.9999	0.00
Neat-5	0.9981	0.00	0.9999	0.60	0.9999	0.60	0.9991	0.00	0.9996	0.00	0.9998	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Neat-7	0.6212	0.00	0.7232	0.00	0.9143	0.00	-0.0004	0.00	0.8877	0.00	0.9991	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Neat-8	0.9999	0.00	1.0000	1.00	1.0000	1.00	0.9787	0.00	0.9995	0.00	0.9995	0.00	0.9967	0.00	0.9997	0.00	0.9980	0.00
Neat-9	0.9917	0.00	0.9999	0.40	1.0000	1.00	0.9883	0.00	0.9962	0.00	0.9860	0.00	0.9999	0.00	0.9951	0.00	0.9998	0.00
Vladislavleva-1	0.9809	0.00	0.9998	0.20	1.0000	1.00	0.9896	0.00	0.9998	0.00	0.9990	0.00	0.9930	0.00	0.9854	0.00	0.9994	0.00
Vladislavleva-3	0.9994	0.00	1.0000	1.00	1.0000	1.00	0.9928	0.00	0.9984	0.00	0.9884	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Vladislavleva-6	0.9998	0.40	1.0000	1.00	1.0000	1.00	0.9990	0.00	0.9993	0.00	0.9994	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Vladislavleva-7	0.2506	0.00	0.8636	0.00	0.9038	0.00	0.7138	0.00	0.9778	0.00	0.9907	0.00	0.9998	0.00	1.0000	1.00	1.0000	1.00
Vladislavleva-8	0.9862	0.00	1.0000	1.00	1.0000	1.00	0.9912	0.00	0.9833	0.00	0.9974	0.00	0.9988	0.00	0.9993	0.00	0.9983	0.00
Pagie-1	0.9985	0.00	0.9978	0.00	0.9994	0.00	0.9478	0.00	0.9888	0.00	0.9918	0.00	0.9927	0.00	0.9637	0.00	0.9965	0.00
Meier-3	0.9997	0.00	1.0000	1.00	1.0000	1.00	0.9946	0.00	0.9953	0.00	0.9948	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Meier-4	0.9999	0.60	1.0000	1.00	1.0000	1.00	0.9804	0.00	0.9950	0.00	0.9999	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.6.2	0.9966	0.00	0.9948	0.00	0.9968	0.00	0.4240	0.00	0.6039	0.00	0.7265	0.00	0.9974	0.00	0.9206	0.00	0.9951	0.00
Feynman-I.12.1	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.12.5	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.14.4	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9998	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.25.13	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9918	0.00	0.9989	0.60	0.9997	0.80	1.0000	1.00	1.0000	1.00	1.0000	0.00
Feynman-I.29.4	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9989	0.00	0.9999	0.20	0.9982	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.34.27	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9999	0.60	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.39.1	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9997	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.8.31	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9997	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.11.28	0.9996	0.20	1.0000	1.00	1.0000	1.00	0.9967	0.00	0.9970	0.00	0.9991	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00

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Table 13 – continued from previous page

Dataset	GP-GOMEA						E2E						PySR					
	Base		+AIF		+Div		Base		+AIF		+Div		Base		+AIF		+Div	
	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec
Feynman-II.27.18	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9999	0.60	0.9999	0.00	0.9994	0.40	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.38.14	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9999	0.20	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9993	0.00	1.0000	1.00
Feynman-III.12.43	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9998	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Korns-2	0.9999	0.20	1.0000	1.00	1.0000	1.00	0.9671	0.00	0.9981	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Korns-8	0.9791	0.00	0.9998	0.80	1.0000	1.00	0.9939	0.00	0.9982	0.00	0.9997	0.00	0.9988	0.00	0.9542	0.00	0.9990	0.00
Korns-9	0.9999	0.00	1.0000	1.00	1.0000	1.00	0.9997	0.00	0.9997	0.00	0.9996	0.00	0.9858	0.00	0.9543	0.00	0.9952	0.00
Vladislavleva-5	0.9883	0.00	1.0000	1.00	1.0000	1.00	0.9996	0.00	0.9997	0.00	1.0000	1.00	0.9701	0.00	1.0000	1.00	1.0000	1.00
Feynman-I.6.20b	0.6867	0.00	0.7831	0.00	0.8847	0.00	0.3847	0.00	0.3487	0.00	0.7117	0.00	0.6738	0.00	0.7376	0.00	0.7362	0.00
Feynman-I.12.4	0.9999	0.80	1.0000	1.00	1.0000	1.00	0.8485	0.00	0.5295	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.14.3	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9997	0.00	1.0000	1.00	0.9999	0.80	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.16.6	0.9703	0.00	0.9925	0.00	0.9876	0.00	0.5432	0.00	0.9032	0.00	0.8489	0.00	0.9214	0.00	0.8491	0.00	1.0000	1.00
Feynman-I.18.12	0.9999	0.80	1.0000	1.00	1.0000	1.00	0.9983	0.00	0.9997	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.27.6	0.9999	0.00	1.0000	1.00	1.0000	1.00	0.8341	0.00	0.9251	0.00	0.9711	0.00	0.9998	0.00	1.0000	1.00	1.0000	1.00
Feynman-I.30.3	0.9998	0.40	1.0000	1.00	1.0000	1.00	0.9993	0.00	0.9984	0.00	0.9991	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.34.10	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.2213	0.00	0.9936	0.00	0.9572	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.34.14	0.9999	0.20	1.0000	1.00	1.0000	1.00	0.8892	0.00	0.8528	0.00	0.9978	0.00	0.5920	0.00	0.9947	0.00	0.9999	0.00
Feynman-I.37.4	0.9853	0.00	0.9960	0.00	0.9978	0.00	0.9763	0.00	0.9938	0.00	0.9949	0.00	0.9991	0.00	1.0000	1.00	1.0000	1.00
Feynman-I.39.11	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9907	0.00	0.9878	0.20	0.9805	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.43.31	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9996	0.40	0.9999	0.60	1.0000	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.48.20	0.9784	0.00	0.9773	0.00	0.9991	1.00	0.7286	0.00	0.9568	0.00	0.8231	0.00	0.8778	0.00	0.9913	0.00	0.9834	0.00
Feynman-II.4.23	0.9999	0.00	1.0000	1.00	1.0000	1.00	0.1657	0.00	0.9735	0.20	0.9999	0.20	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.8.7	0.9896	0.00	1.0000	1.00	1.0000	1.00	0.4925	0.00	0.9895	0.00	1.0000	1.00	0.9999	0.00	1.0000	1.00	1.0000	1.00
Feynman-II.10.9	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9988	0.00	0.9939	0.40	0.9999	0.00	1.0000	1.00	0.9997	0.00	1.0000	1.00
Feynman-II.15.4	0.9999	0.20	1.0000	1.00	1.0000	1.00	0.9990	0.00	0.9999	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.15.5	0.9999	0.40	1.0000	1.00	1.0000	1.00	0.9913	0.00	0.9996	0.00	0.9999	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.24.17	0.9999	0.00	1.0000	1.00	0.9997	1.00	0.9570	0.00	0.9831	0.00	0.9941	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.27.16	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9997	0.00	0.9997	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.34.2a	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9966	0.00	0.9999	0.80	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.34.2	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9985	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.34.11	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.2071	0.00	1.0000	1.00	0.9992	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.34.29a	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9643	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.37.1	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9999	0.00	0.9999	0.20	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00

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Table 13 – continued from previous page

Dataset	GP-GOMEA						E2E						PySR					
	Base		+AIF		+Div		Base		+AIF		+Div		Base		+AIF		+Div	
	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec
Feynman-III.8.54	0.0131	0.00	-1.3171	0.00	-0.7871	0.00	-1.1324	0.00	0.3454	0.00	0.2375	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-III.15.12	0.9997	0.00	1.0000	1.00	1.0000	1.00	0.9940	0.00	0.9995	0.00	1.0000	1.00	0.9999	0.00	1.0000	1.00	1.0000	1.00
Feynman-III.15.14	-0.0189	0.00	1.0000	1.00	1.0000	1.00	-0.0419	0.00	0.8373	0.00	0.9999	0.20	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-III.15.27	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9322	0.00	1.0000	1.00	0.9998	0.00	1.0000	1.00	0.9999	0.00	1.0000	1.00
Feynman-III.17.37	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9994	0.00	0.9994	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
simple_test-1	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9998	0.40	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
simple_test-2	0.9980	0.00	1.0000	1.00	1.0000	1.00	0.9969	0.00	0.9958	0.20	0.9998	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
simple_test-3	0.9854	0.00	0.9919	0.00	1.0000	1.00	0.9944	0.00	0.9934	0.00	0.9997	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
simple_test-4	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9990	0.00	0.9981	0.00	0.9998	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
simple_test-5	0.9953	0.00	0.9973	0.00	0.9988	0.00	0.9947	0.00	0.9988	0.00	0.9995	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
simple_test-6	0.9966	0.00	0.9868	0.00	0.9999	0.00	0.9750	0.00	0.9933	0.00	0.9981	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
simple_test-7	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9425	0.00	0.9984	0.00	0.9979	0.20	1.0000	1.00	1.0000	1.00	1.0000	1.00
simple_test-8	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9887	0.00	0.9936	0.00	0.9948	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
simple_test-9	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9987	0.00	0.9998	0.40	0.9999	0.40	1.0000	1.00	1.0000	1.00	1.0000	1.00
simple_test-10	0.9948	0.00	1.0000	1.00	1.0000	1.00	0.9837	0.00	0.9994	0.00	0.9999	0.80	1.0000	1.00	1.0000	1.00	1.0000	1.00
Korns-3	0.9997	0.00	1.0000	1.00	1.0000	1.00	0.9738	0.00	0.7471	0.00	0.9770	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Korns-10	-1.9741	0.00	0.1472	0.00	0.7829	0.00	0.6588	0.00	0.5697	0.00	0.6523	0.00	1.0000	1.00	0.9996	0.00	1.0000	1.00
Korns-13	0.8990	0.00	0.9280	0.00	0.9992	0.00	0.9370	0.00	0.7587	0.00	0.8646	0.00	-0.0145	0.00	-0.2508	0.00	1.0000	1.00
Korns-14	0.6916	0.00	0.9779	0.00	1.0000	1.00	0.5920	0.00	0.4582	0.00	0.9942	0.00	0.5694	0.00	0.6979	0.00	0.2977	0.00
Korns-15	0.8073	0.00	0.9736	0.00	0.9999	0.80	0.9062	0.00	0.9260	0.00	0.9358	0.00	-0.0115	0.00	0.5608	0.00	0.9972	0.00
Feynman-I.8.14	0.5903	0.00	1.0000	1.00	1.0000	1.00	0.3309	0.00	0.9954	0.00	0.9982	0.00	0.9849	0.00	0.9940	0.00	0.9851	0.00
Feynman-I.12.2	0.9999	0.00	1.0000	1.00	1.0000	1.00	0.4025	0.00	0.9848	0.00	0.9817	0.20	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.13.4	0.9300	0.00	1.0000	1.00	1.0000	1.00	0.9436	0.00	0.9875	0.20	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.15.3x	0.5045	0.00	0.9106	0.00	0.5267	0.00	0.1604	0.00	0.6208	0.00	0.9731	0.00	0.9906	0.00	0.9711	0.00	0.9996	0.00
Feynman-I.15.3t	0.8662	0.00	-0.5369	0.00	0.9934	0.00	0.6895	0.00	0.6114	0.00	0.9443	0.00	0.6556	0.00	0.6488	0.00	0.9965	0.00
Feynman-I.18.4	0.9999	0.60	1.0000	1.00	1.0000	1.00	0.7696	0.00	0.4613	0.00	0.9822	0.00	1.0000	1.00	0.9939	0.00	0.9993	0.00
Feynman-I.18.16	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9931	0.00	0.9998	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.24.6	0.9637	0.00	1.0000	1.00	1.0000	1.00	0.9654	0.00	0.9906	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.29.16	0.9781	0.00	0.9916	0.00	0.9938	0.00	0.9492	0.00	0.6588	0.00	0.9282	0.00	0.9979	0.00	0.9997	0.00	0.9997	0.00
Feynman-I.32.5	0.9735	0.00	1.0000	1.00	1.0000	1.00	-0.0283	0.00	-0.0480	0.00	0.9988	0.00	0.3400	0.00	1.0000	1.00	1.0000	1.00
Feynman-I.34.8	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9545	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.38.12	0.9999	0.80	1.0000	1.00	1.0000	1.00	-0.6487	0.00	0.9987	0.00	0.9879	0.00	0.1072	0.00	1.0000	1.00	1.0000	1.00

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Table 13 – continued from previous page

Dataset	GP-GOMEA						E2E						PySR					
	Base		+AIF		+Div		Base		+AIF		+Div		Base		+AIF		+Div	
	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec
Feynman-I.39.22	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.7578	0.00	1.0000	1.00	0.9989	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.43.16	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.6375	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.43.43	0.9999	0.00	1.0000	1.00	1.0000	1.00	0.3814	0.00	0.8885	0.00	0.7953	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.47.23	0.9999	0.20	1.0000	1.00	1.0000	1.00	0.8823	0.00	0.9872	0.00	0.9993	0.00	0.9839	0.00	1.0000	1.00	1.0000	1.00
Feynman-I.50.26	0.9974	0.00	1.0000	1.00	1.0000	1.00	0.9939	0.00	0.9997	0.00	0.9986	0.00	0.9526	0.00	0.8574	0.00	0.9977	0.00
Feynman-II.6.11	0.9994	0.00	1.0000	1.00	1.0000	1.00	0.2615	0.00	0.0083	0.00	0.0155	0.00	1.0000	1.00	1.0000	1.00	0.9916	0.00
Feynman-II.6.15b	0.2711	0.00	1.0000	1.00	1.0000	1.00	0.2084	0.00	0.0944	0.00	0.8584	0.00	0.8356	0.00	1.0000	1.00	1.0000	1.00
Feynman-II.11.27	0.9999	0.00	1.0000	1.00	1.0000	1.00	0.9469	0.00	0.9995	0.00	0.9999	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.13.17	0.6204	0.00	1.0000	1.00	1.0000	1.00	0.1299	0.00	0.0734	0.00	0.9981	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.13.23	0.9880	0.00	1.0000	1.00	1.0000	1.00	0.9404	0.00	0.7201	0.00	0.9906	0.00	0.4168	0.00	0.9871	0.00	1.0000	1.00
Feynman-II.13.34	0.9996	0.00	1.0000	1.00	1.0000	1.00	0.9826	0.00	0.9046	0.00	0.7876	0.00	0.8264	0.00	0.9815	0.00	0.9646	0.00
Feynman-II.34.29b	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9485	0.00	0.9999	0.60	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.38.3	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9675	0.00	0.9997	0.20	0.9999	0.40	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-III.4.32	0.9999	0.00	1.0000	1.00	1.0000	1.00	0.1392	0.00	0.8023	0.00	0.7619	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-III.4.33	0.9995	0.00	1.0000	1.00	1.0000	1.00	0.9838	0.00	0.9893	0.00	0.9589	0.00	0.9998	0.00	0.9998	0.00	0.9999	0.00
Feynman-III.7.38	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.3831	0.00	0.9987	0.40	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-III.10.19	0.9999	0.20	0.9916	0.00	0.9977	0.20	0.9780	0.00	0.9565	0.00	0.9642	0.00	0.9964	0.00	1.0000	1.00	1.0000	1.00
Feynman-III.13.18	0.9981	0.00	1.0000	1.00	1.0000	1.00	0.3929	0.00	0.9996	0.00	1.0000	1.00	1.0000	1.00	0.9222	0.00	1.0000	1.00
Vladislavleva-4	0.9282	0.00	0.9486	0.00	0.9397	0.00	0.9849	0.00	0.9832	0.00	0.9902	0.00	0.9764	0.00	0.9992	0.00	0.9768	0.00
Feynman-I.12.11	0.9992	0.00	1.0000	1.00	1.0000	1.00	0.9817	0.00	0.9944	0.40	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.13.12	0.9999	0.00	1.0000	1.00	1.0000	1.00	-0.2275	0.00	0.7791	0.00	0.9934	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.41.16	0.2266	0.00	0.9978	0.00	0.9980	0.00	-2.4400	0.00	0.9628	0.00	0.9568	0.00	0.4506	0.00	0.9740	0.00	0.9228	0.00
Feynman-I.44.4	0.9999	0.20	1.0000	1.00	1.0000	1.00	0.4534	0.00	0.7602	0.00	0.9365	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.2.42	0.9999	0.60	1.0000	1.00	1.0000	1.00	0.7445	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.11.3	0.8142	0.00	1.0000	1.00	1.0000	1.00	-0.0190	0.00	-0.0850	0.00	0.9539	0.00	0.0296	0.00	1.0000	1.00	1.0000	1.00
Feynman-II.11.20	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.9239	0.00	0.9937	0.00	0.9992	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-II.21.32	-0.0219	0.00	1.0000	1.00	1.0000	1.00	-0.2049	0.00	-0.3554	0.00	0.6372	0.00	1.0000	1.00	1.0000	1.00	0.8063	0.00
Feynman-II.35.18	0.6870	0.00	1.0000	1.00	1.0000	1.00	0.6569	0.00	0.9986	0.00	0.9968	0.00	0.9989	0.00	0.9896	0.00	0.9959	0.00
Feynman-II.35.21	0.9914	0.00	0.9928	0.00	1.0000	1.00	0.8738	0.00	0.9304	0.00	0.8917	0.00	0.9994	0.00	1.0000	1.00	1.0000	1.00
Feynman-III.14.14	0.9999	0.00	1.0000	1.00	1.0000	1.00	0.7652	0.00	0.9991	0.00	0.9942	0.00	0.2326	0.00	1.0000	1.00	1.0000	1.00
Feynman-III.19.51	-0.0589	0.00	1.0000	1.00	1.0000	1.00	-1.4632	0.00	0.9015	0.00	0.7044	0.00	0.3128	0.00	1.0000	1.00	1.0000	1.00
Feynman-III.21.20	1.0000	1.00	1.0000	1.00	1.0000	1.00	0.7335	0.00	1.0000	1.00	0.9997	0.20	1.0000	1.00	1.0000	1.00	1.0000	1.00

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Table 13 – continued from previous page

Dataset	GP-GOMEA						E2E						PySR					
	Base		+AIF		+Div		Base		+AIF		+Div		Base		+AIF		+Div	
	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec	R^2	Rec
simple_test-11	0.9896	0.00	1.0000	1.00	1.0000	1.00	0.9733	0.00	0.9966	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
complex-test1	0.9938	0.00	0.9957	0.00	1.0000	1.00	0.7746	0.00	0.8574	0.00	0.9851	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
complex-test2	0.9723	0.00	0.9998	0.60	1.0000	1.00	0.7398	0.00	0.9642	0.00	0.9999	0.80	0.2743	0.00	0.8539	0.00	1.0000	1.00
complex-test3	0.9497	0.00	0.8946	0.00	1.0000	1.00	0.2694	0.00	0.7325	0.00	0.9996	0.00	0.9951	0.00	0.9741	0.00	1.0000	1.00
complex-test4	0.9534	0.00	0.9822	0.00	0.9989	0.60	0.9681	0.00	0.9861	0.00	0.9997	0.00	1.0000	1.00	0.9997	0.00	0.9651	0.00
complex-test5	0.9568	0.00	0.8952	0.00	0.9556	0.00	0.8622	0.00	0.9774	0.00	0.9996	0.00	-0.3366	0.00	-0.4401	0.00	1.0000	1.00
Feynman-I.11.19	0.8648	0.00	1.0000	1.00	1.0000	1.00	0.8279	0.00	0.2442	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-I.32.17	-0.0086	0.00	0.2621	0.00	1.0000	1.00	-2.0933	0.00	0.9604	0.00	0.2887	0.00	-1.9648	0.00	0.4238	0.00	0.7821	0.00
Feynman-I.40.1	0.1784	0.00	1.0000	1.00	1.0000	1.00	-0.0178	0.00	0.9790	0.00	0.9999	0.00	-2.3526	0.00	1.0000	1.00	1.0000	1.00
Feynman-II.6.15a	-1.8662	0.00	1.0000	1.00	0.9997	0.60	0.0090	0.00	0.5742	0.00	0.9891	0.00	0.9813	0.00	1.0000	1.00	1.0000	1.00
Feynman-II.11.7	0.9326	0.00	1.0000	1.00	1.0000	1.00	0.6170	0.00	0.9964	0.00	0.9955	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
Feynman-III.9.52	0.9999	0.40	1.0000	1.00	1.0000	1.00	0.7733	0.00	0.9017	0.20	0.9999	0.80	-1.0321	0.00	1.0000	1.00	1.0000	1.00
complex-test6	0.7023	0.00	0.8277	0.00	1.0000	1.00	0.8606	0.00	0.7507	0.00	0.8943	0.00	0.9556	0.00	0.3886	0.00	0.9999	0.00
Feynman-II.36.28	-5.6770	0.00	1.0000	1.00	1.0000	1.00	-0.0053	0.00	0.9759	0.00	1.0000	1.00	-0.5268	0.00	1.0000	1.00	1.0000	1.00
complex-test7	0.2409	0.00	-0.1900	0.00	0.9671	0.00	-0.7962	0.00	0.1286	0.00	0.8584	0.00	0.2050	0.00	0.5923	0.00	1.0000	1.00
Feynman-I.9.18	0.0230	0.00	0.7539	0.00	0.7361	0.00	-0.0121	0.00	0.6351	0.00	0.6959	0.00	-0.7945	0.00	1.0000	1.00	1.0000	1.00
Feynman-I.9.18-variant	0.0155	0.00	-1.5112	0.00	-0.5436	0.00	-0.0393	0.00	0.1674	0.00	0.7294	0.00	-2.6462	0.00	0.1124	0.00	0.9987	0.00
Poly-10	0.4024	0.00	0.6713	0.00	1.0000	1.00	0.7805	0.00	0.7554	0.00	1.0000	1.00	1.0000	1.00	1.0000	1.00	1.0000	1.00
complex-test8	0.1614	0.00	-0.0735	0.00	0.7783	0.00	0.1992	0.00	-0.0303	0.00	0.4905	0.00	-0.2026	0.00	0.7142	0.00	0.9998	0.00
complex-test9	0.4356	0.00	0.6989	0.00	1.0000	1.00	-0.5359	0.00	0.5490	0.00	1.0000	1.00	0.5972	0.00	0.7381	0.00	1.0000	1.00

Table 14: Detailed Separation of DDRSR and AI Feynman in Ablation Experiments of Expression Separation.

Dataset	DDRSR Score	DDRSR Form	AIF Score	AIF Form
Keijzer-10	1.0	exp(f1(x1)*f2(x2))	0.0	f1(x1,x2)
Keijzer-11	0.0	f1(x1,x2)	0.0	f1(x1,x2)
Keijzer-12	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Keijzer-13	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Keijzer-14	0.0	f1(x1,x2)	0.0	f1(x1,x2)
Keijzer-15	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Nguyen-9	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Nguyen-10	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Nguyen-11	1.0	exp(f1(x1)*f2(x2))	0.0	f1(x1,x2)
Nguyen-12	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Nguyen-10c	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Constant-3	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Constant-4	1.0	exp(f1(x1)*f2(x2)+0.9933)	0.0	f1(x1,x2)
Constant-7	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Livermore-5	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Livermore-10	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Livermore-11	0.0	f1(x1,x2)	0.0	f1(x1,x2)
Livermore-12	1.0	(f1(x1))*(f2(x2))	0.0	f1(x1,x2)
Livermore-14	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Livermore-17	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Jin-1	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Jin-2	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Jin-3	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Jin-4	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Jin-5	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Jin-6	0.0	f1(x1,x2)	0.0	f1(x1,x2)
bacterial respiration 1	0.0	f1(x1,x2)	0.0	f1(x1,x2)
bacterial respiration 3	1.0	(f1(x1))*(f2(x2))+10.00	0.0	f1(x1,x2)
bar magnets 1	0.0	f1(x1,x2)	0.0	f1(x1,x2)
bar magnets 2	0.0	f1(x1,x2)	0.0	f1(x1,x2)
glider 1	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
glider 2	0.0	f1(x1,x2)	0.0	f1(x1,x2)
Lotka-Volterra 1	0.0	f1(x1,x2)	0.0	f1(x1,x2)
Lotka-Volterra 2	0.0	f1(x1,x2)	0.0	f1(x1,x2)

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Table 14 – continued from previous page

Dataset	DDRSR Score	DDRSR Form	AIF Score	AIF Form
predator-prey 1	0.0	f1(x1,x2)	0.0	f1(x1,x2)
predator-prey 2	0.0	f1(x1,x2)	0.0	f1(x1,x2)
shear flow 1	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
shear flow 2	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
van der Pol 1	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
van der Pol 2	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Korns-12	1.0	(f1(x1))*(f2(x2))+2.0000	0.0	f1(x1,x2)
Korns-2	1.0	(f1(x1))*(f2(x2))+(f3(x3))*(f4(x2))+0.2300	0.0	f1(x1,x2,x3)
Korns-8	1.0	(f1(x1))*(f2(x3))*(f3(x2))+6.8700	0.0	f1(x1,x2,x3)
Korns-9	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x2))
Korns-3	1.0	((f1(x1)*f2(x4)+f3(x3)*f4(x4))+(f5(x2)*f6(x4)+-5.41))	0.0	f1(x1,x2,x3)
Korns-10	1.0	(f1(x1))*(f2(x3,x4))+f3(x2,x3,x4)	0.0	f1(x1,x2,x3,x4)
Korns-13	1.0	f1(x1)*f2(x4)*f3(x3)*f4(x2)+32.0000	0.0	f1(x1,x2,x3,x4)
Korns-14	1.0	(f1(x1))*(f2(x4))*(f3(x3))+(f4(x2))*(f5(x4))*(f6(x3))+ 22.0000	0.0	f1(x1,x2,x3,x4)
Korns-15	1.0	(f1(x3))*(f2(x2))*(f3(x1))+(f4(x4))*(f5(x2))*(f6(x1))+12.	0.0	f1(x1,x2,x3,x4)
Neat-5	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Neat-7	1.0	(f1(x1))*(f2(x2))+2.	0.0	f1(x1,x2)
Neat-8	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Neat-9	1.0	f1(x1)+f2(x2)	1.0	f1(x1)+f2(x2)
Vladislavleva-1	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Vladislavleva-3	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Vladislavleva-6	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Vladislavleva-7	1.0	(f1(x1))*(f2(x2))+2.0000	0.0	f1(x1,x2)
Vladislavleva-8	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Vladislavleva-5	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x2))*((f2(x1))*(f3(x3)))
Vladislavleva-4	0.0	f1(x1,x2,x3,x4,x5)	0.0	f1(x1,x2,x3,x4,x5)
Pagie-1	1.0	(f1(x1)+f2(x2))	1.0	(f1(x1)+f2(x2))
Meier-3	0.0	f1(x1,x2)	0.0	f1(x1,x2)
Meier-4	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Poly-10	1.0	(f1(x2))*(f2(x1))+(f3(x7))*(f4(x1))*(f5(x8))+(f6(x4))*(f7(x3)) +(f8(x9))*(f9(x3))*(f10(x6))+(f11(x5))*(f12(x6)	0.5	(f1(x1))*(f2(x2)+(f3(x7))*(f4(x8)))+f5(x3,x4,x5,x6,x9)
Feynman I.6.2	0.0	f1(x1,x2)	0.0	f1(x1,x2)
Feynman I.12.1	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Feynman I.12.5	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))

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Table 14 – continued from previous page

Dataset	DDRSR Score	DDRSR Form	AIF Score	AIF Form
Feynman I.14.4	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Feynman I.25.13	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Feynman I.29.4	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Feynman I.34.27	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Feynman I.39.1	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Feynman II.8.31	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Feynman II.11.28	0.0	f1(x1,x2)	0.0	f1(x1,x2)
Feynman II.27.18	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Feynman II.38.14	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Feynman III.12.43	1.0	(f1(x1))*(f2(x2))	1.0	(f1(x1))*(f2(x2))
Feynman I.6.20b	0.0	f1(x1,x2,x3)	0.0	f1(x1,x2,x3)
Feynman I.12.4	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x2))
Feynman I.14.3	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x2))
Feynman-I.16.6	0.0	f1(x1,x2,x3)	0.0	f1(x1,x2,x3)
Feynman-I.18.12	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x2))
Feynman-I.27.6	0.0	f1(x1,x2,x3)	0.0	f1(x1,x2,x3)
Feynman-I.30.3	1.0	(f1(x1))*(f2(x2,x3))	1.0	(f1(x1))*(f2(x2,x3))
Feynman-I.34.10	1.0	(f1(x1))*(f2(x2,x3))	0.0	(f1(x1))*(f2(x2,x3))
Feynman-I.34.14	1.0	(f1(x1,x2))*(f2(x3))	1.0	(f1(x3))*(f2(x1,x2))
Feynman-I.37.4	0.0	f1(x1,x2,x3)	0.0	f1(x1,x2,x3)
Feynman-I.39.11	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x2))
Feynman-I.43.31	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*((f2(x2))*(f3(x3)))
Feynman-I.48.20	1.0	(f1(x1))*(f2(x2,x3))	1.0	(f1(x1))*(f2(x2,x3))
Feynman-II.4.23	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x2))
Feynman-II.8.7	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*((f2(x2))*(f3(x3)))
Feynman-II.10.9	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*((f2(x2))*(f3(x3)))
Feynman-II.15.4	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x2))
Feynman-II.15.5	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x2))
Feynman-II.24.17	1.0	f1(x1,x2,x3)	0.0	f1(x1,x2,x3)
Feynman-II.27.16	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x2))
Feynman-II.34.2a	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x2))
Feynman-II.34.2	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x2))
Feynman-II.34.11	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x2))
Feynman-II.34.29a	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x2))
Feynman-II.37.1	1.0	(f1(x1))*(f2(x3))*(f3(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x2))

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Table 14 – continued from previous page

Dataset	DDRSR Score	DDRSR Form	AIF Score	AIF Form
Feynman-III.8.54	1.0	sin(f1(x1,x2,x3))	0.0	f1(x1,x2,x3)
Feynman-III.15.12	1.0	(f1(x1))*(f2(x2,x3))	1.0	(f1(x1))*(f2(x2,x3))
Feynman-III.15.14	1.0	((f1(x1))*(f2(x2)))*(f3(x3))	1.0	(f1(x1))*((f2(x2))*(f3(x3)))
Feynman-III.15.27	1.0	(f1(x1))*((f2(x2))*(f3(x3)))	1.0	(f1(x1))*(f2(x3))*(f3(x2))
Feynman-III.17.37	1.0	(f1(x1))*((f2(x2))*(f3(x3))+1.)	0.5	(f1(x1))*(f2(x2,x3))
Feynman-I.8.14	0.0	f1(x1,x2,x3,x4)	0.0	f1(x1,x2,x3,x4)
Feynman-I.12.2	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*((f2(x2))*((f3(x3))*(f4(x4))))
Feynman-I.13.4	1.0	(f1(x2))*(f2(x1))+(f3(x4))*(f4(x1))+(f5(x3))*(f6(x1))	0.5	(f1(x1))*(f2(x2)+f3(x3)+f4(x4))
Feynman-I.15.3x	1.0	(f1(x1))*(f2(x2,x4))+(f3(x3))*(f4(x2,x4))	0.0	f1(x1,x2,x3,x4)
Feynman-I.15.3t	1.0	(f1(x1))*(f2(x2,x4))+(f3(x3))*(f4(x2,x4))	0.0	f1(x1,x2,x3,x4)
Feynman-I.18.4	1.0	(f1(x3))*(f2(x1,x2))+(f3(x4))*(f4(x1,x2))	0.0	f1(x1,x2,x3,x4)
Feynman-I.18.16	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	0.0	(f1(x1))*((f2(x2))*((f3(x3))*(f4(x4))))
Feynman-I.24.6	1.0	(f1(x2))*(f2(x4))*(f3(x1))+(f4(x3))*(f5(x4))*(f6(x1))	0.5	(f1(x1))*((f2(x4))*(f3(x2)+f4(x3)))
Feynman-I.29.16	0.0	f1(x1,x2,x3,x4)	0.0	f1(x1,x2,x3,x4)
Feynman-I.32.5	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	1.0	(f1(x2))*((f2(x1))*((f3(x3))*(f4(x4))))
Feynman-I.34.8	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*((f2(x2))*((f3(x3))*(f4(x4))))
Feynman-I.38.12	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*((f2(x2))*((f3(x3))*(f4(x4))))
Feynman-I.39.22	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*((f2(x2))*((f3(x3))*(f4(x4))))
Feynman-I.43.16	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*((f2(x2))*((f3(x3))*(f4(x4))))
Feynman-I.43.43	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*((f2(x2))*((f3(x3))*(f4(x4))))
Feynman-I.47.23	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*((f2(x2))*((f3(x3))*(f4(x4))))
Feynman-I.50.26	1.0	(f1(x1))*(f2(x2,x3,x4))	1.0	(f1(x1))*(f2(x2,x3,x4))
Feynman-II.6.11	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*(f2(x2))*(f3(x3))*(f4(x4))
Feynman-II.6.15b	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*(f2(x2))*(f3(x3))*(f4(x4))
Feynman-II.11.27	1.0	(f1(x1,x2))*(f2(x4))*(f3(x3))	1.0	(f1(x3))*((f2(x4))*(f3(x1,x2)))
Feynman-II.13.17	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*(f2(x2))*(f3(x3))*(f4(x4))
Feynman-II.13.23	1.0	(f1(x1))*(f2(x3,x4))*(f3(x2))	1.0	(f1(x1))*((f2(x2))*(f3(x3,x4)))
Feynman-II.13.34	1.0	(f1(x1))*(f2(x3,x4))*(f3(x2))	1.0	(f1(x1))*((f2(x2))*(f3(x3,x4)))
Feynman-II.34.29b	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*((f2(x2))*((f3(x3))*(f4(x4))))
Feynman-II.38.3	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*((f2(x2))*((f3(x3))*(f4(x4))))
Feynman-III.4.32	0.0	f1(x1,x2,x3,x4)	0.0	f1(x1,x2,x3,x4)
Feynman-III.4.33	0.0	f1(x1,x2,x3,x4)	0.0	f1(x1,x2,x3,x4)
Feynman-III.7.38	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*(f2(x2))*(f3(x3))*(f4(x4))
Feynman-III.10.19	1.0	(f1(x1))*(f2(x2,x3,x4))	1.0	(f1(x1))*(f2(x2,x3,x4))
Feynman-III.13.18	1.0	(f1(x1))*(f2(x4))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*((f2(x2))*((f3(x3))*(f4(x4))))

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Table 14 – continued from previous page

Dataset	DDRSR Score	DDRSR Form	AIF Score	AIF Form
Feynman I.12.11	1.0	(f1(x2))*(f2(x1))+(f3(x3))*(f4(x1))*(f5(x5))*(f6(x4))	1.0	(f1(x1))*(f2(x2)+(f3(x3))*((f4(x4))*(f5(x5))))
Feynman I.13.12	1.0	(f1(x4))*(f2(x3))*(f3(x2))*(f4(x1))+(f5(x5))*(f6(x3))*(f7(x2))*(f8(x1))	1.0	(f1(x1))*((f2(x2))*((f3(x3))*(f4(x4)+f5(x5))))
Feynman I.41.16	1.0	(f1(x1,x2,x4,x5))*(f2(x3))	0.0	f1(x1,x2,x3,x4,x5)
Feynman I.44.4	1.0	(f1(x4))*(f2(x3))*(f3(x2))*(f4(x1))+(f5(x5))*(f6(x3))*(f7(x2))*(f8(x1))	1.0	(f1(x1))*((f2(x2))*((f3(x3))*(f4(x4)+f5(x5))))
Feynman II.2.42	1.0	(f1(x2))*(f2(x5))*(f3(x4))*(f4(x1))+(f5(x3))*(f6(x5))*(f7(x4))*(f8(x1))	1.0	(f1(x1))*((f2(x4))*((f3(x5))*(f4(x2)+f5(x3))))
Feynman II.11.3	1.0	(f1(x1))*(f2(x4,x5))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x3))*(f4(x4,x5))
Feynman II.11.20	1.0	(f1(x1))*(f2(x5))*(f3(x4))*(f4(x3))*(f5(x2))	1.0	(f1(x1))*((f2(x2))*((f3(x3)*((f4(x4))*(f5(x5)))))
Feynman II.21.32	1.0	(f1(x1))*(f2(x4,x5))*(f3(x3))*(f4(x2))	1.0	(f1(x1))*(f2(x3))*(f3(x3))*(f4(x4,x5))
Feynman II.35.18	1.0	(f1(x1))*(f2(x2,x3,x4,x5))	0.0	f1(x1,x2,x3,x4,x5)
Feynman II.35.21	1.0	(f1(x1))*(f2(x2,x3,x4,x5))	1.0	(f1(x1))*(f2(x2,x3,x4,x5))
Feynman III.14.14	1.0	(f1(x1)+f2(x2))*(f3(x3,x4,x5,x2))	0.5	(f1(x1))*(f2(x2,x3,x4,x5))
Feynman III.19.51	1.0	(f1(x1))*(f2(x5))*(f3(x4))*(f4(x3))*(f5(x2))	1.0	(f1(x1))*((f2(x2))*((f3(x3))*((f4(x4))*(f5(x5)))))
Feynman III.21.20	1.0	(f1(x1))*(f2(x5))*(f3(x4))*(f4(x3))*(f5(x2))	1.0	(f1(x1))*((f2(x2))*((f3(x3))*((f4(x4))*(f5(x5)))))
Feynman-I.11.19	1.0	(f1(x1))*(f2(x4))+(f3(x3))*(f4(x6))+(f5(x2))*(f6(x5))	1.0	(f1(x1))*(f2(x4))+(f3(x2))*(f4(x5))+(f5(x3))*(f6(x6))
Feynman-I.32.17	1.0	(f1(x1))*(f2(x5,x6))*(f3(x4))*(f4(x3))*(f5(x2))	0.5	(f1(x3))*(f2(x1,x2,x4,x5,x6))
Feynman-I.40.1	1.0	(f1(x1)+f2(x6)+f3(x3)+f4(x2))*(f5(x4,x5,x2,x3,x6))	0.5	(f1(x1))*(f2(x2,x3,x4,x5,x6))
Feynman-II.6.15a	1.0	(f1(x1))*(f2(x6))*(f3(x5))*(f4(x3,x4))*(f5(x2))	1.0	(f1(x1))*((f2(x2))*((f3(x5))*((f4(x6))*(f5(x3,x4)))))
Feynman-II.11.7	1.0	(f1(x1))*((f2(x2))*(f3(x6))*(f4(x5))*(f5(x4))*(f6(x3))+1.)	0.5	(f1(x1))*(f2(x2,x3,x4,x5,x6))
Feynman-III.9.52	1.0	(f1(x1))*(f2(x6))*(f3(x3,x5))*(f4(x2))+f5(x4)+f6(x6)+f7(x5)+f8(x3)+f9(x2)	0.5	(f1(x1))*(f2(x2))*(f3(x3,x4,x5))*(f4(x6))
Feynman-II.36.28	1.0	(f1(x2))*(f2(x4))*(f3(x3))*(f4(x1))+(f5(x5,x6,x7,x8))*(f6(x4))*(f7(x3))*(f8(x1))	0.5	(f1(x1))*((f2(x3))*((f3(x4))*(f4(x2)+f5(x5,x6,x7,x8))))
Feynman-I.9.18	1.0	(f1(x1))*(f2(x9))*(f3(x3,x4,x5,x6,x7,x8))*(f4(x2))	1.0	(f1(x1))*(f2(x2))*(f3(x9))*(f4(x3,x4,x5,x6,x7,x8))
Feynman-I.9.18-variant	1.0	(f1(x1))*(f2(x9))*(f3(x3,x4,x5,x6,x7,x8))*(f4(x2))	1.0	(f1(x1))*(f2(x2))*(f3(x9))*(f4(x3,x4,x5,x6,x7,x8))
simple_test-1	1.0	(f1(x1))*(f2(x2))+(f3(x3))*(f4(x2))	1.0	(f1(x2))*(f2(x1)+f3(x3))
simple_test-2	1.0	(f1(x1))*(f2(x2))+(f3(x3))*(f4(x2))	1.0	(f1(x2))*(f2(x1)+f3(x3))
simple_test-3	1.0	(f1(x1))*(f2(x2))+(f3(x3))*(f4(x2))	0.0	f1(x1,x2,x3)
simple_test-4	1.0	(f1(x1,x2))*(f2(x3,x2))	0.0	f1(x1,x2,x3)
simple_test-5	1.0	(f1(x1,x2))*(f2(x3,x2))	0.0	f1(x1,x2,x3)
simple_test-6	1.0	(f1(x1,x2))*(f2(x3,x2))	0.0	f1(x1,x2,x3)
simple_test-7	1.0	exp(f1(x1)*f2(x3)*f3(x2))	0.0	f1(x1,x2,x3)
simple_test-8	1.0	log(f1(x1)+f2(x3)+f3(x2))	0.0	f1(x1,x2,x3)

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Table 14 – continued from previous page

Dataset	DDRSR Score	DDRSR Form	AIF Score	AIF Form
simple_test-9	1.0	sin(f1(x1,x2,x3))	0.0	f1(x1,x2,x3)
simple_test-10	1.0	(f1(x1))*(f2(x3))*(f3(x2))+1.2846	0.0	f1(x1,x2,x3)
simple_test-11	1.0	(f1(x2))*(f2(x3))*(f3(x1))+(f4(x5))*(f5(x3))*(f6(x1))+(f7(x4))*(f8(x5))*(f9(x3))	0.5	(f1(x1,x2,x4,x5))*(f2(x3))

Table 15: Detailed Performance (R^2) in Ablation Experiments of Expression Separation.

Dataset	**E2E**			**GP-GOMEA**			**NGGP**			**PySR**			**MetaSymNet**		
	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div
Keijzer-10	0.7149	0.9360	0.9991	1.0000	1.0000	1.0000	0.9948	0.9998	1.0000	0.9784	0.9995	1.0000	0.9524	0.9562	0.9972
Keijzer-11	0.9941	0.9961	0.9937	0.9848	0.9608	0.9415	0.9958	0.9785	0.9992	0.9948	0.9927	0.9962	0.2511	0.6666	0.6403
Keijzer-12	0.9816	0.9783	0.9999	0.9611	0.9998	1.0000	0.9830	1.0000	1.0000	0.9960	0.9980	0.9949	0.9767	0.9999	0.9889
Keijzer-13	0.9993	0.9997	0.9998	0.9992	1.0000	0.9996	0.9997	1.0000	1.0000	0.9999	1.0000	1.0000	0.9969	0.9998	1.0000
Keijzer-14	0.9458	0.9327	0.9413	0.9930	0.9856	0.9934	0.9954	0.9973	0.9996	0.9997	0.9141	0.9996	0.9758	0.9311	0.9897
Keijzer-15	0.9938	0.9994	0.9997	0.9999	1.0000	1.0000	0.9963	1.0000	1.0000	0.9988	0.9980	0.9982	0.9636	1.0000	1.0000
Nguyen-9	0.9891	0.9972	0.9999	0.9968	0.9999	1.0000	0.9997	0.9997	1.0000	1.0000	0.9973	1.0000	0.9891	1.0000	1.0000
Nguyen-10	0.9992	0.9998	0.9994	0.9988	1.0000	1.0000	0.9998	1.0000	0.9873	1.0000	0.9998	0.9999	0.9836	0.9782	0.9992
Nguyen-11	0.8117	0.7973	0.9978	1.0000	1.0000	1.0000	0.9925	0.9992	1.0000	0.9893	0.9828	1.0000	0.9558	0.9566	0.9956
Nguyen-12	0.9562	0.9963	0.9994	0.9743	0.9995	1.0000	0.9756	1.0000	1.0000	0.9919	0.9965	0.9952	0.9852	0.9993	0.9981
Nguyen-10c	0.9974	0.9997	0.9998	0.9984	1.0000	0.9999	0.9999	1.0000	1.0000	0.9991	0.9997	0.9993	0.9977	0.9981	1.0000
Constant-3	0.9966	0.9996	0.9993	0.9956	0.9989	1.0000	0.9994	1.0000	1.0000	0.9984	0.9993	0.9996	0.9964	0.9997	1.0000
Constant-4	0.8952	0.9284	0.9968	1.0000	1.0000	1.0000	0.9951	1.0000	1.0000	0.7653	0.9989	1.0000	0.9311	0.9858	0.9974
Constant-7	0.9982	0.9970	0.9978	0.9980	0.9999	1.0000	0.9999	0.9998	1.0000	0.9999	0.9993	0.9983	0.9799	1.0000	0.9999
Livermore-5	0.8973	0.9918	0.9986	0.9042	0.9993	1.0000	0.9397	0.9482	1.0000	0.9880	0.9956	0.9963	0.8719	0.9985	0.9917
Livermore-10	0.9987	0.9989	0.9995	0.9994	1.0000	1.0000	0.9997	1.0000	1.0000	1.0000	0.9999	0.9995	0.9684	1.0000	1.0000
Livermore-11	0.9903	0.9953	0.9961	0.9999	1.0000	1.0000	0.9999	0.9994	0.9992	0.9959	0.9756	0.9927	0.8726	0.9555	0.9750
Livermore-12	0.9642	0.9806	0.9796	0.9013	1.0000	1.0000	0.9831	0.9987	0.9868	1.0000	1.0000	1.0000	-0.0256	0.9481	0.9534
Livermore-14	0.9974	0.9985	0.9994	0.9998	0.9998	1.0000	0.9938	0.9531	1.0000	1.0000	1.0000	0.9999	0.9986	1.0000	1.0000
Livermore-17	0.9983	0.9997	0.9979	0.9993	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000	0.9995	0.9998	0.9802	1.0000	0.9992
Jin-1	0.9796	0.9975	1.0000	0.9875	0.9999	1.0000	0.9933	0.9942	1.0000	0.9999	0.9998	0.9991	0.8474	0.9998	1.0000
Jin-2	0.9852	0.9993	0.9991	0.9995	1.0000	1.0000	0.9755	0.9816	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	1.0000
Jin-3	0.9432	0.9984	0.9996	0.9960	0.9995	1.0000	0.9952	1.0000	0.9719	0.9991	0.9965	0.9998	0.9829	0.9871	0.9999
Jin-4	0.9984	0.9991	0.9997	0.9978	1.0000	1.0000	0.9989	1.0000	1.0000	1.0000	0.9999	0.9999	0.9966	0.9999	1.0000
Jin-5	0.9952	0.9999	0.9996	0.9994	1.0000	1.0000	0.9996	1.0000	1.0000	1.0000	1.0000	0.9988	0.9999	1.0000	0.9998
Jin-6	0.9940	0.9887	0.9966	0.9466	0.9513	0.9438	0.9999	0.9279	0.9973	0.9998	0.9999	0.9999	0.9739	0.8746	0.9721
bacterial respiration 1	0.9116	0.9859	0.9943	0.9999	0.9997	0.9975	0.9999	0.9997	0.9998	0.9993	1.0000	0.9991	0.9951	0.9902	0.9980
bacterial respiration 3	0.9982	0.9972	0.9998	0.9909	0.9886	1.0000	0.9999	1.0000	1.0000	0.9990	0.9987	0.9996	0.9702	0.9324	1.0000
bar magnets 1	0.9930	0.9984	0.9954	0.9980	0.9889	0.9974	0.9984	0.9972	0.9967	0.9952	0.9969	0.9957	0.9609	0.9928	0.9969
bar magnets 2	0.9947	0.9895	0.9932	0.9968	0.9941	0.9960	0.9950	0.9972	0.9982	0.9977	0.9969	0.9977	0.9910	0.9941	0.9872
glider 1	0.9994	0.9997	0.9996	0.9999	1.0000	1.0000	0.9999	0.9991	1.0000	1.0000	0.9967	1.0000	0.9998	1.0000	1.0000
glider 2	0.4977	0.7419	0.5391	0.9994	0.9961	1.0000	0.9983	1.0000	1.0000	1.0000	1.0000	1.0000	0.9894	0.9434	0.9944
Lotka-Volterra 1	0.9994	0.9995	0.9998	0.9985	0.9973	0.9999	0.9999	0.9997	1.0000	1.0000	0.9999	0.9995	0.8888	0.9039	0.9694

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Table 15 – continued from previous page

Dataset	E2E			GP-GOMEA			NGGP			PySR			MetaSymNet		
	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div
Lotka-Volterra 2	0.9992	0.9985	0.9990	0.9999	1.0000	1.0000	0.9999	1.0000	1.0000	0.9988	0.9997	0.9998	0.8799	0.9273	0.9309
predator-prey 1	0.9993	0.9997	0.9997	0.9999	0.9998	0.9998	0.9998	1.0000	1.0000	1.0000	0.9998	0.9998	0.9916	0.9991	0.9953
predator-prey 2	0.9925	0.9921	0.9936	0.9972	0.9993	0.9973	0.9995	0.9984	1.0000	0.9863	0.9975	0.9970	0.9731	0.9113	0.9938
shear flow 1	0.9911	0.7092	0.9957	0.9986	1.0000	0.9989	0.9998	1.0000	1.0000	1.0000	1.0000	0.9999	0.9978	0.9641	0.9994
shear flow 2	0.9908	0.9991	0.9975	0.9999	1.0000	0.9999	0.9983	1.0000	1.0000	0.9997	0.9987	0.9997	0.9650	0.9987	0.9998
van der Pol 1	0.9996	0.9999	0.9992	0.9999	1.0000	1.0000	0.9990	1.0000	1.0000	1.0000	1.0000	1.0000	0.9990	1.0000	1.0000
van der Pol 2	0.9999	1.0000	0.9999	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000	0.9061	1.0000	0.9992	0.9998	1.0000
Korns-12	0.5815	0.4276	0.9828	0.3539	0.3173	0.8575	0.6606	0.6583	1.0000	0.9962	0.9999	1.0000	0.4158	0.6809	0.9838
Neat-5	0.9991	0.9982	0.9998	0.9981	0.9999	0.9992	0.9998	0.9996	1.0000	0.9995	0.9999	0.9987	0.9976	1.0000	0.9662
Neat-7	-0.0004	0.9684	0.9977	0.6212	0.5296	0.6272	0.5721	0.8584	1.0000	1.0000	1.0000	1.0000	0.8213	0.7433	0.8424
Neat-8	0.9787	0.9996	0.9990	0.9999	1.0000	0.9959	0.9999	1.0000	1.0000	0.9856	0.9994	0.9928	0.9835	0.9999	1.0000
Neat-9	0.9883	0.9701	0.9814	0.9917	0.9976	0.9886	0.9777	1.0000	1.0000	0.9904	0.9977	0.9954	0.9542	0.9978	0.9992
Vladislavleva-1	0.9896	0.9960	0.9964	0.9809	1.0000	0.9999	0.9995	1.0000	1.0000	0.8645	0.9490	0.9958	0.9576	0.9998	0.9991
Vladislavleva-3	0.9928	0.9996	0.9984	0.9994	1.0000	1.0000	0.9993	1.0000	1.0000	0.9846	0.9993	0.9992	0.9981	0.9998	0.9999
Vladislavleva-6	0.9990	0.9997	0.9998	0.9998	1.0000	1.0000	0.9999	0.9991	1.0000	0.9421	1.0000	0.9990	0.9667	1.0000	1.0000
Vladislavleva-7	0.7138	0.6685	0.9993	0.2506	0.6133	0.9375	0.6811	0.9909	1.0000	1.0000	0.9999	1.0000	0.1839	0.9501	0.9672
Vladislavleva-8	0.9912	0.9988	0.9992	0.9862	1.0000	1.0000	0.9999	1.0000	1.0000	0.9457	0.9985	0.9937	0.9551	0.9997	0.9995
Pagie-1	0.9478	0.9736	0.9968	0.9985	0.9988	0.9999	0.9971	0.9995	0.9978	0.9766	0.9899	0.9954	0.9939	0.9911	0.9949
Meier-3	0.9946	0.9859	0.9949	0.9997	1.0000	1.0000	0.9999	0.9982	0.9984	0.9902	0.9961	0.9904	0.9460	0.9741	0.9590
Meier-4	0.9804	0.9556	0.9870	0.9999	0.9998	1.0000	0.7253	0.9982	1.0000	1.0000	1.0000	1.0000	0.0338	0.9752	0.9816
Feynman-I.6.2	0.4240	0.6341	0.2798	0.9966	0.8993	0.9390	0.9744	0.9628	0.9906	0.9976	0.9409	0.9948	0.7862	0.8137	0.7709
Feynman-I.12.1	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	1.0000	0.9974	1.0000	1.0000	0.9760	0.9724	1.0000
Feynman-I.12.5	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000	0.9968	0.9999	0.9658	0.9717	1.0000
Feynman-I.14.4	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	0.9872	1.0000	0.9983	0.9792	0.9787	0.9956
Feynman-I.25.13	0.9918	0.9950	0.9996	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000	1.0000	0.8004	0.9985	1.0000
Feynman-I.29.4	0.9989	0.9997	0.9896	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000	0.9992	1.0000	0.9461	0.9330	1.0000
Feynman-I.34.27	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000	0.9950	1.0000	0.9466	0.9687	1.0000
Feynman-I.39.1	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	0.9635	0.9841	1.0000
Feynman-II.8.31	0.9997	0.9996	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	0.9999	0.9956	0.9999	0.9473	0.9790	0.9995
Feynman-II.11.28	0.9967	0.9963	0.9860	0.9996	0.9997	0.9998	0.9999	1.0000	1.0000	0.9884	0.9994	0.9994	0.9943	0.9869	0.9841
Feynman-II.27.18	0.9999	0.9995	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000	0.9990	1.0000	0.8901	0.9943	1.0000
Feynman-II.38.14	0.9999	1.0000	0.9999	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	0.9974	0.9933	0.9968	0.9952	1.0000	1.0000
Feynman-III.12.43	0.9998	0.9998	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	0.9570	1.0000	1.0000	0.9529	0.9321	1.0000
Korns-2	0.9671	0.9812	0.9941	0.9999	1.0000	1.0000	0.9984	0.9927	1.0000	1.0000	1.0000	1.0000	0.8388	0.7943	0.8515

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Table 15 – continued from previous page

Dataset	E2E			GP-GOMEA			NGGP			PySR			MetaSymNet		
	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div
Korns-8	0.9939	0.9577	0.9991	0.9791	0.9982	1.0000	0.9455	0.9375	1.0000	0.9898	0.9892	0.9970	0.5941	0.8776	0.9986
Korns-9	0.9997	0.9987	0.9996	0.9999	1.0000	1.0000	0.9500	1.0000	1.0000	0.9974	0.9828	0.9996	0.8908	0.9926	0.9995
Vladislavleva-5	0.9996	0.9994	1.0000	0.9883	0.9968	1.0000	0.6825	1.0000	1.0000	0.9993	0.9979	0.9999	0.7421	0.9979	0.9999
Feynman-I.6.20b	0.3847	0.4930	0.4080	0.6867	0.8017	0.7412	-0.2231	0.2767	0.0170	0.5791	0.8382	0.9819	0.2025	0.3276	0.1309
Feynman-I.12.4	0.8485	0.9850	0.9438	0.9999	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000	0.9999	1.0000	-0.0254	0.9133	0.9100
Feynman-I.14.3	0.9997	0.9989	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000	0.9966	1.0000	0.2508	0.9705	1.0000
Feynman-I.16.6	0.5432	0.8614	0.7061	0.9703	0.9665	0.9884	0.6455	0.6594	0.6839	1.0000	0.9022	1.0000	0.7507	0.7944	0.8069
Feynman-I.18.12	0.9983	0.9993	0.9997	0.9999	1.0000	1.0000	1.0000	0.9969	0.9932	0.9969	1.0000	0.9995	0.9858	1.0000	0.9982
Feynman-I.27.6	0.8341	0.6495	0.9541	0.9999	1.0000	1.0000	0.9951	0.9917	0.9932	1.0000	1.0000	1.0000	0.7297	0.6824	0.8315
Feynman-I.30.3	0.9993	0.9991	0.9991	0.9998	0.9994	1.0000	0.9997	1.0000	1.0000	0.8706	0.9896	0.9998	0.2328	0.9353	0.9576
Feynman-I.34.10	0.2213	-0.3413	0.7631	0.9999	1.0000	1.0000	0.9444	0.9659	0.9938	1.0000	0.9998	1.0000	-0.2419	-0.1503	0.0082
Feynman-I.34.14	0.8892	0.9824	0.8942	0.9999	1.0000	1.0000	0.5183	0.9970	0.9766	0.8698	0.8375	0.9990	0.5276	0.5322	0.6586
Feynman-I.37.4	0.9763	0.9947	0.9828	0.9853	0.9905	0.9871	0.9684	0.9665	0.9748	0.9993	0.9990	0.9997	0.9624	0.9717	0.9689
Feynman-I.39.11	0.9907	1.0000	0.9684	0.9999	1.0000	1.0000	0.9999	1.0000	0.9998	0.9999	0.9456	1.0000	-0.0276	0.9981	0.9997
Feynman-I.43.31	0.9996	0.9999	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	0.9967	1.0000	1.0000	0.6402	0.9701	0.9999
Feynman-I.48.20	0.7286	0.8076	0.9744	0.9784	0.9996	0.9914	0.8868	0.9370	0.8432	0.8183	0.9677	0.9962	-0.2346	0.6385	0.9049
Feynman-II.4.23	0.1657	0.9554	0.9999	0.9999	0.9713	1.0000	0.9999	1.0000	1.0000	1.0000	1.0000	0.9857	-0.1220	0.9997	0.9999
Feynman-II.8.7	0.4925	0.9361	0.9832	0.9896	1.0000	1.0000	0.8195	1.0000	1.0000	1.0000	1.0000	1.0000	-0.8602	0.9834	0.9922
Feynman-II.10.9	0.9988	0.9983	0.9995	0.9999	1.0000	1.0000	0.9995	1.0000	1.0000	1.0000	1.0000	1.0000	0.6391	0.3229	1.0000
Feynman-II.15.4	0.9990	0.9999	0.9990	0.9999	1.0000	1.0000	0.9999	0.9999	1.0000	1.0000	0.9999	1.0000	0.9094	0.9996	0.9994
Feynman-II.15.5	0.9913	0.9998	0.9984	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8077	1.0000	1.0000
Feynman-II.24.17	0.9570	0.9394	0.9652	0.9999	1.0000	0.9997	0.9992	0.9738	1.0000	1.0000	1.0000	1.0000	0.9537	0.4093	0.9447
Feynman-II.27.16	0.9997	0.9999	0.9969	1.0000	1.0000	1.0000	0.9999	0.9999	1.0000	0.9952	0.9999	1.0000	0.4986	0.9027	0.9998
Feynman-II.34.2a	0.9966	0.9963	0.9928	0.9999	1.0000	1.0000	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	0.8365	0.4911	1.0000
Feynman-II.34.2	0.9985	0.9994	0.9988	1.0000	1.0000	1.0000	0.9999	0.9999	1.0000	0.9953	1.0000	0.9978	0.7437	0.9351	0.9986
Feynman-II.34.11	0.2071	0.9999	0.9751	0.9999	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	-0.1739	0.9824	0.9956
Feynman-II.34.29a	0.9643	0.9648	0.9738	0.9999	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000	1.0000	0.4203	0.9358	0.9998
Feynman-II.37.1	0.9999	0.9998	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	0.9999	0.9997	1.0000	0.9141	0.9907	0.9989
Feynman-III.8.54	-1.1324	-0.1724	0.0017	0.0131	-0.5903	0.1932	-0.2627	-0.3568	-0.0619	1.0000	1.0000	1.0000	-0.4751	-0.3061	-0.4928
Feynman-III.15.12	0.9940	0.9986	0.9969	0.9997	1.0000	1.0000	0.9998		1.0000	0.9980	0.9954	0.9996	0.6680	0.9305	0.9454
Feynman-III.15.14	-0.0419	0.9811	0.9915	-0.0189	1.0000	1.0000	0.9999	0.9418	0.9997	1.0000	0.9996	1.0000	0.4976	0.8065	0.7217
Feynman-III.15.27	0.9322	0.9239	0.9821	0.9999	1.0000	1.0000	0.9996	1.0000	1.0000	1.0000	0.9993	1.0000	-0.1586	0.8819	0.9973
Feynman-III.17.37	0.9994	0.9992	0.9954	0.9999	0.9993	1.0000	0.9990	1.0000	1.0000	0.9953	0.9995	1.0000	0.9622	0.9991	1.0000
test-1	0.9998	0.9997	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	0.9145	0.9567	0.9687

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Table 15 – continued from previous page

Dataset	E2E			GP-GOMEA			NGGP			PySR			MetaSymNet		
	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div
test-2	0.9969	0.9986	0.9999	0.9980	0.9997	1.0000	0.9791	1.0000	0.9974	0.9993	0.9987	0.9997	0.9558	1.0000	0.9902
test-3	0.9944	0.9906	0.9999	0.9854	0.9941	1.0000	0.9636	0.9718	0.9964	0.9995	0.9999	1.0000	0.9823	0.9681	1.0000
test-4	0.9990	0.9982	0.9995	1.0000	1.0000	1.0000	0.9816	0.9561	1.0000	0.9999	1.0000	0.9999	0.9127	0.9673	0.9994
test-5	0.9947	0.9770	0.9986	0.9953	0.9947	0.9968	0.9797	0.9890	1.0000	1.0000	1.0000	0.9995	0.9652	0.9773	0.9987
test-6	0.9750	0.9944	0.9953	0.9966	0.9971	0.9929	0.9919	0.9894	0.9998	0.9997	0.9998	0.9999	0.9502	0.9783	0.9834
test-7	0.9425	0.9340	1.0000	1.0000	1.0000	1.0000	1.0000	0.9992	1.0000	0.9991	1.0000	1.0000	0.3880	0.3169	1.0000
test-8	0.9887	0.9955	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8787	0.8461	1.0000
test-9	0.9987	0.9998	0.9976	0.9999	0.9995	1.0000	1.0000	1.0000	0.9997	0.9979	1.0000	1.0000	0.7010	0.7570	0.9653
test-10	0.9837	0.9963	1.0000	0.9948	1.0000	1.0000	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	0.6538	0.7743	1.0000
Korns-3	0.9738	0.9027	0.9944	0.9997	1.0000	1.0000	0.9635	0.9746	1.0000	1.0000	1.0000	1.0000	0.5422	0.5223	0.9829
Korns-10	0.6588	0.7133	0.7303	-1.9741	-1.5283	0.7670	0.4192	0.3491	0.9311	1.0000	0.9656	1.0000	-1.9249	0.6593	0.7679
Korns-13	0.9370	0.9052	0.9901	0.8990	0.9404	0.9649	0.5591	0.6575	0.3863	0.9037	0.9833	0.9992	-1.1902	0.1551	0.9438
Korns-14	0.5920	0.8750	0.9987	0.6916	0.9469	1.0000	0.4670	0.9037	1.0000	0.9939	0.9384	1.0000	0.6977	0.7134	0.9999
Korns-15	0.9062	0.9437	0.9797	0.8073	0.9528	1.0000	0.7127	0.7652	0.5944	0.9985	0.9951	0.9998	0.1613	0.0814	0.9995
Feynman-I.8.14	0.3309	0.4381	0.3361	0.5903	0.5810	0.1210	0.2484	-0.1281	0.3568	0.9816	0.9940	0.9987	-0.1238	0.5440	0.3411
Feynman-I.12.2	0.4025	0.9629	0.9880	0.9999	0.9879	1.0000	-0.2207	1.0000	1.0000	0.3420	1.0000	1.0000	-0.0355	0.8493	0.8423
Feynman-I.13.4	0.9436	0.9257	1.0000	0.9300	1.0000	1.0000	0.9359	0.6639	1.0000	0.9985	0.9204	1.0000	0.4642	0.8580	0.9996
Feynman-I.15.3x	0.1604	0.5111	0.5470	0.5045	0.9870	0.9997	0.4226	0.6493	0.2530	0.9804	0.8670	0.9972	0.6608	0.5016	0.6821
Feynman-I.15.3t	0.6895	0.3468	0.7435	0.8662	0.9916	0.9988	0.4357	0.0327	0.8962	0.9970	0.9279	0.9691	0.0260	0.4725	0.5057
Feynman-I.18.4	0.7696	0.8905	0.7725	0.9999	0.9879	1.0000	0.7013	0.6593	1.0000	0.9988	0.9978	1.0000	0.8404	0.7768	0.9500
Feynman-I.18.16	0.9931	0.9900	0.9999	0.9999	1.0000	1.0000	0.9986	0.9791	1.0000	0.9630	0.9950	0.9999	0.2687	0.9945	0.9994
Feynman-I.24.6	0.9654	0.9642	1.0000	0.9637	0.9870	1.0000	0.8062	0.9748	1.0000	0.7498	0.9827	1.0000	0.4498	0.9468	0.9952
Feynman-I.29.16	0.9492	0.8866	0.9201	0.9781	0.9916	0.7299	0.8550	0.9462	0.9116	0.9989	0.9950	0.9799	0.5491	0.9070	0.8591
Feynman-I.32.5	-0.0283	0.9111	0.9986	0.9735	0.9600	1.0000	0.0796	0.9003	0.9998	0.4576	1.0000	1.0000	0.6621	0.4504	0.4036
Feynman-I.34.8	0.9545	0.9997	0.9997	0.9999	1.0000	1.0000	0.9999	0.9961	1.0000	1.0000	1.0000	1.0000	0.5332	0.9368	0.9791
Feynman-I.38.12	-0.6487	0.9855	0.9957	0.9999	1.0000	1.0000	-0.0154	1.0000	1.0000	1.0000	1.0000	1.0000	0.0984	0.7348	0.5422
Feynman-I.39.22	0.7578	0.9970	0.9995	0.9999	1.0000	1.0000	0.9899	1.0000	1.0000	1.0000	1.0000	1.0000	0.3422	0.9597	1.0000
Feynman-I.43.16	0.6375	0.9904	0.9999	0.9999	1.0000	1.0000	0.8433	0.9862	1.0000	1.0000	0.9999	1.0000	-0.1693	0.9769	0.9990
Feynman-I.43.43	0.3814	0.9049	0.9862	0.9999	1.0000	1.0000	0.5589	1.0000	1.0000	1.0000	1.0000	1.0000	0.7370	0.9926	0.9997
Feynman-I.47.23	0.8823	0.9998	0.9962	0.9999	1.0000	1.0000	0.8322	1.0000	0.9895	0.9138	0.9999	0.9998	0.2954	0.9795	0.7977
Feynman-I.50.26	0.9939	0.9992	0.9983	0.9974	0.9884	0.9982	0.9738	0.9920	0.9943	0.9996	0.9974	0.9999	0.7263	0.9736	0.9233
Feynman-II.6.11	0.2615	0.2629	0.9617	0.9994	1.0000	1.0000	0.1371	1.0000	1.0000	1.0000	1.0000	0.9980	-0.4742	0.2601	0.9535
Feynman-II.6.15b	0.2084	0.9992	0.6153	0.2711	1.0000	0.9999	0.1178	1.0000	1.0000	0.9436	1.0000	0.9998	0.9955	0.8353	0.8027
Feynman-II.11.27	0.9469	0.9784	0.9921	0.9999	1.0000	0.9999	0.9430	0.9413	1.0000	0.9919	0.9987	0.9972	0.5615	0.9772	0.9743

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Table 15 – continued from previous page

Dataset	E2E			GP-GOMEA			NGGP			PySR			MetaSymNet		
	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div
Feynman-II.13.17	0.1299	0.5253	0.9963	0.6204	1.0000	1.0000	0.2365	1.0000	1.0000	1.0000	1.0000	1.0000	0.5554	0.7632	0.9202
Feynman-II.13.23	0.9404	0.7958	0.5018	0.9880	1.0000	1.0000	0.5817	0.8776	0.9998	0.9971	0.9901	0.9989	0.7086	0.6859	0.4412
Feynman-II.13.34	0.9826	0.9201	0.8562	0.9996	0.8267	1.0000	-0.3999	0.7442	0.8324	0.9244	0.9282	0.9982	0.1202	0.8719	0.9494
Feynman-II.34.29b	0.9485	0.9038	0.9152	1.0000	1.0000	1.0000	0.9798	1.0000	1.0000	1.0000	1.0000	1.0000	0.2566	0.9167	1.0000
Feynman-II.38.3	0.9675	0.9567	0.9344	1.0000	1.0000	1.0000	0.9930	1.0000	1.0000	1.0000	1.0000	1.0000	-0.0790	0.9276	0.9212
Feynman-III.4.32	0.1392	-0.1402	0.2998	0.9999	1.0000	1.0000	0.8437	0.9106	0.8092	0.9999	1.0000	1.0000	0.3799	0.1545	0.2231
Feynman-III.4.33	0.9838	0.9859	0.9438	0.9995	0.9990	0.9992	0.9058	0.8448	0.9143	0.9958	0.9867	0.9951	0.8117	0.7911	0.8444
Feynman-III.7.38	0.3831	0.8978	0.9508	0.9999	1.0000	1.0000	0.9904	0.9346	1.0000	1.0000	1.0000	1.0000	0.0101	0.9334	0.9994
Feynman-III.10.19	0.9780	0.9712	0.9800	0.9999	0.9946	0.9999	0.9802	0.9871	0.9900	0.9935	0.9991	0.9919	0.8244	0.9473	0.9568
Feynman-III.13.18	0.3929	0.9974	0.9944	0.9981	0.9864	1.0000	0.2722	0.9714	1.0000	1.0000	1.0000	1.0000	0.0054	0.9316	1.0000
Vladislavleva-4	0.9849	0.9754	0.9660	0.9282	0.8601	0.8820	0.6851	0.7159	0.7207	0.8881	0.9081	0.9401	0.6690	0.4038	0.6548
Feynman-I.12.11	0.9817	0.9477	1.0000	0.9992	0.9903	1.0000	0.9172	0.9787	1.0000	0.9983	0.9869	1.0000	0.7878	0.9771	0.9998
Feynman-I.13.12	-0.2275	0.9055	0.9964	0.9999	0.9284	1.0000	-0.1121	0.9022	1.0000	1.0000	1.0000	1.0000	0.4031	0.5106	0.5677
Feynman-I.41.16	-2.4400	0.6909	0.9179	0.2266	0.2467	0.9931	-0.0216	0.1233	-0.0246	-0.0530	0.4609	0.7469	-0.7893	0.3899	0.8288
Feynman-I.44.4	0.4534	0.9738	0.9867	0.9999	1.0000	1.0000	0.5167	0.9114	1.0000	0.9658	0.9738	0.9999	-0.0364	0.9815	0.9216
Feynman-II.2.42	0.7445	0.9897	0.9947	0.9999	1.0000	1.0000	-0.0088	0.9163	1.0000	1.0000	1.0000	0.9998	-1.1301	0.9596	0.9347
Feynman-II.11.3	-0.0190	0.0299	0.2549	0.8142	0.4252	1.0000	-0.0141	-0.0248	0.8685	0.9519	0.6252	0.9951	-0.0616	-0.9373	0.3752
Feynman-II.11.20	0.9239	0.9898	0.9976	0.9999	1.0000	1.0000	0.8740	0.9353	1.0000	1.0000	1.0000	0.9956	-0.0482	0.9537	0.9602
Feynman-II.21.32	-0.2049	-0.1898	0.9853	-0.0219	0.4590	1.0000	-0.0267	-0.0538	0.8197	-0.3160	0.2338	1.0000	-0.0228	0.0107	0.4059
Feynman-II.35.18	0.6569	0.8066	0.9404	0.6870	0.9916	0.8931	0.6576	0.8564	0.8450	0.9976	0.8635	0.9986	0.4802	0.2976	0.6934
Feynman-II.35.21	0.8738	0.7125	0.5272	0.9914	0.9836	0.6548	0.6874	0.6606	0.8350	0.9242	0.9211	0.9677	0.4396	0.6470	0.8271
Feynman-III.14.14	0.7652	-1.2744	0.3639	0.9999	0.9518	1.0000	-1.0318	-0.1305	-0.3642	1.0000	1.0000	1.0000	-0.0526	0.0201	0.2999
Feynman-III.19.51	-1.4632	0.7327	0.4219	-0.0589	0.8092	1.0000	-0.0169	0.9413	0.9615	0.4381	0.9348	1.0000	-0.0179	-0.0048	-0.0266
Feynman-III.21.20	0.7335	0.9661	0.9646	0.9999	1.0000	1.0000	0.6910	0.9215	1.0000	1.0000	1.0000	1.0000	0.7351	0.9938	0.9570
test-11	0.9733	0.9930	1.0000	0.9896	0.9907	1.0000	0.6489	0.9357	1.0000	1.0000	1.0000	1.0000	0.5270	0.8705	0.9987
Feynman-I.11.19	0.8279	0.9987	1.0000	0.8648	1.0000	1.0000	0.7626	1.0000	1.0000	1.0000	0.9985	1.0000	0.4273	0.9794	0.9890
Feynman-I.32.17	-2.0933	-0.0010	0.3331	-0.0086	-0.3468	0.9996	-0.3821	-0.1081	0.0722	-0.2847	-0.2164	0.9964	-0.0461	-0.1061	-0.0373
Feynman-I.40.1	-0.0178	0.5228	0.9519	0.1784	-1.7287	1.0000	0.0802	0.0307	1.0000	1.0000	-0.0887	1.0000	-0.0256	0.4824	0.3427
Feynman-II.6.15a	0.0090	0.9168	0.9620	-1.8662	0.9287	0.9994	-1.5637	0.9974	0.9953	0.9228	1.0000	0.9991	-0.4804	0.8647	0.9090
Feynman-II.11.7	0.6170	0.6657	0.9998	0.9326	0.0151	1.0000	-0.0268	0.8126	1.0000	0.8241	1.0000	0.9989	-0.1473	0.4350	0.9462
Feynman-III.9.52	0.7733	0.2971	0.9823	0.9999	1.0000	0.2495	0.7634	0.9997	0.9750	1.0000	0.9958	1.0000	0.0915	0.3409	0.8354
Feynman-II.36.28	-0.0053	0.2529	0.9742	-5.6770	0.5074	1.0000	-0.1595	-0.2703	0.9996	0.0902	1.0000	0.9999	0.2811	-0.9524	0.5387
Feynman-I.9.18	-0.0121	-0.0376	-0.9426	0.0230	-0.8097	0.1428	0.0286	0.6621	0.5038	-0.0656	-0.0344	-0.0104	-0.0322	-0.1194	-0.1540
Feynman-I.9.18-variant	-0.0393	0.2757	-0.4593	0.0155	-1.6601	0.1826	-0.0487	-0.0901	-0.0463	0.1159	-1.4681	0.3807	-0.0525	0.0104	-0.0843

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Table 15 – continued from previous page

Dataset	E2E			GP-GOMEA			NGGP			PySR			MetaSymNet		
	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div
Poly-10	0.7805	0.9298	1.0000	0.4024	0.9416	1.0000	0.3665	0.8322	1.0000	1.0000	1.0000	1.0000	0.0108	0.7263	0.9999

Table 16: Detailed Separation of DDRSR and AI Feynman in Ablation Experiments of Variable Composition

Dataset	DDRSR Score	DDRSR Form	AIF Score	AIF Form
complex-test1	0.0	f1(x1,x2,x3,x4,x5)	0.0	f1(x1,x2,x3,x4,x5)
complex-test2	0.0	f1(x1,x2,x3,x4,x5)	0.0	f1(x1,x2,x3,x4,x5)
complex-test3	1.0	f1((x1^(1)*x2^(2)*x5^(-1)),x3,x4)	0.5	f1(x1/x5,x2,x3,x4)
complex-test4	1.0	f1((x1+(1.2000)*x2),x3,(x4+(1.1667)*x5))	0.0	f1(x1,x2,x3,x4,x5)
complex-test5	1.0	f1((x1+(2.0000)*x2+(3.0000)*x3),(x4^(1)*x5^(2)))	0.0	f1(x1,x2,x3,x4,x5)
complex-test6	1.0	f1((x1^(1)*x2^(2)),(x3+(1.5000)*x4),(x5^(2)*x6^(3)))	0.0	f1(x1,x2,x3,x4,x5,x6)
complex-test7	1.0	f1(x1,x2,(x3+(2.0000)*x4),((x5+(-0.5000)*x6)^(1)*x7^(-1)*x8^(-2)))	0.0	f1(x1,x2,x3,x4,x5,x6,x7,x8)
complex-test8	1.0	f1(x1,x2,(x3+(-2.0000)*x4),(x5+(-3.0000)*x6),(x7+(-4.0000)*x8),x9)	0.0	f1(x1,x2,x3,x4,x5,x6,x7,x8,x9)
complex-test9	1.0	f1(x1,(x2^(1)*x4^(2)),x3,x5,(x6^(2)*x8^(1)),x7,x9)	0.0	f1(x1,x2,x3,x4,x5,x6,x7,x8,x9)

Table 17: Detailed Performance (R^2) in Ablation Experiments of Variable Composition.

Dataset	**E2E**			**GP-GOMEA**			**NGGP**			**PySR**			**MetaSymNet**		
	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div	Base	+AIF	+Div
complex-test1	0.7746	0.7487	0.7124	0.9938	0.9999	0.9999	0.6533	0.7181	0.6723	0.9908	0.9927	0.9821	0.6264	0.6707	0.5972
complex-test2	0.7398	0.8061	0.8867	0.9723	0.9769	0.9747	0.1501	0.1622	0.2232	0.8969	0.9345	0.9291	0.6514	0.6671	0.6844
complex-test3	0.2694	0.9337	0.9997	0.9497	0.8824	0.9946	-0.9234	0.9445	0.9950	0.9960	0.9930	1.0000	-0.0992	0.0642	0.1164
complex-test4	0.9681	0.9711	0.9985	0.9534	0.9877	0.9871	0.8541	0.7823	0.9532	0.9992	0.9941	0.9999	0.6522	0.8087	0.9726
complex-test5	0.8622	0.7616	0.9997	0.9568	0.9183	0.9832	0.7094	0.7875	0.2109	0.9998	0.9997	1.0000	0.7743	0.8433	0.9844
complex-test6	0.8606	0.6253	0.9558	0.7023	0.7561	1.0000	-0.5006	-0.8065	0.9782	0.9925	0.9790	0.9984	-0.0356	0.3749	0.8935
complex-test7	-0.7962	-0.3475	0.9364	0.2409	-0.5462	0.8854	-0.4746	-0.0007	0.9166	0.0231	-0.0211	0.9167	-0.3776	-0.5273	0.4535
complex-test8	0.1992	-0.0438	0.2381	0.1614	0.2692	0.3756	-0.0297	-0.4877	0.1147	0.1343	0.5240	0.6964	0.0229	-0.1689	-0.4380
complex-test9	-0.5359	0.3703	0.7430	0.4356	0.5984	0.8925	-0.1396	-0.2876	0.1061	0.7707	0.6331	0.5517	-0.5859	-0.3428	-0.0370