CAN MULTI-OBJECTIVE BAYESIAN OPTIMISATION WORK WITHOUT POSTERIOR UNCERTAINTY?

Anonymous authors
Paper under double-blind review

ABSTRACT

Bayesian optimisation (BO) is a sample-efficient approach for solving expensive black-box optimisation problems. It uses a probabilistic surrogate model to provide a posterior distribution over the unknown objective function and an acquisition function to select solutions for evaluation. Acquisition functions commonly use both the model's posterior mean and posterior uncertainty to balance exploitation and exploration, favouring solutions with better predicted objective values while exploring regions where predictions are less certain. Most multi-objective Bayesian optimisation (MOBO) methods follow such a design. In this paper, we argue that it may not be necessary to consider posterior uncertainty in every MOBO design. In single-objective BO, using only the posterior mean may keep the search near the currently predicted best solution, leaving other regions unexplored. In MOBO, however, considering the posterior means across different objectives (i.e., their nondominance relations) can also facilitate exploration. Rather than focusing solely on solutions with the best posterior mean for each objective, the search can identify solutions representing different trade-offs among the objectives. These trade-off solutions may be located in different areas in the search space, allowing exploration to emerge from these areas without explicitly modelling posterior uncertainty. Based on this, we propose a simple MOBO framework that considers only the posterior means of the surrogate models. We investigate two instantiations of this framework: one directly approximates the Pareto front formed by the posterior means, while the other optimises the scalarised posterior means to learn the corresponding Pareto set. We evaluate it on a wide range of benchmark and real-world problems. The results show that it performs very competitively with state-of-the-art methods, even under noisy settings.

1 INTRODUCTION

Bayesian optimisation (BO) is a sample-efficient approach for solving expensive black-box optimisation problems (Garnett, 2023). It has two main components: a surrogate model and an acquisition function. The surrogate model, most commonly a Gaussian process (GP) (Williams & Rasmussen, 2006), is fitted to the observations collected so far and provides a posterior distribution over the unknown objective function, from which the posterior mean and uncertainty can be obtained.

Acquisition functions commonly use both the posterior mean and posterior uncertainty to balance exploitation and exploration, favouring points with better predicted objective values while encouraging exploration in regions where predictions are less certain (Wilson et al., 2018). Common examples include lower confidence bound (LCB) (Lai & Robbins, 1985), probability of improvement (PI) (Kushner, 1964), expected improvement (EI) (Mockus et al., 1978; Jones et al., 1998), and Thompson sampling (TS) (Thompson, 1933).

Most existing multi-objective Bayesian optimisation (MOBO) methods follow this design, incorporating both posterior means and posterior uncertainty when optimising multiple objectives. These methods can be loosely divided into scalarisation and Pareto-based methods. In scalarisation-based methods (Knowles, 2006; Paria et al., 2020; Zhang, 2024), a multi-objective problem is converted into a number of single-objective problems (Chugh, 2020; Jiang et al., 2026a). This allows acquisition functions from single-objective BO to be used for selecting the next point to evaluate, such as LCB, PI, EI, or TS, to be applied (Knowles, 2006; Paria et al., 2020).

Reviewers: please read the Reviewer Guidelines (iclr.cc/Conferences/2027/ReviewerGuidelines) and the AI Policy for Reviewers (iclr.cc/Conferences/2027/AIPolicyForReviewers). If you used AI to expand, edit, or polish your review, please provide the input text to the LLM. Better still, consider skipping the LLM and submitting your original text: we, and the authors, are much more interested in your unedited thoughts than in what an LLM has to say. AI-assisted or not, you are putting your name and reputation behind your review: LLM-generated falsehoods, hallucinations or misrepresentations are subject to disciplinary action, which may include desk-rejecting all papers you have authored.

Pareto-based methods consider Pareto dominance relations over objectives and directly solve a multi-objective problem without converting it into a number of single-objective subproblems. Representative examples include hypervolume-based and information-based methods. Similar to scalarisation-based methods, hypervolume-based methods combine hypervolume with acquisition functions from single-objective BO. EHVI is a representative hypervolume-based method that combines EI with hypervolume (Emmerich et al., 2006; Daulton et al., 2020). Information-based methods select points that are expected to reduce uncertainty about the Pareto set or Pareto front (Hernandez-Lobato et al., 2016; Belakaria et al., 2019; Tu et al., 2022). Although formulated differently, these methods consider posterior uncertainty into the optimisation process.

However, do we really need to consider posterior uncertainty in every MOBO design? In single-objective BO, using only the posterior mean keeps the search near the currently predicted best solution, leaving other regions unexplored. In MOBO, however, considering the posterior means between different objectives (i.e., nondominance relations between solutions) may naturally factor in exploration. That is, rather than focusing solely on solutions with the best posterior mean for each objective, the search can identify solutions representing different trade-offs among the objectives. These trade-off regions also contain uncertain predictions, so searching across them can provide exploration without explicitly using posterior uncertainty to select solutions.

Based on this, we propose a posterior mean-based MOBO (PMOBO) framework that uses only the posterior means. The proposed framework is instantiated with both Pareto-based and scalarisation-based approaches. The Pareto-based approach selects solutions from the Pareto front obtained by optimising the posterior means, while the scalarisation-based approach uses Pareto set learning to learn a mapping from preference weights to their corresponding decision variables. The main contributions can be summarised as follows:

- We propose a posterior mean-based MOBO framework that considers only posterior means, where exploration implicitly arises from the nondominance relations between solutions rather than posterior uncertainty.
- We instantiate the proposed framework using both Pareto-based and scalarisation-based approaches.
- We perform extensive experiments on a wide range of synthetic benchmark and real-world problems under both noiseless and noisy settings. The results show that the proposed method achieves highly competitive performance against state-of-the-art methods, even under noisy settings.
- We further investigate the impact of incorporating posterior uncertainty into the proposed framework. The results show that incorporating posterior uncertainty does not necessarily improve performance and can even degrade the performance of the proposed framework.

2 Background

Multi-Objective Optimisation. Multi-objective optimisation aims to optimise two or more conflicting objectives simultaneously and obtain a set of trade-off solutions instead of a single optimal solution. Without loss of generality, this paper considers a minimisation problem defined as a vector-valued function $\boldsymbol{f}(\boldsymbol{x}) : \mathcal{X} \to \mathbb{R}^m$, where $\boldsymbol{x} \in \mathcal{X}$ ($\mathcal{X} \subset \mathbb{R}^d$) and m denotes the number of objectives. A solution $\boldsymbol{x}^*$ is said to dominate another solution $\boldsymbol{x}'$, denoted by $\boldsymbol{x}^* \prec \boldsymbol{x}'$, if $\forall i \in \{1, \ldots, m\}$, $f_i(\boldsymbol{x}^*) \leq f_i(\boldsymbol{x}')$ and $\exists j \in \{1, \ldots, m\}$, $f_j(\boldsymbol{x}^*) < f_j(\boldsymbol{x}')$. A solution that is not dominated by any other solution is called Pareto optimal. The set of all Pareto optimal solutions is referred to as the Pareto set, while its corresponding objective values form the Pareto front.

Bayesian Optimisation (BO). Bayesian optimisation (BO) is a sample-efficient global optimisation approach that builds a probabilistic surrogate model, typically a Gaussian process (GP), and employs an acquisition function $\alpha : \mathcal{X} \to \mathbb{R}$ to select the next point for evaluation (Garnett, 2023). In this work, each objective is modelled independently using a Gaussian process $f_i \sim \mathcal{GP}(m_i(\boldsymbol{x}), k_i(\boldsymbol{x}, \boldsymbol{x}'))$, where $m_i : \mathcal{X} \to \mathbb{R}$ and $k_i(\cdot, \cdot) : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ denote the mean and covariance functions of the ith objective, respectively. Given n observations $\mathcal{D}^{(n)} = \{(\boldsymbol{x}^{(t)}, \boldsymbol{y}^{(t)})\}_{t=1}^n$, where $\boldsymbol{y}^{(t)} = \boldsymbol{f}(\boldsymbol{x}^{(t)}) + \boldsymbol{\zeta}^{(t)}$ and $\boldsymbol{\zeta}^{(t)} \sim \mathcal{N}(\boldsymbol{0}, \mathrm{diag}(\boldsymbol{\sigma}_\zeta^2))$, the posterior distribution of the ith objective at a new point $\boldsymbol{x}$ is given by

$f_i(\boldsymbol{x})|\mathcal{D}^{(n)} \sim \mathcal{N}(\mu_i(\boldsymbol{x}), \sigma_i^2(\boldsymbol{x}))$, where $\mu_i(\boldsymbol{x})$ and $\sigma_i^2(\boldsymbol{x})$ represent the posterior mean and variance, respectively. The detailed expressions of the posterior mean and variance are provided in Appendix A.

3 Related Work

Over the past decades, a variety of MOBO methods have been proposed (Zuluaga et al., 2013; Shah & Ghahramani, 2016; Daulton et al., 2022a; Tian et al., 2023; Yang et al., 2026; Rashidi et al., 2024; Wang et al., 2023; Binois et al., 2020; Picheny et al., 2019; Zhu et al., 2023; Li et al., 2025a; Jiang et al., 2026b). They can be loosely divided into scalarisation-based and Pareto-based methods. Scalarisation-based methods (Knowles, 2006; Paria et al., 2020; Zhang, 2024) convert a multi-objective problem into multiple single-objective problems (Chugh, 2020; Jiang et al., 2026a). Then, acquisition functions from single-objective BO can be used to select the next point for evaluation, which include lower confidence bound (LCB) (Paria et al., 2020; Zhang & Golovin, 2020; Li et al., 2024; Lin et al., 2022; Li et al., 2025b; Cheng et al., 2026), probability of improvement (PI) (De Ath et al., 2022), expected improvement (EI) (Knowles, 2006; Zhang et al., 2010; Namura et al., 2017; Pires et al., 2026), Thompson sampling (TS) (Paria et al., 2020; Zhang & Golovin, 2020), and knowledge gradient (KG) (Buckingham et al., 2025).

Pareto-based methods directly optimise multiple objectives by considering Pareto dominance relations, without transforming the problem into multiple single-objective subproblems. Representative approaches include hypervolume-based and information-based methods. Hypervolume-based methods (Ponweiser et al., 2008; Shang et al., 2020; Bhatija et al., 2025; Hung et al., 2025; Qing et al., 2023a; Zhang et al., 2025) are widely used because maximising hypervolume is equivalent to obtaining the Pareto front (Zitzler et al., 2007; Li & Yao, 2019). Similar to scalarisation-based methods, most hypervolume-based methods combine hypervolume with acquisition functions from single-objective BO, including lower confidence bound (LCB) (Roussel et al., 2021; Wang et al., 2026), probability of improvement (PI) (Wang et al., 2022; Couckuyt et al., 2014), expected improvement (EI) (Emmerich et al., 2006; Daulton et al., 2020; 2021), Thompson sampling (TS) (Bradford et al., 2018; Daulton et al., 2022b), and knowledge gradient (KG) (Daulton et al., 2023).

Information-based methods use information theory to reduce uncertainty about the Pareto set or Pareto front and guide the search towards promising regions (Qing et al., 2023b; James & Thomas, 2024; Garrido-Merchán, 2025). Representative approaches include predictive entropy search (PES), which reduces uncertainty about the optimal inputs (i.e., the approximated Pareto set) (Garrido-Merchán et al., 2023; Garrido-Merchán & Hernández-Lobato, 2019; Hernandez-Lobato et al., 2016), max-value entropy search (MES), which reduces uncertainty about the optimal outputs (i.e., the approximated Pareto front) (Belakaria et al., 2019; 2021; Suzuki et al., 2020), and joint entropy search (JES), which considers both the Pareto set and Pareto front (Tu et al., 2022; Fernández-Sánchez & Hernández-Lobato, 2025).

Apart from hypervolume-based and information-based methods, other Pareto-based methods select points based on different criteria (Ahmadianshalchi et al., 2024; Park et al., 2024). These methods first optimise multiple acquisition functions for different objectives, such as PI, EI, LCB, and TS, to obtain an approximate Pareto front, and then select points from it using criteria such as maximum uncertainty (Belakaria et al., 2020) or maximin distance (Renganathan & Carlson, 2025).

Although formulated differently, most methods incorporate posterior uncertainty into the optimisation process. It should be noted that, similar to our approach, very few studies do not consider posterior uncertainty (Konakovic Lukovic et al., 2020). For example, instead of using posterior uncertainty, DGEMO (Konakovic Lukovic et al., 2020) promotes exploration through batch selection and explicit consideration of diversity in both the decision and objective spaces. In contrast, our approach does not introduce additional exploration mechanisms (e.g., batch selection and diversity); instead, exploration implicitly arises from the nondominance relations between solutions, the most basic relation in multiobjective optimisation. Building upon this, we systematically investigate whether MOBO can work without posterior uncertainty.

4 The Proposed Method

Unlike most existing MOBO methods, which use both posterior means and posterior uncertainty to guide the search, the proposed framework considers only the posterior means. Algorithm 1 gives

Algorithm 1: Posterior Mean-based Multi-Ojective Bayesian Optimisation (PMOBO) Framework

Input: $\boldsymbol{f}$: Expensive multi-objective problem; $\mathcal{X}$: Search space; T: Maximum number of iterations; $\mathcal{M}$: Posterior mean-based search strategy; $\mathcal{D}^{(0)}$: Initial observations.

for $t = 1$ **to** T **do**

- $GPs^{(t)} \leftarrow \text{TrainGPs}\left(\mathcal{D}^{(t-1)}\right)$ `// Train GP models`
- $\boldsymbol{x}^{(t)} \leftarrow \mathcal{M}\left(GPs^{(t)}, \mathcal{X}\right)$ `// Select a solution using posterior means`
- $\boldsymbol{y}^{(t)} \leftarrow \boldsymbol{f}\left(\boldsymbol{x}^{(t)}\right) + \boldsymbol{\zeta}^{(t)}$ `// Evaluate the selected solution`
- $\mathcal{D}^{(t)} \leftarrow \mathcal{D}^{(t-1)} \cup \left\{\left(\boldsymbol{x}^{(t)}, \boldsymbol{y}^{(t)}\right)\right\}$ `// Update observations`

Output: $\mathcal{D}^{(T)}$: Observed solution set.

the procedure of the proposed posterior mean-based MOBO framework. Starting from an initial observation set $\mathcal{D}^{(0)}$, independent GP models are fitted to the observations of each objective at every iteration. A posterior mean-based search strategy $\mathcal{M}$ is then applied to the GP models to select the next solution $\boldsymbol{x}^{(t)}$ for evaluation. Note that $\mathcal{M}$ uses only the posterior means provided by the GP models and does not consider posterior uncertainty. The selected solution is evaluated on the expensive objective functions, and the resulting observation is added to the dataset for the next iteration. This process is repeated until the evaluation budget is exhausted.

The framework is flexible in how $\mathcal{M}$ uses the posterior means to select solutions. In the following, we consider two instantiations of $\mathcal{M}$: a Pareto-based approach and a scalarisation-based approach.

4.1 Pareto-Based Instantiation

The first instantiation directly treats the posterior means as the objectives and solves the following multi-objective optimisation problem:

$$\min_{\boldsymbol{x} \in \mathcal{X}} \boldsymbol{\mu}(\boldsymbol{x}) = (\mu_1(\boldsymbol{x}), \ldots, \mu_m(\boldsymbol{x}))\,. \tag{1}$$

This produces an approximation to the Pareto set induced by the posterior means, denoted by

$$\widehat{\mathcal{P}}_\mu = \{\boldsymbol{x} \in \mathcal{X} \mid \nexists\, \boldsymbol{x}' \in \mathcal{X} : \boldsymbol{\mu}(\boldsymbol{x}') \prec \boldsymbol{\mu}(\boldsymbol{x})\}\,, \tag{2}$$

with the corresponding predicted Pareto front

$$\widehat{\mathcal{F}}_\mu = \left\{\boldsymbol{\mu}(\boldsymbol{x}) \mid \boldsymbol{x} \in \widehat{\mathcal{P}}_\mu\right\}. \tag{3}$$

Optimising the posterior means in this way generates a set of solutions with different predicted trade-offs among the objectives. Although posterior uncertainty is not explicitly incorporated, these solutions can cover different regions of the search space. To select candidates for expensive evaluation, we calculate the hypervolume contribution of the solutions in $\widehat{\mathcal{P}}_\mu$ with respect to the currently evaluated solutions and choose the solution with the largest contribution.

The optimisation of $\boldsymbol{\mu}(\boldsymbol{x})$ is independent of the general PMOBO framework and can be performed using any suitable multi-objective optimiser. In our implementation, the well-known NSGA-II (Deb et al., 2002) is used to approximate the Pareto set of the posterior means. For better optimisation efficiency, part of the initial population of NSGA-II is constructed from the currently evaluated non-dominated solutions, while the remaining individuals are randomly sampled. This warm-start strategy allows the optimiser to further refine existing promising regions instead of restarting the search from the initial design at each iteration. The ablation study can be found at Appendix C.2.

4.2 Scalarisation-Based Instantiation

The second instantiation considers MOBO from a scalarisation-based perspective. We adopt the idea of Pareto set learning (Lin et al., 2022), which aims to learn a continuous mapping from trade-off preferences to the corresponding decision variables.

Let $\Lambda = \{\boldsymbol{\lambda} \in \mathbb{R}_+^m \mid \sum_{i=1}^m \lambda_i = 1\}$ denote the preference space. The Pareto set is represented by a parameterised model

$$\boldsymbol{x}(\boldsymbol{\lambda}) = h_{\boldsymbol{\theta}}(\boldsymbol{\lambda}), \tag{4}$$

where $h_{\boldsymbol{\theta}} : \Lambda \to \mathcal{X}$ maps a preference vector to a solution in the decision space.

To train the Pareto set model, each preference vector is associated with a scalarised optimisation problem. Following the practice in Lin et al. (2022), we employ the augmented Tchebycheff scalarisation:

$$g_{\text{tch}}\big(\boldsymbol{\mu}(\boldsymbol{x}) \mid \boldsymbol{\lambda}\big) = \max_{1\le i\le m} \{\lambda_i(\mu_i(\boldsymbol{x}) - (z_i - \epsilon))\} + \rho \sum_{i=1}^{m} \lambda_i \mu_i(\boldsymbol{x}), \tag{5}$$

where $\boldsymbol{z}$ is a utopian reference vector and ρ is a small augmentation coefficient.

Different from PSL (Lin et al., 2022), which optimises $\boldsymbol{\mu}(\boldsymbol{x}) - \beta\boldsymbol{\sigma}(\boldsymbol{x})$ as the surrogate objective, our method considers only the posterior means $\boldsymbol{\mu}(\boldsymbol{x})$. The parameters of the Pareto set model are therefore learned by minimising the scalarised posterior means:

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \mathbb{E}_{\boldsymbol{\lambda}\sim\Lambda} \left[g_{\text{tch}} \left(\boldsymbol{\mu} \left(h_{\boldsymbol{\theta}}(\boldsymbol{\lambda}) \right) \mid \boldsymbol{\lambda} \right) \right]. \tag{6}$$

The expectation is approximated by sampling preference vectors from Λ. At each iteration, the sampled preferences are used to update the model parameters through gradient descent:

$$\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t - \eta \nabla_{\boldsymbol{\theta}} \frac{1}{K} \sum_{k=1}^{K} g_{\text{tch}} \left(\boldsymbol{\mu} \left(h_{\boldsymbol{\theta}_t}(\boldsymbol{\lambda}_k) \right) \mid \boldsymbol{\lambda}_k \right), \tag{7}$$

where K is the number of sampled preference vectors, $\boldsymbol{\lambda}_k$ denotes the k-th sampled preference vector from Λ, η is the learning rate, and $\boldsymbol{\theta}_t$ represents the parameters of the Pareto set model at iteration t.

After training, we uniformly sample preference vectors from Λ and generate candidate solutions using the learned Pareto set model. The candidate with the largest hypervolume improvement over the currently evaluated solutions is selected for the next evaluation.

Note that we follow all settings in Lin et al. (2022), e.g., the model architecture, optimiser, and training procedure. The only difference is that original PSL considers posterior uncertainty, whereas we consider only the posterior mean.

5 Experimental Design

Compared Methods. To evaluate the proposed PMOBO, we compare it with nine well-established methods: five Pareto-based methods, three scalarisation-based methods, and one baseline, Sobol (Sobol, 1967). The five Pareto-based methods include three HV-based methods, i.e., DGEMO (Konakovic Lukovic et al., 2020), EHVI (Daulton et al., 2020; Ament et al., 2023) (along with its noisy variant NEHVI (Daulton et al., 2021)), and MOBO-OSD (Ngo et al., 2025); one information-based method, JES (Tu et al., 2022); and one maximin-distance-based method, qPOTS (Renganathan & Carlson, 2025). The three scalarisation-based methods are ParEGO (Knowles, 2006; Ament et al., 2023) (along with its noisy variant NParEGO (Daulton et al., 2021)), TS-TCH (Paria et al., 2020), and PSL (Lin et al., 2022). For all EI-based methods, namely EHVI, NEHVI, ParEGO, and NParEGO, we use their log variants for numerical stability, as suggested in (Ament et al., 2023). Detailed implementations of our method and the peer methods can be found in Appendix B.1.

Benchmarks and Real-World Problems. We evaluate our method on 79 problem instances from benchmark suites and real-world problems. For the benchmark problems, we consider widely used BBOB (Brockhoff et al., 2022) and DTLZ (Deb et al., 2005) suites. Following the common practice in black-box optimisation studies, the BBOB problems are considered with a dimensionality of 10. The DTLZ suite includes both bi-objective and tri-objective problems to evaluate the performance of the proposed method under different levels of objective dimensionality. For the real-world benchmark problems, we use 10 problems from the RE problem suite (Tanabe & Ishibuchi, 2020), covering problems with two to four objectives. These problems have been widely used in the MOBO literature (Bradford et al., 2018; Daulton et al., 2020; 2021; Lin et al., 2022; Ament et al., 2023; Renganathan & Carlson, 2025; Yang et al., 2026). We test the above problems under both noiseless and noisy cases. To make their noise cases, we include additive zero-mean Gaussian noise with a standard deviation set to approximately 10% of the objective ranges, as suggested in (Daulton et al., 2021; Tu et al., 2022).

Statistical Validation. For all the methods, we allow a maximum of 200 evaluations, following the practice in (Daulton et al., 2020; 2021; Konakovic Lukovic et al., 2020). To enable statistical comparisons, each optimisation was repeated 30 times. We use the Wilcoxon rank-sum test (Wilcoxon, 1992) at a significance level of 0.05 and Holm-Bonferroni correction (Holm, 1979) to see if our method differs significantly from each peer method.

Performance Indicator. HV (Zitzler & Thiele, 1999) is used to compare all methods. For the evaluation, we use the commonly used setting in quality evaluation of multi-objective optimisation (Li & Yao, 2019). That is, we collect all solutions obtained by all methods over 30 independent runs and set the reference point $\boldsymbol{r} = \boldsymbol{y}_{\text{nadir}} + 0.1\,|\boldsymbol{y}_{\text{nadir}} - \boldsymbol{y}_{\text{ideal}}|$.

6 Experimental Results

6.1 Comparison with Well-Established Methods

Noiseless Cases. We begin our evaluation in the noiseless setting. Table 1 shows the HV results (mean and standard deviation) of our method and the nine peer methods on the 79 benchmark and real-world problems. As can be seen, PMOBO demonstrates strong performance across both benchmark and real-world problems. Overall, our method is statistically better than its peer methods on average in 60.6 out of the 79 problems, while being statistically worse than them in only 6.1 problems.

To see the overall performance, Figure 1 shows the hypervolume trajectories with respect to the number of evaluations of the ten methods on 12 representative problems. As can be seen, PMOBO generally achieves faster convergence and obtains higher or competitive final hypervolume values compared with the peer methods. Regarding benchmark problems, PMOBO quickly improves the hypervolume during the early optimisation stage and maintains its advantage throughout the optimisation process. For the real-world problems, PMOBO also demonstrates strong convergence behaviour, achieving comparable or better performance than existing methods.

To visually understand the results, Figure 2 shows the solutions obtained by each method on DTLZ1 ($m = 2$). As can be seen, PMOBO obtains solutions with both good convergence and diversity. The obtained solutions are close to the Pareto front and cover a wide range of trade-offs between the two objectives. In contrast, peer methods either fail to approximate the Pareto front or provide solutions concentrated in limited regions of the objective space.

Noisy Cases. Next, we consider the noisy setting. Here we do not include DGEMO Konakovic Lukovic et al. (2020), PSL (Lin et al., 2022), and MOBO-OSD (Ngo et al., 2025), as their original studies focus on noiseless settings. For ParEGO and EHVI, we use their noisy variants (Ament et al., 2023; Daulton et al., 2021). We evaluate the proposed PMOBO on the same 79 benchmark and real-world problems as in the noiseless setting but with observation noise. For performance evaluation, the optimisation algorithms use the noisy observations during the search, while HV is calculated using the corresponding noise-free objective values of all evaluated solutions.

The HV results (mean and standard deviation) of each method can be found in Appendix C.1.1. As can be seen in Table 2, similar to the noiseless setting, PMOBO demonstrates a clear performance advantage over its competitors. Overall, our method is statistically better than its peer methods in 58.8 out of the 79 problems on average, while being statistically worse than its peer methods on only 5.0 problems.

6.2 Impact of Incorporating Posterior Uncertainty into PMOBO

Above, we have seen strong performance of the proposed PMOBO using only posterior means. Now one may wonder how the absence of posterior uncertainty affects the algorithm, i.e., whether incorporating posterior uncertainty could further improve its performance. To answer this question, we extend PMOBO by considering different levels of posterior uncertainty during optimisation and investigate their impact on performance.

Specifically, we study how the level of posterior uncertainty affects PMOBO by incorporating posterior uncertainty through the lower confidence bound (LCB): $\boldsymbol{\alpha}(\boldsymbol{x}) = \boldsymbol{\mu}(\boldsymbol{x}) - \beta\boldsymbol{\sigma}(\boldsymbol{x})$, where

Table 1: The HV results (mean and standard deviation) of the ten methods on the 79 benchmark and real-world problems. The method with the best mean HV is highlighted in bold. The symbols "$-$", "$\sim$" and "$+$" indicate that our method PMOBO is statistically worse than, equivalent to, and better than the peer method, respectively.

Method	BBOB1	BBOB2	BBOB3	BBOB4	BBOB5	BBOB6	BBOB7	BBOB8
Sobol	4.29e+04(2.2e+03)$^+$	9.39e+08(1.0e+08)$^+$	4.70e+08(1.7e+07)$^+$	1.45e+08(8.3e+06)$^+$	6.22e+05(2.3e+04)$^+$	9.53e+04(2.5e+03)$^+$	2.59e+06(1.1e+05)$^+$	4.67e+03(3.8e+02)$^+$
ParEGO	5.85e+04(1.0e+03)$^+$	1.53e+09(9.7e+06)$^+$	5.69e+08(9.8e+06)$^+$	1.85e+08(2.6e+06)$^+$	8.98e+05(1.9e+04)$^+$	1.12e+05(1.5e+03)$^+$	3.03e+06(7.9e+03)$^+$	6.52e+03(4.0e+02)$^+$
TS-TCH	5.45e+04(5.0e+02)$^+$	1.43e+09(1.6e+07)$^+$	5.58e+08(3.9e+06)$^+$	1.78e+08(1.7e+06)$^+$	8.09e+05(1.2e+04)$^+$	1.07e+05(1.4e+03)$^+$	2.96e+06(2.6e+04)$^+$	6.47e+03(2.4e+02)$^+$
PSL	5.89e+04(2.4e+03)$^+$	1.53e+09(1.8e+07)$^+$	5.76e+08(6.6e+06)$^+$	1.87e+08(1.7e+06)$^+$	**9.37e+05(3.2e+03)**$^-$	1.15e+05(4.6e+02)$^+$	3.04e+06(5.0e+03)$^+$	6.80e+03(2.8e+02)$^+$
JES	5.84e+04(8.5e+02)$^+$	1.42e+09(5.8e+07)$^+$	5.62e+08(1.2e+07)$^+$	1.76e+08(6.5e+06)$^+$	9.18e+05(5.3e+03)$^+$	1.10e+05(2.8e+03)$^+$	3.02e+06(1.2e+04)$^+$	6.87e+03(3.1e+02)$^+$
DGEMO	**5.95e+04(1.9e+01)**$^-$	**1.55e+09(8.2e+05)**$^-$	5.66e+08(7.2e+06)$^+$	1.77e+08(7.3e+06)$^+$	9.20e+05(5.2e+03)$^+$	1.10e+05(2.4e+03)$^+$	3.02e+06(9.9e+03)$^+$	6.95e+03(3.1e+02)$^+$
EHVI	5.91e+04(1.5e+02)$^+$	1.53e+09(1.2e+07)$^+$	5.76e+08(5.6e+06)$^+$	1.84e+08(2.5e+06)$^+$	8.95e+05(1.3e+04)$^+$	1.13e+05(1.1e+03)$^+$	3.04e+06(1.1e+04)$^+$	7.09e+03(2.7e+02)$^+$
MOBO-OSD	5.92e+04(2.1e+02)$^+$	1.45e+09(4.3e+07)$^+$	5.70e+08(7.4e+06)$^+$	1.79e+08(7.5e+06)$^+$	9.21e+05(3.4e+03)$^+$	1.10e+05(2.8e+03)$^+$	3.02e+06(1.3e+04)$^+$	6.93e+03(3.3e+02)$^+$
qPOTS	5.47e+04(4.9e+02)$^+$	1.43e+09(1.6e+07)$^+$	5.58e+08(4.6e+06)$^+$	1.77e+08(3.0e+06)$^+$	8.04e+05(1.2e+04)$^+$	1.07e+05(9.4e+02)$^+$	2.97e+06(2.7e+04)$^+$	6.41e+03(2.7e+02)$^+$
PMOBO	5.95e+04(4.0e+01)	1.55e+09(2.3e+06)	**5.83e+08(5.7e+05)**	**1.89e+08(1.4e+05)**	9.24e+05(3.7e+03)	**1.16e+05(3.4e+02)**	**3.05e+06(6.8e+03)**	**7.34e+03(2.0e+02)**

Method	BBOB9	BBOB10	BBOB11	BBOB12	BBOB13	BBOB14	BBOB15	BBOB16
Sobol	7.27e+06(1.1e+06)$^+$	4.59e+03(1.0e+03)$^+$	2.91e+14(3.2e+12)$^+$	1.01e+13(9.5e+11)$^+$	6.66e+12(1.9e+11)$^+$	1.50e+09(7.2e+08)$^+$	2.23e+07(1.4e+07)$^+$	3.89e+09(2.3e+08)$^+$
ParEGO	1.40e+07(1.7e+05)$^\sim$	1.33e+04(7.4e+02)$^+$	3.01e+14(2.5e+12)$^+$	1.34e+13(2.0e+10)$^+$	7.26e+12(1.0e+10)$^+$	7.81e+09(1.4e+08)$^\sim$	7.61e+07(3.6e+06)$^+$	5.22e+09(9.2e+07)$^+$
TS-TCH	1.26e+07(1.4e+05)$^+$	8.93e+03(1.2e+03)$^+$	3.00e+14(2.9e+12)$^+$	1.32e+13(6.6e+10)$^+$	7.23e+12(2.0e+10)$^+$	5.60e+09(3.0e+08)$^+$	7.03e+07(3.1e+06)$^+$	5.16e+09(1.3e+08)$^+$
PSL	**1.41e+07(1.1e+05)**$^-$	1.27e+04(5.9e+02)$^+$	3.06e+14(1.8e+12)$^-$	1.33e+13(9.8e+10)$^\sim$	7.25e+12(4.2e+10)$^\sim$	7.20e+09(1.2e+09)$^\sim$	7.30e+07(6.8e+06)$^+$	5.22e+09(1.2e+08)$^+$
JES	1.32e+07(7.0e+05)$^+$	1.13e+04(1.6e+03)$^+$	2.99e+14(3.7e+12)$^+$	1.29e+13(4.5e+11)$^+$	7.22e+12(5.8e+10)$^+$	5.43e+09(1.3e+09)$^+$	6.77e+07(7.0e+06)$^+$	5.16e+09(1.2e+08)$^+$
DGEMO	1.36e+07(4.6e+05)$^+$	1.15e+04(1.1e+03)$^+$	**3.06e+14(2.5e+11)**$^-$	1.27e+13(5.5e+11)$^+$	7.22e+12(6.5e+10)$^+$	5.79e+09(1.5e+09)$^+$	7.02e+07(8.1e+06)$^+$	5.17e+09(9.1e+07)$^+$
EHVI	1.40e+07(1.8e+05)$^+$	1.32e+04(5.8e+02)$^+$	3.02e+14(2.3e+12)$^+$	1.34e+13(1.7e+10)$^+$	7.26e+12(1.7e+10)$^+$	7.77e+09(1.3e+08)$^+$	7.49e+07(3.1e+06)$^+$	**5.44e+09(9.2e+07)**$^-$
MOBO-OSD	1.38e+07(5.0e+05)$^\sim$	1.26e+04(1.1e+03)$^+$	3.02e+14(3.4e+12)$^+$	1.28e+13(5.0e+11)$^+$	7.26e+12(4.5e+10)$^+$	6.23e+09(1.8e+09)$^+$	7.40e+07(9.3e+06)$^+$	5.20e+09(1.7e+08)$^+$
qPOTS	1.25e+07(1.8e+05)$^+$	8.25e+03(1.4e+03)$^+$	2.99e+14(2.6e+12)$^+$	1.32e+13(5.2e+10)$^+$	7.22e+12(2.4e+10)$^+$	5.76e+09(3.4e+08)$^+$	6.93e+07(4.0e+06)$^+$	5.16e+09(7.8e+07)$^+$
PMOBO	1.41e+07(5.3e+04)	**1.38e+04(9.8e+02)**	3.05e+14(1.2e+12)	**1.34e+13(7.5e+09)**	**7.28e+12(6.1e+09)**	**7.87e+09(2.6e+08)**	**8.35e+07(3.0e+06)**	5.34e+09(9.9e+07)

Method	BBOB17	BBOB18	BBOB19	BBOB20	BBOB21	BBOB22	BBOB23	BBOB24
Sobol	5.07e+07(2.3e+07)$^+$	6.33e+11(2.9e+10)$^+$	7.54e+08(1.9e+08)$^+$	1.37e+11(2.0e+10)$^+$	1.55e+11(1.0e+10)$^+$	1.17e+09(8.0e+07)$^+$	5.81e+07(3.1e+06)$^+$	1.19e+09(3.4e+07)$^+$
ParEGO	1.14e+08(1.2e+07)$^+$	7.24e+11(3.0e+09)$^+$	**1.68e+09(1.0e+08)**$^\sim$	2.12e+11(5.1e+08)$^\sim$	1.92e+11(1.7e+09)$^+$	**1.79e+09(2.9e+07)**$^\sim$	7.09e+07(1.8e+06)$^+$	1.33e+09(2.1e+07)$^+$
TS-TCH	1.06e+08(1.3e+07)$^+$	7.22e+11(1.9e+09)$^+$	1.21e+09(2.3e+08)$^+$	2.10e+11(9.3e+08)$^+$	1.91e+11(1.4e+09)$^+$	1.60e+09(2.5e+07)$^+$	6.86e+07(2.0e+06)$^+$	1.33e+09(1.5e+07)$^+$
PSL	1.01e+08(1.1e+07)$^+$	7.20e+11(1.3e+10)$^\sim$	1.31e+09(1.6e+08)$^+$	2.11e+11(2.4e+09)$^+$	1.94e+11(1.6e+09)$^+$	1.73e+09(1.3e+08)$^\sim$	6.91e+07(4.0e+06)$^+$	1.31e+09(2.3e+07)$^+$
JES	9.93e+07(1.5e+07)$^+$	7.15e+11(1.1e+10)$^+$	1.32e+09(1.6e+08)$^+$	2.12e+11(1.3e+08)$^\sim$	1.91e+11(2.3e+09)$^+$	1.74e+09(5.2e+07)$^+$	6.84e+07(2.3e+06)$^+$	1.32e+09(2.3e+07)$^+$
DGEMO	1.04e+08(1.3e+07)$^+$	7.20e+11(6.9e+09)$^+$	1.24e+09(1.7e+08)$^+$	2.12e+11(2.7e+08)$^\sim$	1.92e+11(2.3e+09)$^+$	1.74e+09(6.3e+07)$^\sim$	6.77e+07(3.0e+06)$^+$	1.32e+09(2.8e+07)$^+$
EHVI	**1.28e+08(8.9e+06)**$^\sim$	7.26e+11(1.5e+09)$^+$	1.67e+09(1.2e+08)$^\sim$	**2.12e+11(6.9e+07)**$^\sim$	1.94e+11(1.0e+09)$^+$	1.78e+09(3.2e+07)$^\sim$	6.89e+07(2.3e+06)$^+$	**1.35e+09(1.7e+07)**$^\sim$
MOBO-OSD	1.05e+08(1.2e+07)$^+$	7.22e+11(5.0e+09)$^+$	1.39e+09(1.9e+08)$^+$	2.12e+11(1.5e+08)$^\sim$	1.93e+11(1.8e+09)$^+$	1.75e+09(8.4e+07)$^\sim$	6.83e+07(3.3e+06)$^+$	1.32e+09(2.2e+07)$^+$
qPOTS	1.09e+08(1.0e+07)$^+$	7.23e+11(1.9e+09)$^+$	1.24e+09(2.4e+08)$^+$	2.10e+11(1.2e+09)$^+$	1.90e+11(1.8e+09)$^+$	1.59e+09(2.3e+07)$^+$	6.90e+07(2.3e+06)$^+$	1.33e+09(1.4e+07)$^+$
PMOBO	1.28e+08(1.1e+07)	**7.27e+11(1.1e+09)**	1.66e+09(1.3e+08)	2.12e+11(8.2e+07)	**1.95e+11(8.8e+08)**	1.77e+09(2.6e+07)	**7.42e+07(2.0e+06)**	1.35e+09(1.2e+07)

Method	BBOB25	BBOB26	BBOB27	BBOB28	BBOB29	BBOB30	BBOB31	BBOB32
Sobol	1.63e+07(1.3e+06)$^+$	0.00e+00(0.0e+00)$^+$	1.03e+07(5.8e+06)$^+$	1.29e+09(8.3e+08)$^+$	6.47e+08(4.9e+07)$^+$	4.00e+06(1.1e+06)$^+$	2.32e+08(1.2e+07)$^+$	0.00e+00(0.0e+00)$^+$
ParEGO	2.05e+07(1.7e+06)$^+$	8.56e+07(2.4e+07)$^\sim$	**4.05e+07(3.3e+06)**$^\sim$	3.96e+09(5.2e+07)$^\sim$	1.05e+09(9.4e+06)$^+$	7.14e+06(2.4e+05)$^+$	2.99e+08(8.2e+06)$^+$	1.67e+04(6.4e+03)$^-$
TS-TCH	2.10e+07(1.6e+06)$^+$	1.97e+07(1.9e+07)$^+$	2.04e+07(6.8e+06)$^+$	3.83e+09(6.9e+07)$^+$	9.17e+08(2.4e+07)$^+$	6.97e+06(2.2e+05)$^+$	2.94e+08(5.0e+06)$^+$	6.40e+03(5.6e+03)$^+$
PSL	2.08e+07(1.9e+06)$^+$	**1.15e+08(2.6e+06)**$^-$	3.21e+07(4.3e+06)$^+$	**4.03e+09(4.3e+07)**$^-$	9.85e+08(1.1e+08)$^+$	7.23e+06(7.5e+05)$^+$	2.98e+08(6.5e+06)$^+$	**2.33e+04(7.2e+03)**$^-$
JES	2.20e+07(1.6e+06)$^+$	6.72e+07(3.8e+07)$^+$	2.91e+07(8.6e+06)$^+$	3.95e+09(5.0e+07)$^\sim$	9.66e+08(6.4e+07)$^+$	7.15e+06(2.8e+05)$^+$	2.99e+08(7.4e+06)$^+$	1.80e+04(4.5e+03)$^-$
DGEMO	2.11e+07(2.1e+06)$^+$	6.57e+07(4.6e+07)$^\sim$	2.71e+07(6.6e+06)$^+$	3.93e+09(5.9e+07)$^\sim$	9.89e+08(6.8e+07)$^+$	7.09e+06(2.7e+05)$^+$	2.97e+08(8.0e+06)$^+$	1.75e+04(7.0e+03)$^-$
EHVI	2.26e+07(1.5e+06)$^+$	1.02e+08(1.9e+07)$^-$	4.00e+07(4.5e+06)$^-$	3.95e+09(3.7e+07)$^\sim$	1.04e+09(1.9e+07)$^+$	7.20e+06(3.5e+05)$^+$	**3.11e+08(4.9e+06)**$^-$	2.25e+04(6.3e+03)$^-$
MOBO-OSD	2.12e+07(1.8e+06)$^+$	6.29e+07(5.0e+07)$^\sim$	2.87e+07(8.0e+06)$^+$	3.95e+09(4.9e+07)$^\sim$	1.05e+09(1.3e+07)$^+$	7.16e+06(3.1e+05)$^+$	2.96e+08(8.6e+06)$^+$	1.62e+04(6.2e+03)$^-$
qPOTS	2.09e+07(1.3e+06)$^+$	1.80e+07(2.0e+07)$^+$	2.16e+07(7.6e+06)$^+$	3.84e+09(6.0e+07)$^+$	9.14e+08(2.4e+07)$^+$	7.02e+06(3.2e+05)$^+$	2.96e+08(6.4e+06)$^+$	6.82e+03(5.3e+03)$^+$
PMOBO	**2.44e+07(1.1e+06)**	9.62e+07(9.2e+06)	3.95e+07(5.2e+06)	3.95e+09(7.6e+07)	**1.08e+09(7.8e+06)**	**8.18e+06(2.3e+05)**	3.07e+08(6.3e+06)	1.23e+04(7.4e+03)

Method	BBOB33	BBOB34	BBOB35	BBOB36	BBOB37	BBOB38	BBOB39	BBOB40
Sobol	5.51e+09(1.8e+09)$^+$	1.31e+07(4.2e+06)$^+$	3.43e+06(2.3e+05)$^+$	2.11e+05(1.1e+04)$^+$	8.37e+06(3.6e+05)$^+$	4.16e+04(3.4e+03)$^+$	1.42e+08(1.6e+07)$^+$	7.65e+04(1.6e+04)$^+$
ParEGO	9.60e+09(1.3e+08)$^+$	**3.78e+07(2.8e+06)**$^\sim$	6.21e+06(1.1e+05)$^+$	3.37e+05(1.1e+04)$^+$	1.13e+07(1.7e+05)$^\sim$	5.80e+04(5.1e+03)$^+$	2.76e+08(2.4e+06)$^+$	2.16e+05(8.6e+03)$^\sim$
TS-TCH	9.62e+09(1.8e+08)$^+$	2.28e+07(5.0e+06)$^+$	5.15e+06(1.0e+05)$^+$	2.97e+05(9.6e+03)$^+$	1.04e+07(2.0e+05)$^+$	5.32e+04(3.3e+03)$^+$	2.38e+08(7.4e+06)$^+$	1.44e+05(1.8e+04)$^+$
PSL	**9.93e+09(7.0e+07)**$^\sim$	3.13e+07(5.6e+06)$^+$	**6.41e+06(4.9e+05)**$^-$	**3.56e+05(5.0e+03)**$^-$	1.12e+07(2.0e+05)$^+$	6.30e+04(3.8e+03)$^\sim$	2.79e+08(1.3e+07)$^\sim$	2.02e+05(2.4e+04)$^+$
JES	9.60e+09(1.9e+08)$^+$	2.64e+07(6.1e+06)$^+$	6.41e+06(5.3e+04)$^-$	3.39e+05(1.2e+04)$^+$	1.10e+07(2.5e+05)$^+$	6.01e+04(4.7e+03)$^+$	2.63e+08(7.6e+06)$^+$	1.82e+05(2.5e+04)$^+$
DGEMO	9.62e+09(1.5e+08)$^+$	2.60e+07(4.3e+06)$^+$	6.41e+06(2.8e+04)$^-$	3.39e+05(9.5e+03)$^+$	1.10e+07(2.1e+05)$^+$	6.22e+04(3.3e+03)$^\sim$	2.60e+08(6.1e+06)$^+$	1.85e+05(2.3e+04)$^+$
EHVI	9.59e+09(1.7e+08)$^+$	3.76e+07(1.2e+06)$^\sim$	6.08e+06(1.2e+05)$^-$	3.37e+05(9.2e+03)$^+$	1.13e+07(1.7e+05)$^\sim$	6.29e+04(3.7e+03)$^\sim$	2.74e+08(3.0e+06)$^+$	2.15e+05(7.5e+03)$^\sim$
MOBO-OSD	9.65e+09(1.5e+08)$^+$	2.65e+07(4.9e+06)$^+$	6.39e+06(3.7e+04)$^-$	3.50e+05(8.2e+03)$^\sim$	1.11e+07(2.3e+05)$^+$	6.20e+04(3.0e+03)$^\sim$	2.63e+08(1.1e+07)$^+$	1.94e+05(1.7e+04)$^+$
qPOTS	9.58e+09(1.6e+08)$^+$	2.20e+07(6.3e+06)$^+$	5.19e+06(9.4e+04)$^+$	2.95e+05(7.5e+03)$^+$	1.03e+07(2.2e+05)$^+$	5.22e+04(4.6e+03)$^+$	2.36e+08(9.1e+06)$^+$	1.46e+05(2.0e+04)$^+$
PMOBO	9.89e+09(9.0e+07)	3.77e+07(3.7e+06)	6.34e+06(5.4e+04)	3.48e+05(7.9e+03)	**1.13e+07(2.3e+05)**	**6.43e+04(3.6e+03)**	**2.80e+08(4.5e+06)**	**2.20e+05(1.4e+04)**

Method	BBOB41	BBOB42	BBOB43	BBOB44	BBOB45	BBOB46	BBOB47	BBOB48
Sobol	2.15e+04(8.1e+02)$^+$	1.55e+05(4.6e+03)$^+$	2.39e+02(1.3e+02)$^+$	3.76e+05(2.1e+05)$^+$	6.97e+02(5.3e+02)$^+$	8.74e+05(2.9e+04)$^+$	1.83e+03(5.9e+02)$^+$	1.28e+08(5.3e+06)$^+$
ParEGO	2.40e+04(5.6e+02)$^+$	1.81e+05(3.0e+03)$^+$	8.36e+02(1.6e+02)$^\sim$	1.36e+06(5.1e+04)$^+$	**3.70e+03(2.6e+02)**$^\sim$	1.11e+06(2.8e+04)$^+$	5.44e+03(4.8e+02)$^\sim$	1.51e+08(3.1e+06)$^+$
TS-TCH	2.35e+04(5.4e+02)$^+$	1.74e+05(3.5e+03)$^+$	7.10e+02(1.4e+02)$^+$	1.24e+06(5.6e+04)$^+$	2.25e+03(7.2e+02)$^+$	1.07e+06(2.0e+04)$^+$	4.68e+03(4.9e+02)$^+$	1.51e+08(3.1e+06)$^+$
PSL	2.49e+04(5.6e+02)$^+$	1.84e+05(3.8e+03)$^+$	**9.70e+02(1.2e+02)**$^-$	1.42e+06(8.7e+04)$^\sim$	3.70e+03(4.1e+02)$^\sim$	1.10e+06(2.9e+04)$^+$	5.79e+03(4.3e+02)$^\sim$	1.46e+08(5.0e+06)$^+$
JES	2.41e+04(8.3e+02)$^+$	1.80e+05(4.1e+03)$^+$	8.96e+02(1.4e+02)$^\sim$	1.40e+06(6.8e+04)$^\sim$	3.43e+03(4.3e+02)$^+$	1.11e+06(2.1e+04)$^+$	5.38e+03(5.9e+02)$^\sim$	1.53e+08(4.3e+06)$^+$
DGEMO	2.42e+04(5.5e+04)$^+$	1.81e+05(4.1e+03)$^+$	8.88e+02(1.2e+02)$^\sim$	1.40e+06(7.3e+04)$^\sim$	3.23e+03(4.9e+02)$^+$	1.10e+06(2.9e+04)$^+$	5.48e+03(5.1e+02)$^\sim$	1.52e+08(5.0e+06)$^+$
EHVI	2.40e+04(6.6e+02)$^+$	1.82e+05(4.3e+03)$^+$	9.41e+02(1.1e+02)$^\sim$	1.35e+06(4.9e+04)$^+$	3.61e+03(2.7e+02)$^\sim$	**1.13e+06(3.3e+04)**$^\sim$	**5.81e+03(5.2e+02)**$^\sim$	1.56e+08(3.4e+06)$^\sim$
MOBO-OSD	2.43e+04(8.1e+02)$^+$	1.80e+05(3.7e+03)$^+$	8.93e+02(1.5e+02)$^\sim$	**1.43e+06(7.7e+04)**$^\sim$	3.13e+03(6.6e+02)$^+$	1.10e+06(2.7e+04)$^+$	5.39e+03(4.4e+02)$^\sim$	1.51e+08(4.1e+06)$^+$
qPOTS	2.35e+04(7.7e+02)$^+$	1.74e+05(3.4e+03)$^+$	6.87e+02(8.3e+01)$^+$	1.23e+06(5.1e+04)$^+$	2.09e+03(7.7e+02)$^+$	1.07e+06(2.0e+04)$^+$	4.69e+03(4.4e+02)$^+$	1.51e+08(3.3e+06)$^+$
PMOBO	**2.59e+04(4.8e+02)**	**1.86e+05(3.0e+03)**	8.50e+02(1.6e+02)	1.41e+06(6.4e+04)	3.58e+03(6.8e+02)	1.13e+06(2.2e+04)	5.63e+03(5.9e+02)	**1.58e+08(2.9e+06)**

Method	BBOB49	BBOB50	BBOB51	BBOB52	BBOB53	BBOB54	BBOB55	DTLZ1 ($m = 2$)
Sobol	3.39e+04(7.8e+03)$^+$	8.70e+02(5.9e+01)$^+$	9.55e+05(1.5e+05)$^+$	1.54e+02(7.0e+01)$^+$	1.19e+10(3.8e+08)$^+$	1.49e+06(6.1e+05)$^+$	1.01e+03(2.5e+02)$^+$	2.02e+00(8.4e+00)$^+$
ParEGO	7.71e+04(7.7e+03)$^\sim$	1.17e+03(7.2e+01)$^+$	1.37e+06(1.2e+05)$^+$	6.89e+02(1.5e+02)$^\sim$	1.35e+10(4.1e+07)$^+$	4.97e+06(2.3e+05)$^\sim$	5.14e+03(1.0e+03)$^\sim$	8.10e+00(3.1e+01)$^+$
TS-TCH	5.17e+04(1.0e+04)$^+$	1.03e+03(5.5e+01)$^+$	1.33e+06(1.2e+05)$^+$	3.20e+02(1.5e+02)$^+$	1.35e+10(3.2e+07)$^+$	3.57e+06(6.2e+05)$^+$	1.96e+03(5.8e+02)$^+$	0.00e+00(0.0e+00)$^+$
PSL	7.18e+04(1.1e+04)$^\sim$	1.19e+03(6.4e+01)$^+$	1.37e+06(1.2e+05)$^+$	6.74e+02(1.5e+02)$^\sim$	1.35e+10(9.0e+07)$^+$	4.49e+06(6.2e+05)$^+$	3.60e+03(7.6e+02)$^+$	9.03e+01(1.4e+02)$^+$
JES	7.83e+04(7.3e+03)$^\sim$	1.16e+03(7.6e+01)$^+$	1.39e+06(1.5e+05)$^+$	7.38e+02(1.5e+02)$^\sim$	1.34e+10(5.5e+07)$^+$	4.32e+06(5.1e+05)$^+$	5.00e+03(1.0e+03)$^\sim$	0.00e+00(0.0e+00)$^+$
DGEMO	7.05e+04(9.0e+03)$^+$	1.16e+03(6.3e+01)$^+$	1.38e+06(1.3e+05)$^+$	6.98e+02(1.4e+02)$^\sim$	1.34e+10(5.0e+07)$^+$	4.47e+06(4.6e+05)$^+$	5.01e+03(1.2e+03)$^\sim$	0.00e+00(0.0e+00)$^+$
EHVI	**8.19e+04(5.5e+03)**$^\sim$	1.17e+03(7.2e+01)$^+$	1.52e+06(1.4e+05)$^+$	**7.82e+02(1.1e+02)**$^-$	1.35e+10(2.4e+07)$^+$	**5.09e+06(1.5e+05)**$^\sim$	**5.65e+03(9.9e+02)**$^\sim$	1.32e-01(6.5e-01)$^+$
MOBO-OSD	6.90e+04(1.2e+04)$^+$	1.17e+03(8.0e+01)$^+$	1.32e+06(1.4e+05)$^+$	6.96e+02(1.5e+02)$^\sim$	1.35e+10(4.1e+07)$^+$	4.35e+06(4.2e+05)$^+$	5.14e+03(8.7e+02)$^\sim$	1.05e+00(4.0e+00)$^+$
qPOTS	5.06e+04(1.2e+04)$^+$	1.03e+03(5.9e+01)$^+$	1.33e+06(1.3e+05)$^+$	3.73e+02(1.5e+02)$^+$	1.35e+10(3.6e+07)$^+$	3.37e+06(7.5e+05)$^+$	2.05e+03(7.2e+02)$^+$	0.00e+00(0.0e+00)$^+$
PMOBO	7.82e+04(1.1e+04)	**1.25e+03(6.5e+01)**	**1.71e+06(8.9e+04)**	6.29e+02(2.3e+02)	**1.35e+10(4.7e+06)**	5.03e+06(1.8e+05)	5.06e+03(1.6e+03)	**1.48e+02(1.4e+02)**

Method	DTLZ2 ($m = 2$)	DTLZ3 ($m = 2$)	DTLZ4 ($m = 2$)	DTLZ5 ($m = 2$)	DTLZ6 ($m = 2$)	DTLZ7 ($m = 2$)	DTLZ1 ($m = 3$)	DTLZ2 ($m = 3$)
Sobol	2.63e-02(2.1e-02)$^+$	0.00e+00(0.0e+00)$^+$	1.90e-02(1.6e-02)$^+$	2.30e-02(2.2e-02)$^+$	0.00e+00(0.0e+00)$^+$	0.00e+00(0.0e+00)$^+$	2.60e+03(5.4e+03)$^+$	6.43e-02(3.2e-02)$^+$
ParEGO	4.04e-01(2.7e-03)$^+$	9.19e+02(1.1e+03)$^+$	4.04e-01(2.3e-03)$^+$	4.04e-01(3.3e-03)$^+$	2.91e-01(9.3e-02)$^\sim$	6.13e-01(2.3e-01)$^+$	3.83e+03(8.4e+03)$^+$	**6.03e-01(6.9e-02)**$^-$
TS-TCH	1.56e-01(2.5e-02)$^+$	0.00e+00(0.0e+00)$^+$	1.51e-01(2.3e-02)$^+$	1.50e-01(2.1e-02)$^+$	0.00e+00(0.0e+00)$^+$	0.00e+00(0.0e+00)$^+$	3.83e+03(3.3e+03)$^+$	8.71e-02(5.1e-02)$^+$
PSL	3.93e-01(1.1e-02)$^+$	1.05e+03(1.6e+03)$^\sim$	1.17e-01(2.1e-02)$^+$	3.94e-01(3.1e-02)$^+$	0.00e+00(0.0e+00)$^+$	4.92e-01(1.6e-01)$^+$	1.05e+04(9.0e+03)$^\sim$	2.74e-01(1.2e-01)$^+$
JES	3.99e-01(3.9e-03)$^+$	1.11e+03(1.3e+03)$^\sim$	3.99e-01(3.4e-03)$^+$	3.99e-01(2.4e-03)$^+$	2.88e-01(7.4e-02)$^\sim$	6.23e-01(1.9e-01)$^+$	**1.52e+04(4.6e+03)**$^-$	5.29e-01(7.7e-02)$^-$
DGEMO	4.04e-01(2.4e-03)$^+$	1.60e+03(1.4e+03)$^\sim$	4.04e-01(1.7e-03)$^+$	4.04e-01(2.2e-03)$^+$	2.71e-01(4.3e-02)$^\sim$	2.05e-01(4.6e-02)$^+$	1.32e+04(5.6e+03)$^\sim$	3.32e-01(1.0e-01)$^+$
EHVI	4.08e-01(1.2e-03)$^+$	1.41e+03(1.8e+03)$^\sim$	4.08e-01(1.6e-03)$^+$	4.08e-01(1.4e-03)$^+$	2.82e-01(4.3e-02)$^\sim$	1.89e-01(3.2e-02)$^+$	1.37e+04(6.1e+03)$^-$	3.20e-01(1.1e-01)$^+$
MOBO-OSD	4.00e-01(2.6e-03)$^+$	1.08e+03(9.6e+02)$^\sim$	4.00e-01(2.3e-03)$^+$	4.00e-01(2.8e-03)$^+$	2.33e-01(3.6e-02)$^\sim$	2.71e-01(6.9e-02)$^+$	1.38e+04(5.6e+03)$^\sim$	2.77e-01(9.6e-02)$^+$
qPOTS	1.51e-01(2.6e-02)$^+$	0.00e+00(0.0e+00)$^+$	1.54e-01(1.9e-02)$^+$	1.59e-01(2.3e-02)$^+$	0.00e+00(0.0e+00)$^+$	0.00e+00(0.0e+00)$^+$	9.73e+02(3.4e+03)$^+$	9.80e-02(4.7e-02)$^+$
PMOBO	**4.14e-01(1.3e-03)**	**3.29e+03(3.9e+03)**	**4.15e-01(8.1e-04)**	**4.14e-01(9.8e-04)**	**2.95e-01(9.2e-02)**	**6.82e-01(1.3e-01)**	1.09e+04(1.4e+04)	4.25e-01(1.0e-01)

Method	DTLZ3 ($m = 3$)	DTLZ4 ($m = 3$)	DTLZ5 ($m = 3$)	DTLZ6 ($m = 3$)	DTLZ7 ($m = 3$)	RE21	RE23	RE24
Sobol	4.46e+04(1.2e+05)$^+$	1.66e-01(5.0e-02)$^+$	2.95e-03(3.9e-03)$^+$	5.65e+02(3.5e+00)$^+$	0.00e+00(0.0e+00)$^+$	4.66e+01(5.6e-01)$^+$	5.31e+09(1.9e+09)$^+$	2.06e+04(4.7e+02)$^+$
ParEGO	8.81e+06(4.9e+06)$^\sim$	**9.52e-01(8.6e-02)**$^-$	1.21e-01(4.5e-03)$^+$	1.16e+03(1.1e+00)$^\sim$	7.24e-01(3.0e-01)$^+$	5.37e+01(5.6e-01)$^+$	**1.13e+10(2.3e+07)**$^-$	2.18e+04(1.0e+01)$^-$
TS-TCH	0.00e+00(0.0e+00)$^+$	2.13e-01(7.5e-02)$^+$	2.13e-02(1.0e-02)$^+$	6.38e+02(5.5e+00)$^+$	0.00e+00(0.0e+00)$^+$	5.20e+01(7.0e-01)$^+$	1.07e+10(2.3e+08)$^+$	2.17e+04(6.5e+01)$^-$
PSL	1.43e+07(1.6e+06)$^-$	3.19e-01(8.8e-02)$^+$	1.26e-01(5.3e-03)$^+$	1.10e+03(1.3e+01)$^+$	2.69e-01(2.1e-01)$^+$	5.40e+01(3.7e-02)$^+$	1.12e+10(1.6e+08)$^-$	2.18e+04(7.4e+00)$^-$
JES	1.52e+07(1.3e+06)$^-$	8.62e-01(6.4e-02)$^-$	1.13e-01(6.3e-03)$^+$	1.16e+03(2.7e+00)$^+$	7.95e-01(1.9e-01)$^+$	5.35e+01(

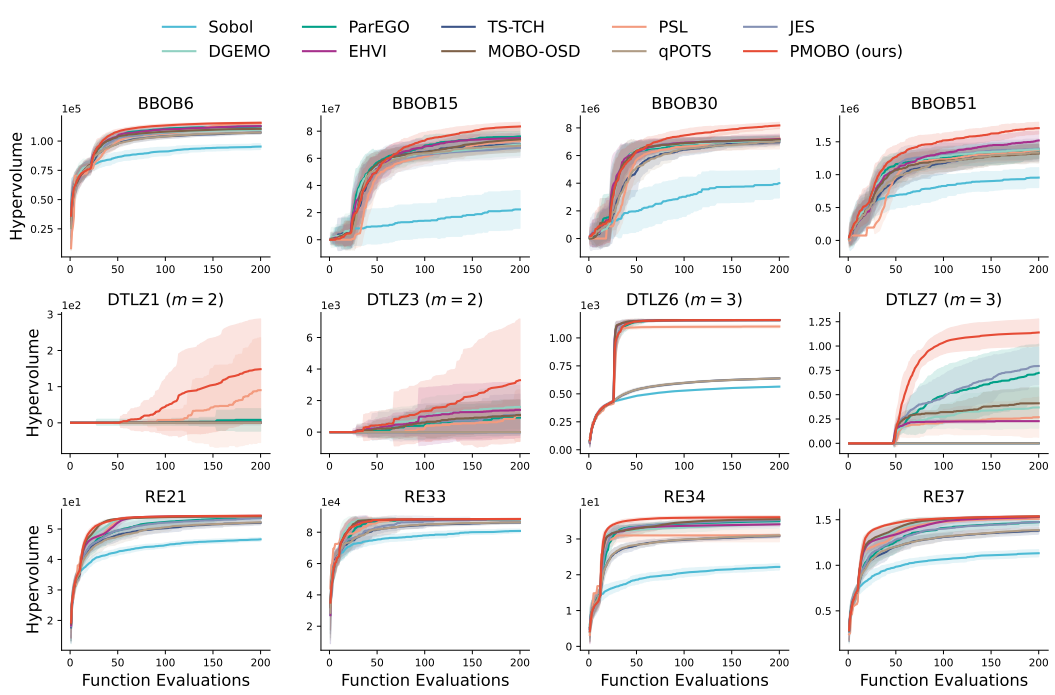

Figure 1: Mean hypervolume trajectories of the ten methods on 12 representative problems over 30 independent runs. The shaded regions indicate one standard deviation around the mean.

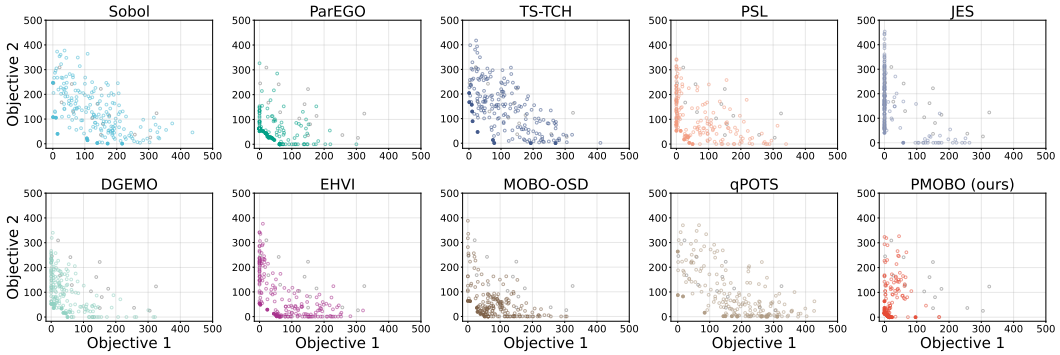

Figure 2: Solutions obtained by the proposed PMOBO and the nine peer methods on 2-objective DTLZ1. All methods start from the same $2(d+1)$ initial points generated by a scrambled Sobol sequence, i.e., gray points. Solid points are the non-dominated solutions found.

$\boldsymbol{\mu}(\boldsymbol{x})$ and $\boldsymbol{\sigma}(\boldsymbol{x})$ denote the posterior mean and standard deviation vectors, respectively, and β controls the level of posterior uncertainty. We then solve the following multi-objective optimisation problem: $\min_{\boldsymbol{x}\in\mathcal{X}} \boldsymbol{\alpha}(\boldsymbol{x})$. We vary β over $\{0, 0.1, 1, 10\}$. The case $\beta = 0$ corresponds to PMOBO, which relies only on the posterior mean. For $\beta > 0$, we obtain uncertainty-aware variants, denoted as U-MOBO. All U-MOBO variants use the same settings as PMOBO, with the only difference being the incorporation of posterior uncertainty.

The HV results (mean and standard deviation) of each method can be found in Appendix C.3. As can be seen, compared with the proposed mean-based method (PMOBO), U-MOBO ($\beta = 0.1$), U-MOBO ($\beta = 1$), and U-MOBO ($\beta = 10$) are statistically better on 0, 1, and 1 problems, respectively, while being statistically worse on 8, 49, and 72 problems, respectively. These results show that increasing the level of posterior uncertainty considered in PMOBO does not improve optimisation performance and can even lead to degraded performance.

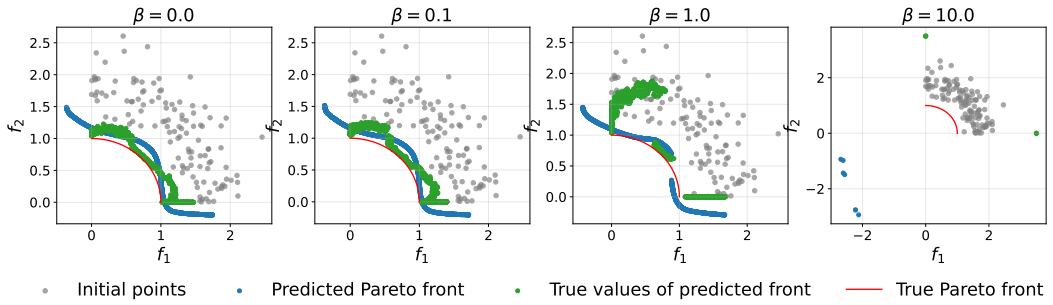

Figure 3: Effect of incorporating different levels of posterior uncertainty into PMOBO on the two-objective DTLZ2 problem. Results obtained with different values of β are shown. Blue points denote the Pareto front obtained by optimising $\min_{\boldsymbol{x}\in\mathcal{X}} (\boldsymbol{\mu}(\boldsymbol{x}) - \beta\boldsymbol{\sigma}(\boldsymbol{x}))$, green points denote the corresponding true objective values after evaluation, red curves show the true Pareto front, and gray points indicate the initial points.

To further investigate the reason behind the performance degradation, we visualise the predicted Pareto fronts and the corresponding true objective values after evaluation under different uncertainty levels in Figure 3. When $\beta = 0$, optimising only the posterior means produces a set of non-dominated solutions representing different trade-offs among the objectives, and the corresponding evaluated solutions cover a large portion of the optimal trade-off region. As β increases, the predicted Pareto fronts become further away from the true Pareto front, which may lead the selected solutions to lie further away from the true Pareto front. This suggests that placing greater weight on posterior uncertainty does not necessarily help PMOBO identify better solutions and may even degrade its performance.

6.3 Using the Scalarisation-Based Instantiation

As introduced in Section 4, PMOBO can be instantiated using both Pareto-based and scalarisation-based strategies. The experiments above used the Pareto-based instantiation. We now consider the scalarisation-based instantiation in Section 4, namely, PSL with the posterior means, denoted as PMOBO$_{PSL}$.

The HV results (mean and standard deviation) of each method can be found in Appendix C.4. As shown in Table 5, PMOBO$_{PSL}$ achieves competitive or even superior performance compared with PSL. Specifically, PMOBO$_{PSL}$ is statistically better than PSL on 25 problems, equivalent on 51 problems, and worse on 3 problems. It is worth noting that we follow the original PSL settings in Lin et al. (2022) (including the model architecture, optimiser, and training procedure). The only difference is that PSL considers posterior uncertainty, whereas we consider only the posterior mean.

7 Conclusion

Most MOBO methods use both posterior means and posterior uncertainty to guide the search. The latter is usually used for exploration. However, in this paper, we argue that it may not be necessary to consider posterior uncertainty in MOBO. The nondominance relations between solutions (through the posterior means) can implicitly encourage exploration. Based on this, we propose a posterior mean-based MOBO framework that uses only the posterior means, with both Pareto-based and scalarisation-based instantiations. Extensive experiments on a wide range of benchmark and real-world problems under both noiseless and noisy settings demonstrated that considering only the posterior means can achieve very competitive performance against existing MOBO methods that incorporate posterior uncertainty. One caveat of the proposed framework is that, because exploration is achieved through different trade-offs among the objectives, it may not work well with MOBO methods that consider only a very limited number of trade-offs, such as random scalarisation with a single direction. Future work could investigate how the proposed framework can be extended to other scenarios in multi-objective optimisation, such as constrained, high-dimensional, and multi-fidelity optimisation.

AI USE STATEMENT

Generative AI tools were used only for language editing, e.g., grammar, spelling, and word choice.

REFERENCES

Alaleh Ahmadianshalchi, Syrine Belakaria, and Janardhan Rao Doppa. Pareto front-diverse batch multi-objective Bayesian optimization. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 38, pp. 10784–10794, 2024.

Sebastian Ament, Samuel Daulton, David Eriksson, Maximilian Balandat, and Eytan Bakshy. Unexpected improvements to expected improvement for Bayesian optimization. In *Advances in Neural Information Processing Systems*, volume 36, pp. 20577–20612. Curran Associates, Inc., 2023.

Maximilian Balandat, Brian Karrer, Daniel Jiang, Samuel Daulton, Ben Letham, Andrew G Wilson, and Eytan Bakshy. Botorch: A framework for efficient monte-carlo Bayesian optimization. In *Advances in Neural Information Processing Systems*, volume 33, pp. 21524–21538. Curran Associates, Inc., 2020.

Syrine Belakaria, Aryan Deshwal, and Janardhan Rao Doppa. Max-value entropy search for multi-objective Bayesian optimization. In *Advances in Neural Information Processing Systems*, volume 32. Curran Associates, Inc., 2019.

Syrine Belakaria, Aryan Deshwal, Nitthilan Kannappan Jayakodi, and Janardhan Rao Doppa. Uncertainty-aware search framework for multi-objective Bayesian optimization. *Proceedings of the AAAI Conference on Artificial Intelligence*, 34(06):10044–10052, 2020.

Syrine Belakaria, Aryan Deshwal, and Janardhan Rao Doppa. Output space entropy search framework for multi-objective Bayesian optimization. *Journal of artificial intelligence research*, 72:667–715, 2021.

Shriya Bhatija, Paul-David Zuercher, Jakob Thumm, and Thomas Bohné. Multi-objective causal Bayesian optimization. *arXiv preprint arXiv:2502.14755*, 2025.

Mickael Binois, Victor Picheny, Patrick Taillandier, and Abderrahmane Habbal. The kalai-smorodinsky solution for many-objective Bayesian optimization. *Journal of Machine Learning Research*, 21(150):1–42, 2020.

Eric Bradford, Artur M Schweidtmann, and Alexei Lapkin. Efficient multiobjective optimization employing gaussian processes, spectral sampling and a genetic algorithm. *Journal of global optimization*, 71(2):407–438, 2018.

Dimo Brockhoff, Anne Auger, Nikolaus Hansen, and Tea Tušar. Using well-understood single-objective functions in multiobjective black-box optimization test suites. *Evolutionary computation*, 30(2):165–193, 2022.

Jack M Buckingham, Sebastian Rojas-Gonzalez, and Juergen Branke. Knowledge gradient for multi-objective Bayesian optimization with decoupled evaluations. In *International Conference on Evolutionary Multi-Criterion Optimization*, pp. 117–132. Springer, 2025.

Ji Cheng, Bo Xue, and Qingfu Zhang. Parametric pareto set learning for expensive multi-objective optimization. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 40, pp. 36829–36837, 2026.

Tinkle Chugh. Scalarizing functions in Bayesian multiobjective optimization. In *2020 IEEE Congress on Evolutionary Computation (CEC)*, pp. 1–8. IEEE, 2020.

Ivo Couckuyt, Dirk Deschrijver, and Tom Dhaene. Fast calculation of multiobjective probability of improvement and expected improvement criteria for Pareto optimization. *Journal of Global Optimization*, 60(3):575–594, 2014.

Sam Daulton, Maximilian Balandat, and Eytan Bakshy. Hypervolume knowledge gradient: A lookahead approach for multi-objective Bayesian optimization with partial information. In *Proceedings of the 40th International Conference on Machine Learning*, volume 202 of *Proceedings of Machine Learning Research*, pp. 7167–7204. PMLR, 2023.

Samuel Daulton, Maximilian Balandat, and Eytan Bakshy. Differentiable expected hypervolume improvement for parallel multi-objective Bayesian optimization. In *Advances in Neural Information Processing Systems*, volume 33, pp. 9851–9864. Curran Associates, Inc., 2020.

Samuel Daulton, Maximilian Balandat, and Eytan Bakshy. Parallel Bayesian optimization of multiple noisy objectives with expected hypervolume improvement. In *Advances in Neural Information Processing Systems*, volume 34. Curran Associates, Inc., 2021.

Samuel Daulton, Sait Cakmak, Maximilian Balandat, Michael A Osborne, Enlu Zhou, and Eytan Bakshy. Robust multi-objective Bayesian optimization under input noise. In *International Conference on Machine Learning*, pp. 4831–4866. PMLR, 2022a.

Samuel Daulton, David Eriksson, Maximilian Balandat, and Eytan Bakshy. Multi-objective Bayesian optimization over high-dimensional search spaces. In *Uncertainty in Artificial Intelligence*, pp. 507–517. PMLR, 2022b.

George De Ath, Richard M Everson, Alma AM Rahat, and Jonathan E Fieldsend. Greed is good: exploration and exploitation trade-offs in Bayesian optimisation. *ACM Transactions on Evolutionary Learning and Optimization*, 1(1):1–22, 2021.

George De Ath, Tinkle Chugh, and Alma AM Rahat. MBORE: multi-objective Bayesian optimisation by density-ratio estimation. In *Proceedings of the Genetic and Evolutionary Computation Conference*, pp. 776–785, 2022.

Kalyanmoy Deb and Himanshu Jain. An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part I: solving problems with box constraints. *IEEE transactions on Evolutionary Computation*, 18(4):577–601, 2013.

Kalyanmoy Deb, Amrit Pratap, Sameer Agarwal, and TAMT Meyarivan. A fast and elitist multiobjective genetic algorithm: Nsga-ii. *IEEE Transactions on Evolutionary Computation*, 6(2):182–197, 2002.

Kalyanmoy Deb, Lothar Thiele, Marco Laumanns, and Eckart Zitzler. Scalable test problems for evolutionary multiobjective optimization. In *Evolutionary multiobjective optimization: theoretical advances and applications*, pp. 105–145. Springer, 2005.

Michael TM Emmerich, Kyriakos C Giannakoglou, and Boris Naujoks. Single-and multiobjective evolutionary optimization assisted by Gaussian random field metamodels. *IEEE Transactions on Evolutionary Computation*, 10(4):421–439, 2006.

Daniel Fernández-Sánchez and Daniel Hernández-Lobato. Joint entropy search for multi-objective Bayesian optimization with constraints and multiple fidelities. *Neurocomputing*, pp. 131674, 2025.

Jacob Gardner, Geoff Pleiss, Kilian Q Weinberger, David Bindel, and Andrew G Wilson. Gpytorch: Blackbox matrix-matrix Gaussian process inference with GPU acceleration. In *Advances in Neural Information Processing Systems*, volume 31. Curran Associates, Inc., 2018.

Roman Garnett. *Bayesian optimization*. Cambridge University Press, 2023.

Eduardo C Garrido-Merchán. Information-theoretic Bayesian optimization: survey and tutorial. *arXiv preprint arXiv:2502.06789*, 2025.

Eduardo C Garrido-Merchán, Daniel Fernández-Sánchez, and Daniel Hernández-Lobato. Parallel predictive entropy search for multi-objective Bayesian optimization with constraints applied to the tuning of machine learning algorithms. *Expert Systems with Applications*, 215:119328, 2023.

Eduardo C. Garrido-Merchán and Daniel Hernández-Lobato. Predictive entropy search for multi-objective Bayesian optimization with constraints. *Neurocomputing*, 361:50–68, 2019.

Daniel Hernandez-Lobato, Jose Hernandez-Lobato, Amar Shah, and Ryan Adams. Predictive entropy search for multi-objective Bayesian optimization. In *Proceedings of The 33rd International Conference on Machine Learning*, volume 48 of *Proceedings of Machine Learning Research*, pp. 1492–1501, New York, USA, 2016. PMLR.

Sture Holm. A simple sequentially rejective multiple test procedure. *Scandinavian Journal of Statistics*, pp. 65–70, 1979.

Yu-Heng Hung, Kai-Jie Lin, Yu-Heng Lin, Chien-Yi Wang, Cheng Sun, and Ping-Chun Hsieh. Boformer: Learning to solve multi-objective Bayesian optimization via non-markovian RL. *arXiv preprint arXiv:2505.21974*, 2025.

Himanshu Jain and Kalyanmoy Deb. An evolutionary many-objective optimization algorithm using reference-point based nondominated sorting approach, part ii: Handling constraints and extending to an adaptive approach. *IEEE Transactions on evolutionary computation*, 18(4):602–622, 2013.

Thomas James and Sinnu Thomas. Multi objective regionalized Bayesian optimization via entropy search. In *OPT 2024: Optimization for Machine Learning*, 2024.

Chao Jiang and Miqing Li. Multi-objectivising acquisition functions in Bayesian optimisation. *ACM Transactions on Evolutionary Learning and Optimization*, 5(2), 2025a.

Chao Jiang and Miqing Li. Trading off quality and uncertainty through multi-objective optimisation in batch Bayesian optimisation. *Proceedings of the AAAI Conference on Artificial Intelligence*, 39 (25):27027–27035, 2025b.

Chao Jiang, Jingyu Huang, and Miqing Li. Do we really need to approach the entire pareto front in many-objective Bayesian optimisation? *arXiv preprint arXiv:2604.09417*, 2026a.

Chao Jiang, Yueling Huang, and Miqing Li. Converge then diversify: Decoupling convergence and diversity in multi-objective Bayesian optimisation. *arXiv preprint arXiv:2609.13396*, 2026b.

Donald R Jones, Matthias Schonlau, and William J Welch. Efficient global optimization of expensive black-box functions. *Journal of Global Optimization*, 13(4):455–492, 1998.

Joshua Knowles. ParEGO: A hybrid algorithm with on-line landscape approximation for expensive multiobjective optimization problems. *IEEE transactions on Evolutionary Computation*, 10(1): 50–66, 2006.

Mina Konakovic Lukovic, Yunsheng Tian, and Wojciech Matusik. Diversity-guided multi-objective Bayesian optimization with batch evaluations. In *Advances in Neural Information Processing Systems*, volume 33, pp. 17708–17720. Curran Associates, Inc., 2020.

Harold J Kushner. A new method of locating the maximum point of an arbitrary multipeak curve in the presence of noise. *Journal Basic Engineering*, 86(1):97–106, 1964.

Tze Leung Lai and Herbert Robbins. Asymptotically efficient adaptive allocation rules. *Advances in Applied Mathematics*, 6(1):4–22, 1985.

Bingdong Li, Zixiang Di, Yongfan Lu, Hong Qian, Feng Wang, Peng Yang, Ke Tang, and Aimin Zhou. Expensive multi-objective Bayesian optimization based on diffusion models. *Proceedings of the AAAI Conference on Artificial Intelligence*, 39(25):27063–27071, 2025a.

Diantong Li, Fengxue Zhang, Chong Liu, and Yuxin Chen. Constrained multi-objective Bayesian optimization through optimistic constraints estimation. *arXiv preprint arXiv:2411.03641*, 2024.

Diantong Li, Fengxue Zhang, Chong Liu, and Yuxin Chen. Constrained multi-objective Bayesian optimization through optimistic constraints estimation. In *International Conference on Artificial Intelligence and Statistics*, pp. 370–378. PMLR, 2025b.

Ke Li, Kalyanmoy Deb, Qingfu Zhang, and Sam Kwong. An evolutionary many-objective optimization algorithm based on dominance and decomposition. *IEEE transactions on Evolutionary Computation*, 19(5):694–716, 2014.

Miqing Li and Xin Yao. Quality evaluation of solution sets in multiobjective optimisation: A survey. *ACM Computing Surveys (CSUR)*, 52(2):1–38, 2019.

Xi Lin, Zhiyuan Yang, Xiaoyuan Zhang, and Qingfu Zhang. Pareto set learning for expensive multi-objective optimization. In *Advances in Neural Information Processing Systems*, volume 35, pp. 19231–19247. Curran Associates, Inc., 2022.

Jonas Mockus, Vytautas Tiesis, and Antanas Zilinskas. The application of Bayesian methods for seeking the extremum. *Towards Global Optimization*, 2(2):117–129, 1978.

Nobuo Namura, Koji Shimoyama, and Shigeru Obayashi. Expected improvement of penalty-based boundary intersection for expensive multiobjective optimization. *IEEE Transactions on Evolutionary Computation*, 21(6):898–913, 2017.

Lam Ngo, Huong Ha, Jeffrey Chan, and Hongyu Zhang. Mobo-osd: Batch multi-objective Bayesian optimization via orthogonal search directions. *Advances in neural information processing systems*, 2025.

Biswajit Paria, Kirthevasan Kandasamy, and Barnabás Póczos. A flexible framework for multi-objective Bayesian optimization using random scalarizations. In Ryan P. Adams and Vibhav Gogate (eds.), *Proceedings of The 35th Uncertainty in Artificial Intelligence Conference*, volume 115 of *Proceedings of Machine Learning Research*, pp. 766–776. PMLR, 2020.

Ji Won Park, Natasa Tagasovska, Michael Maser, Stephen Ra, and Kyunghyun Cho. BOtied: Multi-objective Bayesian optimization with tied multivariate ranks. In *Proceedings of the 41st International Conference on Machine Learning*, volume 235 of *Proceedings of Machine Learning Research*, pp. 39813–39833. PMLR, 2024.

Victor Picheny, Mickael Binois, and Abderrahmane Habbal. A Bayesian optimization approach to find nash equilibria. *Journal of Global Optimization*, 73:171–192, 2019.

Nogueira Pires, RP Cardoso Coelho, and FM Andrade Pires. Composite Bayesian optimisation for multi-objective problems with smooth Tchebycheff scalarisation. *Structural and Multidisciplinary Optimization*, 69(2), 2026.

Wolfgang Ponweiser, Tobias Wagner, Dirk Biermann, and Markus Vincze. Multiobjective optimization on a limited budget of evaluations using model-assisted $\mathcal{S}$-metric selection. In *Parallel Problem Solving from Nature – PPSN X*, pp. 784–794. Springer, 2008.

Jixiang Qing, Ivo Couckuyt, and Tom Dhaene. A robust multi-objective Bayesian optimization framework considering input uncertainty. *Journal of Global Optimization*, 86(3):693–711, 2023a.

Jixiang Qing, Henry B. Moss, Tom Dhaene, and Ivo Couckuyt. {PF}^{2}ES: Parallel feasible pareto frontier entropy search for multi-objective Bayesian optimization. In *Proceedings of The 26th International Conference on Artificial Intelligence and Statistics*, volume 206 of *Proceedings of Machine Learning Research*, pp. 2565–2588. PMLR, 2023b.

Bahador Rashidi, Kerrick Johnstonbaugh, and Chao Gao. Cylindrical thompson sampling for high-dimensional Bayesian optimization. In *International Conference on Artificial Intelligence and Statistics*, pp. 3502–3510. PMLR, 2024.

Ashwin Renganathan and Kade Carlson. qPOTS: Efficient batch multiobjective Bayesian optimization via pareto optimal Thompson sampling. In *Proceedings of The 28th International Conference on Artificial Intelligence and Statistics*, volume 258 of *Proceedings of Machine Learning Research*, pp. 4051–4059. PMLR, 03–05 May 2025.

Ryan Roussel, Adi Hanuka, and Auralee Edelen. Multiobjective Bayesian optimization for online accelerator tuning. *Physical Review Accelerators and Beams*, 24(6):062801, 2021.

Amar Shah and Zoubin Ghahramani. Pareto frontier learning with expensive correlated objectives. In *International conference on machine learning*, pp. 1919–1927. PMLR, 2016.

Ke Shang, Hisao Ishibuchi, Linjun He, and Lie Meng Pang. A survey on the hypervolume indicator in evolutionary multiobjective optimization. *IEEE Transactions on Evolutionary Computation*, 25 (1):1–20, 2020.

Ilya M Sobol. The distribution of points in a cube and the approximate evaluation of integrals. *USSR Computational Mathematics and Mathematical Physics*, 7(4):86–112, 1967.

Shinya Suzuki, Shion Takeno, Tomoyuki Tamura, Kazuki Shitara, and Masayuki Karasuyama. Multi-objective Bayesian optimization using Pareto-frontier entropy. In *Proceedings of the 37th International Conference on Machine Learning*, volume 119 of *Proceedings of Machine Learning Research*, pp. 9279–9288. PMLR, 2020.

Ryoji Tanabe and Hisao Ishibuchi. An easy-to-use real-world multi-objective optimization problem suite. *Applied Soft Computing*, 89:106078, 2020.

William R Thompson. On the likelihood that one unknown probability exceeds another in view of the evidence of two samples. *Biometrika*, 25(3/4):285–294, 1933.

Yunsheng Tian, Pavle Vanja Konakovic, Beichen Li, Ane Zuniga, Michael Foshey, Timothy Erps, Wojciech Matusik, and Mina Konakovic Lukovic. Autodex: Automated optimal design of experiments platform with data-and time-efficient multi-objective optimization. In *NeurIPS 2023 Workshop on Adaptive Experimental Design and Active Learning in the Real World*, 2023.

Ben Tu, Axel Gandy, Nikolas Kantas, and Behrang Shafei. Joint entropy search for multi-objective Bayesian optimization. In *Advances in Neural Information Processing Systems*, volume 35, pp. 9922–9938. Curran Associates, Inc., 2022.

Hao Wang, Kaifeng Yang, and Michael Affenzeller. Probability distribution of hypervolume improvement in bi-objective Bayesian optimization. *arXiv preprint arXiv:2205.05505*, 2022.

Hongyan Wang, Hua Xu, and Zeqiu Zhang. High-dimensional multi-objective Bayesian optimization with block coordinate updates: Case studies in intelligent transportation system. *IEEE Transactions on Intelligent Transportation Systems*, 25(1):884–895, 2023.

Yanhong Wang, Yungai Tang, Cong-Feng Wu, and Guangyao Feng. Multiobjective Bayesian optimization for the shape design of rf cavity in particle accelerators. *Physical Review Accelerators and Beams*, 29(3):034601, 2026.

Frank Wilcoxon. Individual comparisons by ranking methods. In *Breakthroughs in Statistics: Methodology and Distribution*, pp. 196–202. Springer, 1992.

Christopher KI Williams and Carl Edward Rasmussen. *Gaussian processes for machine learning*. MIT press Cambridge, MA, 2006.

James Wilson, Frank Hutter, and Marc Deisenroth. Maximizing acquisition functions for Bayesian optimization. In *Advances in Neural Information Processing Systems*, volume 31, pp. 9884–9895. Curran Associates, Inc., 2018.

Yaohong Yang, Sammie Katt, and Samuel Kaski. Multi-objective Bayesian optimization via adaptive ε-constraints decomposition. *arXiv preprint arXiv:2604.15959*, 2026.

Qingfu Zhang, Wudong Liu, Edward Tsang, and Botond Virginas. Expensive multiobjective optimization by MOEA/D with Gaussian process model. *IEEE Transactions on Evolutionary Computation*, 14(3):456–474, 2010.

Qiuyi (Richard) Zhang. Optimal scalarizations for sublinear hypervolume regret. In *Proceedings of the 38th International Conference on Neural Information Processing Systems*, NIPS '24, Red Hook, NY, USA, 2024. Curran Associates Inc. ISBN 9798331314385.

Richard Zhang and Daniel Golovin. Random hypervolume scalarizations for provable multi-objective black box optimization. In *Proceedings of the 37th International Conference on Machine Learning*, volume 119 of *Proceedings of Machine Learning Research*, pp. 11096–11105. PMLR, 2020.

Xinyu Zhang, Conor Hassan, Julien Martinelli, Daolang Huang, and Samuel Kaski. In-context multi-objective optimization. *arXiv preprint arXiv:2512.11114*, 2025.

Yiheng Zhu, Jialu Wu, Chaowen Hu, Jiahuan Yan, Tingjun Hou, Jian Wu, et al. Sample-efficient multi-objective molecular optimization with gflownets. *Advances in Neural Information Processing Systems*, 36:79667–79684, 2023.

Eckart Zitzler and Lothar Thiele. Multiobjective evolutionary algorithms: A comparative case study and the strength Pareto approach. *IEEE Transactions on Evolutionary Computation*, 3(4):257–271, 1999.

Eckart Zitzler, Dimo Brockhoff, and Lothar Thiele. The hypervolume indicator revisited: On the design of pareto-compliant indicators via weighted integration. In *International conference on evolutionary multi-criterion optimization*, pp. 862–876. Springer, 2007.

Marcela Zuluaga, Guillaume Sergent, Andreas Krause, and Markus Püschel. Active learning for multi-objective optimization. In *International conference on Machine Learning*, pp. 462–470. PMLR, 2013.

Appendix to:

Can Multi-Objective Bayesian Optimisation Work Without Posterior Uncertainty?

A Multi-Objective Bayesian Optimisation (MOBO)

MOBO consists of two main steps, i.e., training m Gaussian process models based on the observed solutions and optimising an acquisition function $\alpha : \mathcal{X} \to \mathbb{R}$ to select a solution for evaluation. In this work, we model each objective with an independent Gaussian process $f_i \sim \mathcal{GP}(m_i(\boldsymbol{x}), k_i(\boldsymbol{x}, \boldsymbol{x}'))$, where $m_i : \mathcal{X} \to \mathbb{R}$ is the ith mean function, and $k_i(\cdot, \cdot) : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ is the ith covariance function. We use the notation $K(\mathrm{A}, \mathrm{B})$ to represent the covariance matrix at all pairs of solutions in set A and in set B. Given n observed points $\mathcal{D}^{(n)} = \{(\boldsymbol{x}^{(t)}, \boldsymbol{y}^{(t)})\}_{t=1}^{n}$, where $\boldsymbol{y}^{(t)} = \boldsymbol{f}(\boldsymbol{x}^{(t)}) + \boldsymbol{\zeta}^{(t)}$ and the noise $\boldsymbol{\zeta}^{(t)} \sim \mathcal{N}(\mathbf{0}, \mathrm{diag}(\boldsymbol{\sigma}_\zeta^2))$, the posterior distribution of the ith objective at a new location $\boldsymbol{x}$ is Gaussian:

$$f_i(\boldsymbol{x}) \mid \mathcal{D}^{(n)} \sim \mathcal{N}(\mu_i(\boldsymbol{x}), \sigma_i^2(\boldsymbol{x})) \tag{8}$$

$$\mu_i(\boldsymbol{x}) = K(\boldsymbol{x}, X^{(n)})(K(X^{(n)}, X^{(n)}) + \sigma_{\zeta_i}^2 \mathbf{I})^{-1} Y_i^{(n)} \tag{9}$$

$$\sigma_i^2(\boldsymbol{x}) = K(\boldsymbol{x}, \boldsymbol{x}) - K(\boldsymbol{x}, X^{(n)})(K(X^{(n)}, X^{(n)}) + \sigma_{\zeta_i}^2 \mathbf{I})^{-1} K(X^{(n)}, \boldsymbol{x}) \tag{10}$$

where $\mu_i(\boldsymbol{x})$ and $\sigma_i^2(\boldsymbol{x})$ are the mean and variance at $\boldsymbol{x}$, respectively; $X^{(n)} = (\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(n)}) \in \mathbb{R}^{n \times d}$ and $Y_i^{(n)} = (y_i^{(1)}, \ldots, y_i^{(n)}) \in \mathbb{R}^n$ are the matrix of evaluated solutions and the corresponding vector of y values, respectively; $\sigma_{\zeta_i}^2$ is the variance of the observation noise $\zeta_i \sim \mathcal{N}(0, \sigma_{\zeta_i}^2)$, and corresponds to the i-th diagonal entry of the noise covariance matrix $\mathrm{diag}(\boldsymbol{\sigma}_\zeta^2)$;

Based on the posterior means and variances, an acquisition function is optimised to determine the next solution to evaluate. The lower confidence bound (LCB) (Lai & Robbins, 1985) is a commonly used acquisition function (De Ath et al., 2021; Lin et al., 2022; Jiang & Li, 2025a;b; Cheng et al., 2026). Considering a single-objective case, LCB is defined as $\alpha_{\mathrm{LCB}}(\boldsymbol{x}) = \mu(\boldsymbol{x}) - \beta\sigma(\boldsymbol{x})$, where $\beta \geq 0$ controls the trade-off between exploitation and exploration. A larger β places greater emphasis on posterior uncertainty, while setting $\beta = 0$ reduces LCB to the posterior mean. LCB can be readily extended to the multi-objective setting by applying it to each objective independently.

B Experiment Settings

B.1 Implementation details

For all the methods, we sample $2(d+1)$ initial points from a scrambled Sobol sequence, following the practice in (Daulton et al., 2020; 2021; Konakovic Lukovic et al., 2020). The detailed implementation are as follows.

PMOBO. For the Pareto-based instantiation of PMOBO, we directly optimise the posterior mean vectors. Our implementation is based on the open-source Python framework BoTorch (Balandat et al., 2020), which builds on GPyTorch (Gardner et al., 2018) for Gaussian process modelling. The well-known NSGA-II Deb et al. (2002) is used to optimise the posterior mean vectors. We use a population size of 100 and run NSGA-II for 500 generations. Simulated binary crossover (SBX) is used with a crossover probability of 1.0 and $\eta = 20$, while polynomial mutation (PM) is applied with $\eta = 20$ and mutation probability $1/d$, where d is the number of decision variables. To warm start NSGA-II, five individuals in the initial population are selected from the currently evaluated non-dominated solutions, while the remaining individuals are randomly sampled from the search space. We first randomly select one non-dominated solution and then iteratively select the

other four using the max-min distance criterion to encourage diversity. After NSGA-II obtains the predicted Pareto set, we select the solution that provides the largest hypervolume improvement over the currently evaluated solutions.

For the scalarisation-based instantiation of PMOBO, we follow the original PSL settings (Lin et al., 2022), including the model architecture, optimiser, and training procedure. The only difference is that PMOBO$_{PSL}$ uses only the posterior mean without posterior uncertainty. Detailed parameter settings can be found in the original PSL paper.

The data and code are available at an anonymised repository: `https://anonymous.4open.science/r/PMOBO-CA57`.

ParEGO (Knowles, 2006) and NParEGO (Daulton et al., 2021). We implement ParEGO and NParEGO based on the implementation provided by BoTorch.[1] Both methods use the augmented Tchebycheff scalarisation function and their log version, following the recommendation in (Ament et al., 2023).

TS-TCH (Paria et al., 2020). We implement TS-TCH based on the implementation provided by BoTorch.[2] The augmented Tchebycheff scalarisation function is used. After converting to single-objective optimisation problem, Thompson sampling is used as the acquisition function. We draw a sample from the joint posterior over a discrete set of $1000d$ points sampled from a scrambled Sobol sequence, suggested by (Daulton et al., 2020).

JES (Tu et al., 2022). We implement JES using the open-source implementation provided by the authors[3].

DGEMO (Konakovic Lukovic et al., 2020). We implement DGEMO using the open-source implementation provided by the authors[4] and set the batch size to one.

EHVI (Daulton et al., 2020) and NEHVI (Daulton et al., 2021). We implement EHVI and NEHVI based on the implementation provided by BoTorch.[5]. For both methods, we use their log versions, following the recommendation of (Ament et al., 2023).

PSL (Lin et al., 2022). We implement PSL using the open-source implementation provided by the authors[6] and set the batch size to one.

MOBO-OSD (Ngo et al., 2025). We implement MOBO-OSD using the open-source implementation provided by the authors[7] and set the batch size to one.

qPOTS (Renganathan & Carlson, 2025). We implement qPOTS using the open-source implementation provided by the authors[8] and set the batch size to one.

For all the problems, we normalise the input variables and standardise the objective values before training Gaussian processes. We assume an independent surrogate model for each objective, using a constant mean function and a Matérn 5/2 ARD kernel.

[1] EI, NEI, and the BoTorch multi-objective tutorial.
[2] TS and BoTorch multi-objective tutorial
[3] JES implementation
[4] DGEMO implementation
[5] EHVI, NEHVI, and BoTorch multi-objective tutorial
[6] PSL implementation
[7] MOBO-OSD implementation
[8] qPOTS implementation

B.2 Problem Setting

For BBOB problems, we set the dimensionality to 10 (Brockhoff et al., 2022). Regarding DTLZ problems, each problem is considered with 2 and 3 objectives, following the practice in (Deb & Jain, 2013; Jain & Deb, 2013; Li et al., 2014). We set the dimensionality to $d = m + 4$ for DTLZ1, $d = m + 9$ for DTLZ2–DTLZ6, and $d = m + 19$ for DTLZ7, according to the original paper (Deb et al., 2005). For the ten real-world problems, we follow the original problem definitions and settings in the corresponding literature (see Tanabe & Ishibuchi (2020)).

C Additional Experimental Results

C.1 Comparison with Well-Established Methods

C.1.1 Noisy Cases.

Table 2: The HV results (mean and standard deviation) of the proposed PMOBO and the peer methods on the 79 benchmark and real-world problems. The method with the best mean HV is highlighted in bold. The symbols "$-$", "$\sim$" and "$+$" indicate that our method PMOBO is statistically worse than, equivalent to, and better than the peer method, respectively.

Method	BBOB1	BBOB2	BBOB3	BBOB4	BBOB5	BBOB6	BBOB7	BBOB8
Sobol	4.02e+04(2.6e+03)$^+$	8.02e+08(1.4e+08)$^+$	4.48e+08(2.0e+07)$^+$	1.37e+08(9.3e+06)$^+$	5.93e+05(3.1e+04)$^+$	9.11e+04(3.6e+03)$^+$	2.52e+06(1.1e+05)$^+$	4.12e+03(5.3e+02)$^+$
ParEGO	5.72e+04(1.8e+03)$^+$	1.51e+09(2.5e+07)$^+$	5.56e+08(1.4e+07)$^+$	1.80e+08(4.7e+06)$^+$	8.34e+05(3.2e+04)$^+$	1.10e+05(2.2e+03)$^+$	3.03e+06(9.2e+03)$^+$	6.42e+03(3.9e+02)$^+$
TS-TCH	5.28e+04(7.8e+02)$^+$	1.40e+09(2.4e+07)$^+$	5.47e+08(8.2e+06)$^+$	1.73e+08(4.6e+06)$^+$	7.68e+05(2.1e+04)$^+$	1.04e+05(1.9e+03)$^+$	2.95e+06(3.2e+04)$^+$	6.11e+03(3.1e+02)$^+$
JES	5.69e+04(2.0e+03)$^+$	1.40e+09(6.0e+07)$^+$	5.53e+08(1.6e+07)$^+$	1.72e+08(6.4e+06)$^+$	8.35e+05(5.9e+04)$^+$	1.07e+05(3.6e+03)$^+$	3.01e+06(2.3e+04)$^+$	6.70e+03(2.9e+02)$^\sim$
EHVI	5.58e+04(2.4e+03)$^+$	1.51e+09(2.5e+07)$^+$	5.65e+08(1.2e+07)$^+$	1.81e+08(3.8e+06)$^+$	8.25e+05(3.4e+04)$^+$	1.10e+05(2.2e+03)$^+$	3.02e+06(1.8e+04)$^+$	6.79e+03(3.1e+02)$^\sim$
qPOTS	5.32e+04(9.2e+02)$^+$	1.40e+09(2.3e+07)$^+$	5.47e+08(7.3e+06)$^+$	1.74e+08(2.9e+06)$^+$	7.68e+05(2.2e+04)$^+$	1.04e+05(1.6e+03)$^+$	2.94e+06(2.7e+04)$^+$	6.03e+03(3.6e+02)$^+$
PMOBO	**5.93e+04(1.8e+02)**	**1.54e+09(2.5e+06)**	**5.80e+08(1.6e+06)**	**1.88e+08(8.1e+05)**	**9.07e+05(5.9e+03)**	**1.13e+05(1.2e+03)**	**3.04e+06(9.7e+03)**	**6.80e+03(3.2e+02)**

Method	BBOB9	BBOB10	BBOB11	BBOB12	BBOB13	BBOB14	BBOB15	BBOB16
Sobol	6.45e+06(1.0e+06)$^+$	3.71e+03(9.4e+02)$^+$	2.85e+14(5.1e+12)$^+$	9.50e+12(1.2e+12)$^+$	6.40e+12(2.4e+11)$^+$	1.01e+09(7.8e+08)$^+$	1.40e+07(1.2e+07)$^+$	3.57e+09(3.6e+08)$^+$
ParEGO	1.35e+07(4.3e+05)$^+$	1.11e+04(1.6e+03)$^\sim$	2.96e+14(4.0e+12)$^+$	1.33e+13(7.4e+10)$^+$	7.24e+12(2.0e+10)$^\sim$	7.24e+09(3.4e+08)$^\sim$	6.94e+07(7.5e+06)$^\sim$	5.14e+09(9.1e+07)$^\sim$
TS-TCH	1.21e+07(2.3e+05)$^+$	6.93e+03(1.9e+03)$^+$	2.96e+14(3.5e+12)$^+$	1.30e+13(2.0e+11)$^+$	7.18e+12(3.1e+10)$^+$	5.11e+09(4.0e+08)$^+$	6.47e+07(5.4e+06)$^+$	4.96e+09(1.4e+08)$^+$
JES	1.27e+07(7.1e+05)$^+$	8.71e+03(2.4e+03)$^+$	2.94e+14(7.7e+12)$^+$	1.28e+13(4.6e+11)$^+$	7.20e+12(6.1e+10)$^+$	4.65e+09(8.6e+08)$^+$	6.26e+07(8.2e+06)$^+$	5.09e+09(1.4e+08)$^\sim$
EHVI	1.35e+07(4.5e+05)$^+$	1.13e+04(1.5e+03)$^\sim$	2.98e+14(4.6e+12)$^+$	1.33e+13(3.7e+10)$^+$	**7.25e+12(3.3e+10)**$^\sim$	7.17e+09(2.1e+08)$^+$	6.88e+07(4.7e+06)$^+$	**5.31e+09(1.0e+08)**$^-$
qPOTS	1.20e+07(2.5e+05)$^+$	6.13e+03(1.1e+03)$^+$	2.95e+14(4.1e+12)$^+$	1.31e+13(1.1e+11)$^+$	7.19e+12(3.7e+10)$^+$	5.12e+09(3.8e+08)$^+$	6.37e+07(5.9e+06)$^+$	5.02e+09(1.2e+08)$^+$
PMOBO	**1.38e+07(2.2e+05)**	**1.15e+04(2.7e+03)**	**3.04e+14(8.9e+11)**	**1.34e+13(3.5e+10)**	7.24e+12(2.5e+10)	**7.32e+09(3.0e+08)**	**7.30e+07(6.2e+06)**	5.13e+09(1.3e+08)

Method	BBOB17	BBOB18	BBOB19	BBOB20	BBOB21	BBOB22	BBOB23	BBOB24
Sobol	4.00e+07(2.4e+07)$^+$	5.96e+11(4.2e+10)$^+$	6.49e+08(2.0e+08)$^+$	1.20e+11(2.6e+10)$^+$	1.42e+11(1.4e+10)$^+$	1.08e+09(9.4e+07)$^+$	5.36e+07(5.0e+06)$^+$	1.15e+09(3.4e+07)$^+$
ParEGO	1.00e+08(1.4e+07)$^+$	7.21e+11(4.6e+09)$^+$	1.49e+09(1.8e+08)$^\sim$	2.10e+11(3.3e+09)$^\sim$	1.90e+11(2.6e+09)$^+$	1.68e+09(5.2e+07)$^\sim$	6.72e+07(2.9e+06)$^+$	1.31e+09(2.3e+07)$^+$
TS-TCH	9.27e+07(1.6e+07)$^+$	7.18e+11(3.6e+09)$^+$	9.43e+08(2.3e+08)$^+$	2.08e+11(2.1e+09)$^+$	1.85e+11(2.6e+09)$^+$	1.50e+09(3.5e+07)$^+$	6.52e+07(3.7e+06)$^+$	1.30e+09(2.0e+07)$^+$
JES	8.90e+07(2.0e+07)$^+$	7.13e+11(1.2e+10)$^+$	1.18e+09(2.0e+08)$^+$	2.11e+11(1.4e+09)$^\sim$	1.89e+11(2.6e+09)$^+$	1.64e+09(9.3e+07)$^+$	6.59e+07(3.7e+06)$^+$	1.31e+09(2.2e+07)$^+$
EHVI	**1.12e+08(1.5e+07)**$^\sim$	7.24e+11(2.7e+09)$^\sim$	1.48e+09(2.1e+08)$^\sim$	**2.11e+11(6.5e+08)**$^-$	1.92e+11(2.3e+09)$^\sim$	1.66e+09(8.0e+07)$^\sim$	6.51e+07(3.5e+06)$^+$	**1.33e+09(1.8e+07)**$^\sim$
qPOTS	9.53e+07(1.6e+07)$^+$	7.20e+11(3.4e+09)$^+$	1.00e+09(2.8e+08)$^+$	2.06e+11(3.4e+09)$^+$	1.86e+11(2.9e+09)$^+$	1.49e+09(3.7e+07)$^+$	6.53e+07(3.9e+06)$^+$	1.30e+09(2.3e+07)$^+$
PMOBO	1.09e+08(1.8e+07)	**7.24e+11(1.7e+09)**	**1.56e+09(1.8e+08)**	2.10e+11(2.9e+09)	**1.92e+11(1.7e+09)**	**1.70e+09(3.5e+07)**	**7.03e+07(2.3e+06)**	1.33e+09(1.4e+07)

Method	BBOB25	BBOB26	BBOB27	BBOB28	BBOB29	BBOB30	BBOB31	BBOB32
Sobol	1.48e+07(2.1e+06)$^+$	0.00e+00(0.0e+00)$^+$	8.56e+06(5.8e+06)$^+$	9.30e+08(8.5e+08)$^+$	6.04e+08(7.0e+07)$^+$	3.02e+06(1.4e+06)$^+$	2.22e+08(1.6e+07)$^+$	0.00e+00(0.0e+00)$^+$
ParEGO	1.98e+07(2.0e+06)$^+$	4.54e+07(4.2e+07)$^+$	**3.59e+07(5.1e+06)**$^\sim$	**3.83e+09(1.1e+08)**$^\sim$	1.00e+09(2.5e+07)$^+$	6.79e+06(4.2e+05)$^+$	2.92e+08(1.0e+07)$^\sim$	7.13e+03(5.7e+03)$^\sim$
TS-TCH	1.92e+07(2.2e+06)$^+$	8.90e+06(1.7e+07)$^+$	1.39e+07(6.7e+06)$^+$	3.63e+09(1.5e+08)$^+$	8.73e+08(3.4e+07)$^+$	6.43e+06(4.2e+05)$^+$	2.86e+08(9.8e+06)$^+$	1.59e+03(2.5e+03)$^+$
JES	2.12e+07(1.9e+06)$^\sim$	4.20e+07(4.1e+07)$^+$	2.17e+07(9.2e+06)$^+$	3.78e+09(1.2e+08)$^\sim$	8.90e+08(3.5e+07)$^+$	6.89e+06(3.8e+05)$^+$	2.91e+08(1.0e+07)$^\sim$	8.42e+03(6.4e+03)$^\sim$
EHVI	2.09e+07(1.8e+06)$^+$	6.59e+07(3.8e+07)$^\sim$	3.15e+07(7.0e+06)$^+$	3.82e+09(1.0e+08)$^\sim$	9.88e+08(2.1e+07)$^+$	6.93e+06(3.7e+05)$^+$	**3.01e+08(6.8e+06)**$^\sim$	**1.10e+04(6.9e+03)**$^\sim$
qPOTS	1.87e+07(1.8e+06)$^+$	7.63e+06(1.6e+07)$^+$	1.46e+07(8.2e+06)$^+$	3.67e+09(1.4e+08)$^+$	8.77e+08(2.9e+07)$^+$	6.47e+06(3.7e+05)$^+$	2.85e+08(9.1e+06)$^+$	2.56e+03(3.5e+03)$^\sim$
PMOBO	**2.20e+07(1.6e+06)**	**7.73e+07(1.9e+07)**	3.54e+07(7.7e+06)	3.79e+09(1.2e+08)	**1.03e+09(1.8e+07)**	**7.45e+06(4.3e+05)**	2.97e+08(7.6e+06)	4.75e+03(5.4e+03)

Method	BBOB33	BBOB34	BBOB35	BBOB36	BBOB37	BBOB38	BBOB39	BBOB40
Sobol	3.98e+09(2.0e+09)$^+$	1.00e+07(4.3e+06)$^+$	3.13e+06(2.7e+05)$^+$	1.91e+05(1.6e+04)$^+$	8.00e+06(4.7e+05)$^+$	3.61e+04(3.8e+03)$^+$	1.30e+08(2.1e+07)$^+$	6.42e+04(1.5e+04)$^+$
ParEGO	9.36e+09(3.5e+08)$^+$	3.38e+07(5.2e+06)$^\sim$	5.50e+06(4.6e+05)$^+$	3.11e+05(1.4e+04)$^+$	1.07e+07(3.6e+05)$^+$	5.42e+04(5.4e+03)$^+$	2.92e+08(1.0e+07)$^\sim$	**1.85e+05(2.3e+04)**$^\sim$
TS-TCH	9.33e+09(2.7e+08)$^+$	1.62e+07(5.9e+06)$^+$	4.79e+06(1.8e+05)$^+$	2.77e+05(1.3e+04)$^+$	9.93e+06(3.9e+05)$^+$	4.94e+04(3.8e+03)$^+$	2.29e+08(7.3e+06)$^+$	1.11e+05(2.6e+04)$^+$
JES	9.25e+09(4.5e+08)$^+$	2.37e+07(6.9e+06)$^+$	5.38e+06(6.3e+05)$^+$	3.06e+05(1.5e+04)$^+$	1.06e+07(3.3e+05)$^+$	5.49e+04(5.1e+03)$^+$	2.55e+08(9.7e+06)$^+$	1.46e+05(2.8e+04)$^+$
EHVI	9.43e+09(2.4e+08)$^+$	3.19e+07(5.5e+06)$^\sim$	5.41e+06(3.5e+05)$^+$	3.12e+05(1.6e+04)$^+$	1.06e+07(5.1e+05)$^+$	5.53e+04(4.8e+03)$^+$	2.68e+08(3.4e+06)$^\sim$	1.85e+05(2.2e+04)$^\sim$
qPOTS	9.27e+09(2.7e+08)$^+$	1.51e+07(4.7e+06)$^+$	4.82e+06(1.8e+05)$^+$	2.78e+05(1.1e+04)$^+$	9.86e+06(2.3e+05)$^+$	4.73e+04(3.8e+03)$^+$	2.28e+08(9.0e+06)$^+$	1.04e+05(3.0e+04)$^+$
PMOBO	**9.74e+09(2.9e+08)**	**3.39e+07(5.6e+06)**	**5.96e+06(1.1e+05)**	**3.23e+05(1.1e+04)**	**1.11e+07(3.4e+05)**	**5.76e+04(3.8e+03)**	2.66e+08(7.0e+06)	1.74e+05(3.3e+04)

Method	BBOB41	BBOB42	BBOB43	BBOB44	BBOB45	BBOB46	BBOB47	BBOB48
Sobol	2.06e+04(9.3e+02)$^+$	1.47e+05(6.5e+03)$^+$	1.51e+02(1.2e+02)$^+$	2.58e+05(2.4e+05)$^+$	4.87e+02(5.4e+02)$^+$	8.31e+05(3.8e+04)$^+$	1.40e+03(7.7e+02)$^+$	1.22e+08(8.1e+06)$^+$
ParEGO	2.31e+04(8.0e+02)$^+$	1.75e+05(4.3e+03)$^+$	7.35e+02(1.8e+02)$^-$	1.28e+06(1.0e+05)$^-$	3.07e+03(6.2e+02)$^\sim$	1.08e+06(3.9e+04)$^\sim$	**4.71e+03(6.6e+02)**$^\sim$	1.48e+08(3.9e+06)$^+$
TS-TCH	2.23e+04(8.4e+02)$^+$	1.67e+05(5.1e+03)$^+$	4.72e+02(1.6e+02)$^+$	1.10e+06(9.8e+04)$^+$	1.52e+03(7.0e+02)$^+$	1.02e+06(3.5e+04)$^+$	3.27e+03(7.5e+02)$^+$	1.47e+08(3.5e+06)$^+$
JES	2.32e+04(9.2e+02)$^+$	1.73e+05(4.7e+03)$^+$	7.25e+02(1.9e+02)$^-$	**1.31e+06(7.8e+04)**$^-$	2.84e+03(7.0e+02)$^\sim$	1.07e+06(3.9e+04)$^+$	4.50e+03(8.8e+02)$^\sim$	1.51e+08(4.2e+06)$^\sim$
EHVI	2.31e+04(1.0e+03)$^+$	1.75e+05(5.2e+03)$^+$	**7.83e+02(1.6e+02)**$^-$	1.25e+06(1.0e+05)$^-$	**3.07e+03(6.2e+02)**$^\sim$	1.08e+06(3.5e+04)$^\sim$	4.55e+03(5.8e+02)$^\sim$	1.51e+08(5.4e+06)$^\sim$
qPOTS	2.24e+04(9.6e+02)$^+$	1.68e+05(4.3e+03)$^+$	4.93e+02(1.7e+02)$^\sim$	1.10e+06(8.9e+04)$^+$	1.23e+03(6.7e+02)$^+$	1.01e+06(3.1e+04)$^+$	3.49e+03(6.9e+02)$^+$	1.47e+08(3.1e+06)$^+$
PMOBO	**2.40e+04(7.4e+02)**	**1.80e+05(3.8e+03)**	5.92e+02(1.8e+02)	1.18e+06(1.3e+05)	2.68e+03(1.1e+03)	**1.10e+06(3.3e+04)**	4.47e+03(9.1e+02)	**1.53e+08(5.0e+06)**

Method	BBOB49	BBOB50	BBOB51	BBOB52	BBOB53	BBOB54	BBOB55	DTLZ1 ($m=2$)
Sobol	2.68e+04(7.4e+03)$^+$	8.24e+02(7.7e+01)$^+$	8.27e+05(1.7e+05)$^+$	1.22e+02(7.5e+01)$^+$	1.14e+10(6.6e+08)$^+$	1.07e+06(5.6e+05)$^+$	8.08e+02(2.5e+02)$^+$	5.23e-01(2.3e+00)$^+$
ParEGO	7.04e+04(1.0e+04)$^\sim$	1.09e+03(7.0e+01)$^\sim$	1.26e+06(1.6e+05)$^+$	5.40e+02(1.9e+02)$^\sim$	1.34e+10(6.7e+07)$^+$	4.54e+06(5.0e+05)$^\sim$	3.70e+03(1.4e+03)$^\sim$	1.52e+00(8.3e+00)$^+$
TS-TCH	3.79e+04(1.2e+04)$^+$	9.15e+02(8.0e+01)$^+$	1.18e+06(1.6e+05)$^+$	2.14e+02(1.3e+02)$^+$	1.34e+10(7.3e+07)$^+$	2.78e+06(7.9e+05)$^+$	1.15e+03(3.8e+02)$^+$	0.00e+00(0.0e+00)$^+$
JES	5.78e+04(1.5e+04)$^\sim$	1.05e+03(9.2e+01)$^+$	1.31e+06(1.7e+05)$^+$	5.27e+02(2.0e+02)$^\sim$	1.34e+10(6.1e+07)$^+$	3.84e+06(7.5e+05)$^+$	3.84e+03(1.3e+03)$^\sim$	0.00e+00(0.0e+00)$^+$
EHVI	**7.17e+04(1.2e+04)**$^\sim$	1.07e+03(7.7e+01)$^+$	1.33e+06(1.6e+05)$^+$	**6.25e+02(1.4e+02)**$^\sim$	1.35e+10(6.9e+07)$^+$	**4.78e+06(3.7e+05)**$^\sim$	**4.27e+03(1.5e+03)**$^\sim$	0.00e+00(0.0e+00)$^+$
qPOTS	3.74e+04(1.0e+04)$^+$	9.31e+02(7.5e+01)$^+$	1.19e+06(1.6e+05)$^+$	2.23e+02(1.2e+02)$^+$	1.34e+10(9.3e+07)$^+$	2.65e+06(8.6e+05)$^+$	1.38e+03(6.2e+02)$^+$	0.00e+00(0.0e+00)$^+$
PMOBO	6.44e+04(1.5e+04)	**1.11e+03(9.4e+01)**	**1.52e+06(1.1e+05)**	4.83e+02(2.5e+02)	**1.35e+10(2.2e+07)**	4.44e+06(9.4e+05)	4.15e+03(1.8e+03)	**4.45e+01(7.6e+01)**

Method	DTLZ2 ($m=2$)	DTLZ3 ($m=2$)	DTLZ4 ($m=2$)	DTLZ5 ($m=2$)	DTLZ6 ($m=2$)	DTLZ7 ($m=2$)	DTLZ1 ($m=3$)	DTLZ2 ($m=3$)
Sobol	1.87e-02(1.8e-02)$^+$	0.00e+00(0.0e+00)$^+$	6.47e-03(8.8e-03)$^+$	1.70e-02(2.1e-02)$^+$	0.00e+00(0.0e+00)$^+$	0.00e+00(0.0e+00)$^+$	1.35e+03(3.7e+03)$^+$	4.01e-02(2.7e-02)$^-$
ParEGO	**2.77e-01(8.6e-02)**$^-$	4.52e+02(8.8e+02)$^\sim$	**2.68e-01(9.5e-02)**$^-$	2.70e-01(1.0e-01)$^-$	**2.72e-01(9.3e-02)**$^\sim$	3.69e-01(2.5e-01)$^+$	3.25e+03(7.1e+03)$^\sim$	**1.57e-01(1.3e-01)**$^-$
TS-TCH	5.21e-02(4.9e-02)$^+$	0.00e+00(0.0e+00)$^+$	3.14e-02(3.6e-02)$^+$	3.00e-02(3.4e-02)$^+$	0.00e+00(0.0e+00)$^+$	0.00e+00(0.0e+00)$^+$	5.93e+02(2.5e+03)$^+$	1.35e-02(1.7e-02)$^\sim$
JES	2.26e-01(7.9e-02)$^\sim$	6.48e+02(1.1e+03)$^\sim$	2.48e-01(7.0e-02)$^\sim$	**2.73e-01(4.2e-02)**$^-$	2.59e-01(8.1e-02)$^\sim$	4.21e-01(2.2e-01)$^+$	8.13e+03(5.3e+03)$^-$	1.09e-01(9.5e-02)$^-$
EHVI	1.58e-01(1.1e-01)$^\sim$	9.83e+02(1.8e+03)$^\sim$	1.82e-01(1.0e-01)$^\sim$	1.40e-01(1.2e-01)$^+$	2.38e-01(4.6e-02)$^\sim$	1.78e-01(2.9e-02)$^+$	**9.61e+03(5.0e+03)**$^-$	5.22e-02(6.9e-02)$^\sim$
qPOTS	3.52e-02(3.6e-02)$^+$	0.00e+00(0.0e+00)$^+$	2.75e-02(3.0e-02)$^+$	4.71e-02(4.1e-02)$^+$	0.00e+00(0.0e+00)$^+$	0.00e+00(0.0e+00)$^+$	5.20e+02(2.6e+03)$^+$	2.46e-02(2.8e-02)$^\sim$
PMOBO	2.10e-01(1.0e-01)	**1.33e+03(1.9e+03)**	1.99e-01(9.4e-02)	2.07e-01(8.4e-02)	2.53e-01(1.1e-01)	**6.70e-01(1.3e-01)**	4.31e+03(8.2e+03)	2.36e-02(3.7e-02)

Method	DTLZ3 ($m=3$)	DTLZ4 ($m=3$)	DTLZ5 ($m=3$)	DTLZ6 ($m=3$)	DTLZ7 ($m=3$)	RE21	RE23	RE24
Sobol	2.33e+04(9.7e+04)$^+$	1.14e-01(3.2e-02)$^\sim$	1.61e-03(2.7e-03)$^+$	5.29e+02(4.3e+00)$^+$	0.00e+00(0.0e+00)$^+$	4.47e+01(7.8e-01)$^+$	4.07e+09(2.0e+09)$^+$	1.97e+04(8.3e+02)$^+$
ParEGO	6.63e+06(5.2e+06)$^\sim$	**3.21e-01(2.5e-01)**$^-$	**7.94e-02(3.0e-02)**$^-$	1.16e+03(3.8e+00)$^\sim$	4.90e-01(2.9e-01)$^+$	5.24e+01(1.4e+00)$^+$	**1.12e+10(6.5e+07)**$^-$	2.18e+04(2.7e+01)$^-$
TS-TCH	0.00e+00(0.0e+00)$^+$	5.44e-02(3.5e-02)$^+$	2.71e-03(5.5e-03)$^+$	5.97e+02(6.3e+00)$^+$	0.00e+00(0.0e+00)$^+$	5.02e+01(1.4e+00)$^+$	1.04e+10(2.7e+08)$^+$	2.15e+04(2.1e+02)$^\sim$
JES	1.30e+07(2.0e+06)$^-$	2.53e-01(1.5e-01)$^-$	4.40e-02(3.0e-02)$^\sim$	1.16e+03(3.7e+00)$^+$	4.70e-01(2.3e-01)$^+$	5.21e+01(1.5e+00)$^+$	1.11e+10(1.1e+08)$^\sim$	2.17e+04(2.7e+01)$^-$
EHVI	**1.38e+07(2.0e+06)**$^-$	1.32e-01(1.2e-01)$^\sim$	3.29e-02(3.3e-02)$^\sim$	1.16e+03(3.1e+00)$^+$	2.24e-01(1.6e-02)$^+$	5.41e+01(4.0e-02)$^+$	1.11e+10(8.2e+07)$^\sim$	**2.18e+04(2.2e+01)**$^-$
qPOTS	0.00e+00(0.0e+00)$^+$	8.05e-02(4.4e-02)$^\sim$	2.52e-03(5.3e-03)$^+$	5.98e+02(8.0e+00)$^+$	0.00e+00(0.0e+00)$^+$	5.06e+01(1.2e+00)$^+$	1.05e+10(2.7e+08)$^+$	2.15e+04(1.6e+02)$^\sim$
PMOBO	8.53e+06(3.6e+06)	1.07e-01(7.9e-02)	4.07e-02(3.1e-02)	**1.16e+03(5.4e+00)**	**1.04e+00(1.2e-01)**	**5.41e+01(2.3e-02)**	1.11e+10(1.1e+08)	2.15e+04(1.3e+02)

Method	RE31	RE32	RE33	RE34	RE37	RE41	RE42	Summary
Sobol	1.67e+17(2.5e+15)$^+$	3.53e+14(5.0e+12)$^+$	7.80e+04(1.9e+03)$^+$	2.06e+01(1.2e+00)$^+$	1.07e+00(4.8e-02)$^+$	2.92e+02(8.2e+00)$^+$	2.78e+11(3.9e+10)$^+$	1/1/77
ParEGO	1.72e+17(3.5e+15)$^+$	**3.63e+14(6.7e+11)**$^-$	8.80e+04(3.3e+02)$^+$	3.43e+01(6.2e-01)$^+$	1.41e+00(7.4e-02)$^+$	**4.24e+02(1.3e+01)**$^\sim$	8.46e+11(7.4e+10)$^+$	11/29/39
TS-TCH	1.73e+17(5.1e+14)$^+$	3.60e+14(2.1e+12)$^\sim$	8.46e+04(1.8e+03)$^+$	2.98e+01(7.2e-01)$^+$	1.31e+00(3.9e-02)$^+$	3.65e+02(1.1e+01)$^+$	6.17e+11(4.0e+10)$^+$	0/3/76
JES	1.72e+17(1.0e+15)$^+$	3.63e+14(6.3e+11)$^-$	8.69e+04(3.1e+03)$^+$	3.45e+01(6.2e-01)$^+$	1.40e+00(8.0e-02)$^+$	4.23e+02(8.0e+00)$^\sim$	7.77e+11(9.5e+10)$^+$	10/19/50
EHVI	1.67e+17(5.5e+15)$^+$	3.62e+14(2.0e+12)$^\sim$	8.81e+04(6.7e+02)$^\sim$	3.37e+01(7.4e-01)$^+$	1.48e+00(3.7e-02)$^+$	3.85e+02(2.1e+01)$^+$	7.33e+11(1.4e+11)$^+$	8/33/38
qPOTS	1.73e+17(6.3e+14)$^+$	3.60e+14(2.4e+12)$^\sim$	8.50e+04(1.6e+03)$^+$	2.99e+01(5.7e-01)$^+$	1.31e+00(4.4e-02)$^+$	3.67e+02(1.1e+01)$^+$	6.09e+11(3.4e+10)$^+$	0/6/73
PMOBO	**1.73e+17(1.0e+15)**	3.61e+14(4.2e+12)	**8.83e+04(2.0e+02)**	**3.59e+01(4.3e-01)**	**1.51e+00(3.6e-03)**	4.20e+02(9.4e+00)	**8.98e+11(1.1e+10)**	–/–/–

C.2 ABLATION STUDY

We conduct an ablation study to investigate the effectiveness of using the currently evaluated non-dominated solutions to initialise part of the NSGA-II population. In PMOBO, five individuals in the initial population are constructed from the evaluated non-dominated solutions. Specifically, we first randomly select one non-dominated solution and then iteratively select the remaining four solutions based on the max-min distance criterion to encourage diversity. The rest of the population is randomly sampled from the search space.

To evaluate the contribution of this warm-start strategy, we compare PMOBO with a variant, PMOBO (w/o warm), where all individuals in the initial population are randomly sampled without using the evaluated non-dominated solutions. Table 3 shows the HV results (mean and standard deviation) of the two methods on the 79 benchmark and real-world problems.

Table 3: The HV results (mean and standard deviation) of the two methods on the 79 benchmark and real-world problems. The method with the best mean HV is highlighted in bold. The symbols "$-$", "$\sim$" and "$+$" indicate that PMOBO is statistically worse than, equivalent to, and better than PMOBO (w/o warm), respectively.

Method	BBOB1	BBOB2	BBOB3	BBOB4	BBOB5	BBOB6	BBOB7	BBOB8
PMOBO (w/o warm)	**5.95e+04(2.6e+01)**$^{\sim}$	**1.55e+09(3.4e+06)**$^{\sim}$	5.82e+08(1.1e+06)$^{+}$	1.89e+08(1.7e+05)$^{\sim}$	**9.29e+05(2.8e+03)**$^{-}$	1.16e+05(4.8e+02)$^{\sim}$	**3.05e+06(6.6e+03)**$^{\sim}$	7.25e+03(2.8e+02)$^{\sim}$
PMOBO	5.95e+04(4.0e+01)	1.55e+09(2.3e+06)	**5.83e+08(5.7e+05)**	**1.89e+08(1.4e+05)**	9.24e+05(3.7e+03)	**1.16e+05(3.4e+02)**	3.05e+06(6.8e+03)	**7.34e+03(2.0e+02)**
Method	**BBOB9**	**BBOB10**	**BBOB11**	**BBOB12**	**BBOB13**	**BBOB14**	**BBOB15**	**BBOB16**
PMOBO (w/o warm)	1.40e+07(8.9e+04)$^{+}$	**1.40e+04(8.2e+02)**$^{\sim}$	**3.05e+14(1.1e+12)**$^{\sim}$	1.34e+13(1.3e+10)$^{\sim}$	7.28e+12(1.1e+10)$^{+}$	7.86e+09(1.5e+08)$^{\sim}$	8.19e+07(2.5e+06)$^{+}$	5.33e+09(7.7e+07)$^{\sim}$
PMOBO	**1.41e+07(5.3e+04)**	1.38e+04(9.8e+02)	3.05e+14(1.2e+12)	**1.34e+13(7.5e+09)**	**7.28e+12(6.1e+09)**	**7.87e+09(2.6e+08)**	**8.35e+07(3.0e+06)**	**5.34e+09(9.9e+07)**
Method	**BBOB17**	**BBOB18**	**BBOB19**	**BBOB20**	**BBOB21**	**BBOB22**	**BBOB23**	**BBOB24**
PMOBO (w/o warm)	**1.35e+08(1.2e+07)**$^{\sim}$	7.27e+11(1.3e+09)$^{\sim}$	**1.67e+09(1.3e+08)**$^{\sim}$	2.12e+11(4.3e+08)$^{+}$	1.95e+11(7.6e+08)$^{+}$	1.75e+09(3.2e+07)$^{+}$	7.35e+07(1.6e+06)$^{+}$	**1.35e+09(1.3e+07)**$^{\sim}$
PMOBO	1.28e+08(1.1e+07)	**7.27e+11(1.1e+09)**	1.66e+09(1.3e+08)	**2.12e+11(8.2e+07)**	**1.95e+11(8.8e+08)**	**1.77e+09(2.6e+07)**	**7.42e+07(2.0e+06)**	1.35e+09(1.2e+07)
Method	**BBOB25**	**BBOB26**	**BBOB27**	**BBOB28**	**BBOB29**	**BBOB30**	**BBOB31**	**BBOB32**
PMOBO (w/o warm)	2.43e+07(1.2e+06)$^{\sim}$	8.69e+07(1.5e+07)$^{+}$	3.80e+07(7.7e+06)$^{\sim}$	**3.96e+09(4.4e+07)**$^{\sim}$	1.07e+09(1.2e+07)$^{+}$	8.01e+06(4.1e+05)$^{\sim}$	3.06e+08(5.6e+06)$^{\sim}$	9.61e+03(6.5e+03)$^{\sim}$
PMOBO	**2.44e+07(1.1e+06)**	**9.62e+07(9.2e+06)**	**3.95e+07(5.2e+06)**	3.95e+09(7.6e+07)	**1.08e+09(7.8e+06)**	**8.18e+06(2.3e+05)**	**3.07e+08(6.3e+06)**	**1.23e+04(7.4e+03)**
Method	**BBOB33**	**BBOB34**	**BBOB35**	**BBOB36**	**BBOB37**	**BBOB38**	**BBOB39**	**BBOB40**
PMOBO (w/o warm)	9.89e+09(7.1e+07)$^{\sim}$	**3.78e+07(2.9e+06)**$^{\sim}$	**6.39e+06(5.6e+04)**$^{-}$	**3.53e+05(6.0e+03)**$^{\sim}$	**1.14e+07(2.3e+05)**$^{\sim}$	**6.46e+04(3.6e+03)**$^{\sim}$	2.72e+08(7.0e+06)$^{+}$	**2.24e+05(1.2e+04)**$^{\sim}$
PMOBO	**9.89e+09(9.0e+07)**	3.77e+07(3.7e+06)	6.34e+06(5.4e+04)	3.48e+05(7.9e+03)	1.13e+07(2.3e+05)	6.43e+04(3.6e+03)	**2.80e+08(4.5e+06)**	2.20e+05(1.4e+04)
Method	**BBOB41**	**BBOB42**	**BBOB43**	**BBOB44**	**BBOB45**	**BBOB46**	**BBOB47**	**BBOB48**
PMOBO (w/o warm)	**2.59e+04(4.5e+02)**$^{\sim}$	**1.86e+05(3.2e+03)**$^{\sim}$	8.39e+02(1.2e+02)$^{\sim}$	1.37e+06(7.7e+04)$^{+}$	**3.66e+03(5.4e+02)**$^{\sim}$	**1.14e+06(2.2e+04)**$^{\sim}$	5.49e+03(5.9e+02)$^{\sim}$	1.58e+08(2.5e+06)$^{\sim}$
PMOBO	2.59e+04(4.8e+02)	1.86e+05(3.0e+03)	**8.50e+02(1.6e+02)**	**1.41e+06(6.4e+04)**	3.58e+03(6.8e+02)	1.13e+06(2.2e+04)	**5.63e+03(5.9e+02)**	**1.58e+08(2.9e+06)**
Method	**BBOB49**	**BBOB50**	**BBOB51**	**BBOB52**	**BBOB53**	**BBOB54**	**BBOB55**	**DTLZ1** ($m=2$)
PMOBO (w/o warm)	**7.95e+04(9.1e+03)**$^{\sim}$	**1.25e+03(6.8e+01)**$^{\sim}$	1.68e+06(1.3e+05)$^{\sim}$	**6.63e+02(2.2e+02)**$^{\sim}$	1.35e+10(7.8e+06)$^{+}$	4.88e+06(3.6e+05)$^{\sim}$	4.89e+03(1.2e+03)$^{\sim}$	1.38e+02(1.3e+02)$^{\sim}$
PMOBO	7.82e+04(1.1e+04)	1.25e+03(6.5e+01)	**1.71e+06(8.9e+04)**	6.29e+02(2.3e+02)	**1.35e+10(4.7e+06)**	**5.03e+06(1.8e+05)**	**5.06e+03(1.6e+03)**	**1.48e+02(1.4e+02)**
Method	**DTLZ2** ($m=2$)	**DTLZ3** ($m=2$)	**DTLZ4** ($m=2$)	**DTLZ5** ($m=2$)	**DTLZ6** ($m=2$)	**DTLZ7** ($m=2$)	**DTLZ1** ($m=3$)	**DTLZ2** ($m=3$)
PMOBO (w/o warm)	**4.14e-01(9.5e-04)**$^{\sim}$	7.49e+02(1.6e+03)$^{+}$	4.15e-01(1.0e-03)$^{\sim}$	4.14e-01(9.8e-04)$^{\sim}$	2.81e-01(8.6e-02)$^{\sim}$	6.80e-01(1.3e-01)$^{\sim}$	1.06e+04(1.4e+04)$^{\sim}$	4.13e-01(1.1e-01)$^{\sim}$
PMOBO	4.14e-01(1.3e-03)	**3.29e+03(3.9e+03)**	**4.15e-01(8.1e-04)**	**4.14e-01(9.8e-04)**	**2.95e-01(9.2e-02)**	**6.82e-01(1.3e-01)**	**1.09e+04(1.4e+04)**	**4.25e-01(1.0e-01)**
Method	**DTLZ3** ($m=3$)	**DTLZ4** ($m=3$)	**DTLZ5** ($m=3$)	**DTLZ6** ($m=3$)	**DTLZ7** ($m=3$)	**RE21**	**RE23**	**RE24**
PMOBO (w/o warm)	**1.07e+07(4.7e+06)**$^{\sim}$	7.45e-01(1.6e-01)$^{\sim}$	1.30e-01(4.0e-03)$^{\sim}$	**1.16e+03(1.0e+00)**$^{\sim}$	1.04e+00(8.6e-02)$^{+}$	5.44e+01(4.1e-02)$^{\sim}$	1.12e+10(7.5e+07)$^{\sim}$	**2.17e+04(6.5e+01)**$^{\sim}$
PMOBO	1.05e+07(3.9e+06)	**7.82e-01(1.4e-01)**	**1.30e-01(2.5e-03)**	1.16e+03(1.0e+00)	**1.14e+00(1.4e-01)**	**5.44e+01(3.3e-02)**	**1.12e+10(5.5e+07)**	2.17e+04(6.0e+01)
Method	**RE31**	**RE32**	**RE33**	**RE34**	**RE37**	**RE41**	**RE42**	**Summary**
PMOBO (w/o warm)	**1.73e+17(3.7e+14)**$^{\sim}$	**3.62e+14(2.0e+12)**$^{\sim}$	8.84e+04(1.2e+02)$^{\sim}$	**3.61e+01(4.6e-01)**$^{\sim}$	1.54e+00(1.7e-03)$^{\sim}$	4.42e+02(4.9e+00)$^{\sim}$	9.16e+11(6.6e+09)$^{\sim}$	5/59/15
PMOBO	1.73e+17(9.2e+14)	3.61e+14(3.4e+12)	**8.85e+04(1.1e+02)**	3.61e+01(4.6e-01)	**1.54e+00(2.1e-03)**	**4.42e+02(4.1e+00)**	**9.20e+11(7.9e+09)**	–/–/–

C.3 Impact of Incorporating Posterior Uncertainty into PMOBO

Table 4: The HV results (mean and standard deviation) of the four methods on the 79 benchmark and real-world problems. The method with the best mean HV is highlighted in bold. The symbols "−", "∼" and "+" indicate that our method PMOBO is statistically worse than, equivalent to, and better than the peer method, respectively.

Method	BBOB1	BBOB2	BBOB3	BBOB4	BBOB5	BBOB6	BBOB7	BBOB8
U-MOBO ($\beta = 0.1$)	5.95e+04(5.4e+01)$^{\sim}$	1.55e+09(2.9e+06)$^{\sim}$	5.83e+08(5.0e+05)$^{\sim}$	**1.89e+08(1.2e+05)**$^{\sim}$	9.23e+05(3.8e+03)$^{\sim}$	1.16e+05(3.2e+02)$^{\sim}$	3.05e+06(6.2e+03)$^{\sim}$	**7.39e+03(2.6e+02)**$^{\sim}$
U-MOBO ($\beta = 1.0$)	5.95e+04(4.3e+01)$^{\sim}$	**1.55e+09(3.1e+06)**$^{\sim}$	5.82e+08(7.9e+05)$^{+}$	1.88e+08(7.8e+05)$^{+}$	9.13e+05(3.8e+03)$^{+}$	1.14e+05(8.0e+02)$^{+}$	3.04e+06(9.1e+03)$^{\sim}$	7.04e+03(2.4e+02)$^{+}$
U-MOBO ($\beta = 10$)	5.89e+04(7.8e+01)$^{+}$	1.53e+09(6.4e+06)$^{+}$	5.33e+08(1.8e+07)$^{+}$	1.63e+08(3.4e+06)$^{+}$	7.63e+05(1.8e+04)$^{+}$	9.38e+04(3.3e+03)$^{+}$	2.80e+06(7.4e+04)$^{+}$	4.75e+03(4.8e+02)$^{+}$
PMOBO	**5.95e+04(4.0e+01)**	1.55e+09(2.3e+06)	**5.83e+08(5.7e+05)**	1.89e+08(1.4e+05)	**9.24e+05(3.7e+03)**	**1.16e+05(3.4e+02)**	**3.05e+06(6.8e+03)**	7.34e+03(2.0e+02)

Method	BBOB9	BBOB10	BBOB11	BBOB12	BBOB13	BBOB14	BBOB15	BBOB16
U-MOBO ($\beta = 0.1$)	1.41e+07(7.6e+04)$^{+}$	**1.39e+04(1.1e+03)**$^{\sim}$	3.04e+14(1.2e+12)$^{\sim}$	1.34e+13(8.3e+09)$^{\sim}$	**7.28e+12(5.4e+09)**$^{\sim}$	7.85e+09(2.5e+08)$^{\sim}$	8.21e+07(3.7e+06)$^{\sim}$	5.34e+09(8.8e+07)$^{\sim}$
U-MOBO ($\beta = 1.0$)	1.38e+07(2.3e+05)$^{+}$	1.35e+04(1.2e+03)$^{\sim}$	3.04e+14(9.6e+11)$^{\sim}$	1.34e+13(2.1e+10)$^{\sim}$	7.25e+12(1.7e+10)$^{+}$	7.51e+09(2.0e+08)$^{+}$	7.54e+07(4.7e+06)$^{+}$	5.19e+09(1.1e+08)$^{+}$
U-MOBO ($\beta = 10$)	9.94e+06(5.4e+05)$^{+}$	7.93e+03(2.0e+03)$^{+}$	2.99e+14(1.4e+12)$^{+}$	1.30e+13(1.4e+11)$^{+}$	6.85e+12(2.0e+11)$^{+}$	5.03e+09(5.4e+08)$^{+}$	2.05e+07(1.3e+07)$^{+}$	3.49e+09(3.3e+08)$^{+}$
PMOBO	**1.41e+07(5.3e+04)**	1.38e+04(9.8e+02)	**3.05e+14(1.2e+12)**	**1.34e+13(7.5e+09)**	7.28e+12(6.1e+09)	**7.87e+09(2.6e+08)**	**8.35e+07(3.0e+06)**	**5.34e+09(9.9e+07)**

Method	BBOB17	BBOB18	BBOB19	BBOB20	BBOB21	BBOB22	BBOB23	BBOB24
U-MOBO ($\beta = 0.1$)	**1.30e+08(1.1e+07)**$^{\sim}$	7.27e+11(1.5e+09)$^{\sim}$	1.61e+09(1.4e+08)$^{\sim}$	2.12e+11(2.3e+08)$^{\sim}$	1.95e+11(6.7e+08)$^{\sim}$	1.75e+09(3.5e+07)$^{+}$	7.41e+07(1.8e+06)$^{\sim}$	**1.36e+09(1.3e+07)**$^{\sim}$
U-MOBO ($\beta = 1.0$)	1.14e+08(1.6e+07)$^{+}$	7.25e+11(1.7e+09)$^{+}$	1.66e+09(1.5e+08)$^{\sim}$	2.12e+11(4.1e+08)$^{+}$	1.92e+11(1.3e+09)$^{+}$	1.70e+09(3.3e+07)$^{+}$	7.21e+07(1.4e+06)$^{+}$	1.34e+09(1.3e+07)$^{+}$
U-MOBO ($\beta = 10$)	3.03e+07(1.7e+07)$^{+}$	6.88e+11(1.4e+10)$^{+}$	1.01e+09(3.0e+08)$^{+}$	1.87e+11(1.2e+10)$^{+}$	1.44e+11(1.3e+10)$^{+}$	1.21e+09(5.1e+07)$^{+}$	4.98e+07(4.1e+06)$^{+}$	1.08e+09(5.6e+07)$^{+}$
PMOBO	1.28e+08(1.1e+07)	**7.27e+11(1.1e+09)**	**1.66e+09(1.3e+08)**	**2.12e+11(8.2e+07)**	**1.95e+11(8.8e+08)**	**1.77e+09(2.6e+07)**	**7.42e+07(2.0e+06)**	1.35e+09(1.2e+07)

Method	BBOB25	BBOB26	BBOB27	BBOB28	BBOB29	BBOB30	BBOB31	BBOB32
U-MOBO ($\beta = 0.1$)	**2.44e+07(1.2e+06)**$^{\sim}$	**9.65e+07(1.1e+07)**$^{\sim}$	**4.03e+07(4.8e+06)**$^{\sim}$	3.94e+09(6.0e+07)$^{\sim}$	**1.08e+09(4.3e+06)**$^{\sim}$	7.98e+06(3.7e+05)$^{\sim}$	3.05e+08(5.2e+06)$^{\sim}$	8.79e+03(5.6e+03)$^{+}$
U-MOBO ($\beta = 1.0$)	2.23e+07(1.2e+06)$^{+}$	7.63e+07(1.4e+07)$^{+}$	3.61e+07(7.5e+06)$^{\sim}$	3.85e+09(6.5e+07)$^{+}$	1.05e+09(1.2e+07)$^{+}$	7.40e+06(3.6e+05)$^{+}$	3.01e+08(5.5e+06)$^{+}$	3.25e+03(4.0e+03)$^{+}$
U-MOBO ($\beta = 10$)	1.14e+07(2.0e+06)$^{+}$	0.00e+00(0.0e+00)$^{+}$	5.35e+06(4.9e+06)$^{+}$	1.71e+09(9.9e+08)$^{+}$	7.76e+08(4.0e+07)$^{+}$	2.56e+06(1.1e+06)$^{+}$	2.05e+08(2.3e+07)$^{+}$	0.00e+00(0.0e+00)$^{+}$
PMOBO	2.44e+07(1.1e+06)	9.62e+07(9.2e+06)	3.95e+07(5.2e+06)	**3.95e+09(7.6e+07)**	1.08e+09(7.8e+06)	**8.18e+06(2.3e+05)**	**3.07e+08(6.3e+06)**	**1.23e+04(7.4e+03)**

Method	BBOB33	BBOB34	BBOB35	BBOB36	BBOB37	BBOB38	BBOB39	BBOB40
U-MOBO ($\beta = 0.1$)	**9.90e+09(6.1e+07)**$^{\sim}$	**3.87e+07(3.0e+06)**$^{\sim}$	6.30e+06(7.0e+04)$^{+}$	**3.49e+05(5.2e+03)**$^{\sim}$	1.13e+07(2.3e+05)$^{\sim}$	**6.53e+04(3.2e+03)**$^{\sim}$	**2.81e+08(3.3e+06)**$^{\sim}$	2.25e+05(1.1e+04)$^{\sim}$
U-MOBO ($\beta = 1.0$)	9.86e+09(5.8e+07)$^{+}$	3.65e+07(3.9e+06)$^{\sim}$	6.17e+06(6.6e+04)$^{+}$	3.30e+05(9.0e+03)$^{+}$	1.12e+07(2.6e+05)$^{\sim}$	5.81e+04(4.1e+03)$^{+}$	2.71e+08(6.6e+06)$^{+}$	**2.26e+05(7.4e+03)**$^{\sim}$
U-MOBO ($\beta = 10$)	7.18e+09(1.7e+09)$^{+}$	8.09e+06(3.2e+06)$^{+}$	4.19e+06(1.4e+05)$^{+}$	2.04e+05(1.4e+04)$^{+}$	8.44e+06(4.1e+05)$^{+}$	3.15e+04(5.0e+03)$^{+}$	2.10e+08(1.1e+07)$^{+}$	8.84e+04(3.1e+04)$^{+}$
PMOBO	9.89e+09(9.0e+07)	3.77e+07(3.7e+06)	**6.34e+06(5.4e+04)**	3.48e+05(7.9e+03)	**1.13e+07(2.3e+05)**	6.43e+04(3.6e+03)	2.80e+08(4.5e+06)	2.20e+05(1.4e+04)

Method	BBOB41	BBOB42	BBOB43	BBOB44	BBOB45	BBOB46	BBOB47	BBOB48
U-MOBO ($\beta = 0.1$)	**2.59e+04(4.3e+02)**$^{\sim}$	1.86e+05(2.6e+03)$^{\sim}$	7.95e+02(1.4e+02)$^{\sim}$	1.40e+06(5.2e+04)$^{\sim}$	**3.81e+03(3.6e+02)**$^{\sim}$	**1.13e+06(2.8e+04)**$^{\sim}$	5.34e+03(4.9e+02)$^{\sim}$	1.57e+08(3.9e+06)$^{\sim}$
U-MOBO ($\beta = 1.0$)	2.43e+04(6.5e+02)$^{+}$	1.82e+05(3.2e+03)$^{+}$	5.99e+02(1.8e+02)$^{+}$	1.23e+06(8.5e+04)$^{+}$	3.56e+03(4.0e+02)$^{\sim}$	1.13e+06(2.3e+04)$^{\sim}$	4.57e+03(7.8e+02)$^{+}$	1.56e+08(2.1e+06)$^{\sim}$
U-MOBO ($\beta = 10$)	1.78e+04(1.5e+03)$^{+}$	1.27e+05(1.1e+04)$^{+}$	6.88e+01(1.0e+02)$^{+}$	1.48e+05(1.8e+05)$^{+}$	2.95e+02(2.6e+02)$^{+}$	7.00e+05(7.2e+04)$^{+}$	2.96e+02(3.9e+02)$^{+}$	1.13e+08(9.7e+06)$^{+}$
PMOBO	2.59e+04(4.8e+02)	**1.86e+05(3.0e+03)**	**8.50e+02(1.6e+02)**	**1.41e+06(6.4e+04)**	3.58e+03(6.8e+02)	1.13e+06(2.2e+04)	**5.63e+03(5.9e+02)**	**1.58e+08(2.9e+06)**

Method	BBOB49	BBOB50	BBOB51	BBOB52	BBOB53	BBOB54	BBOB55	DTLZ1 ($m = 2$)
U-MOBO ($\beta = 0.1$)	**7.93e+04(8.4e+03)**$^{\sim}$	1.24e+03(6.8e+01)$^{\sim}$	**1.72e+06(9.5e+04)**$^{\sim}$	6.33e+02(1.9e+02)$^{\sim}$	**1.35e+10(8.0e+06)**$^{\sim}$	5.02e+06(2.7e+05)$^{\sim}$	5.42e+03(1.4e+03)$^{\sim}$	1.46e+02(1.6e+02)$^{\sim}$
U-MOBO ($\beta = 1.0$)	7.71e+04(8.2e+03)$^{\sim}$	1.12e+03(8.3e+01)$^{+}$	1.58e+06(1.4e+05)$^{+}$	**6.64e+02(1.9e+02)**$^{\sim}$	1.35e+10(1.1e+07)$^{\sim}$	5.00e+06(2.3e+05)$^{\sim}$	**5.45e+03(1.1e+03)**$^{\sim}$	0.00e+00(0.0e+00)$^{+}$
U-MOBO ($\beta = 10$)	2.21e+04(5.2e+03)$^{+}$	6.87e+02(1.0e+02)$^{+}$	5.82e+05(2.0e+05)$^{+}$	5.72e+01(4.4e+01)$^{+}$	1.25e+10(3.7e+08)$^{+}$	1.75e+06(9.9e+05)$^{+}$	1.14e+03(4.8e+02)$^{+}$	0.00e+00(0.0e+00)$^{+}$
PMOBO	7.82e+04(1.1e+04)	**1.25e+03(6.5e+01)**	1.71e+06(8.9e+04)	6.29e+02(2.3e+02)	1.35e+10(4.7e+06)	**5.03e+06(1.8e+05)**	5.06e+03(1.6e+03)	**1.48e+02(1.4e+02)**

Method	DTLZ2 ($m = 2$)	DTLZ3 ($m = 2$)	DTLZ4 ($m = 2$)	DTLZ5 ($m = 2$)	DTLZ6 ($m = 2$)	DTLZ7 ($m = 2$)	DTLZ1 ($m = 3$)	DTLZ2 ($m = 3$)
U-MOBO ($\beta = 0.1$)	4.14e-01(1.4e-03)$^{\sim}$	1.78e+03(2.6e+03)$^{\sim}$	4.14e-01(9.7e-04)$^{+}$	4.14e-01(1.2e-03)$^{+}$	3.18e-01(8.2e-02)$^{\sim}$	6.29e-01(1.4e-01)$^{\sim}$	5.22e+03(6.2e+03)$^{\sim}$	3.99e-01(1.1e-01)$^{\sim}$
U-MOBO ($\beta = 1.0$)	4.06e-01(2.1e-03)$^{+}$	1.03e+03(8.5e+02)$^{\sim}$	4.06e-01(1.8e-03)$^{+}$	4.07e-01(1.9e-03)$^{+}$	**3.20e-01(9.5e-02)**$^{\sim}$	7.27e-01(1.1e-01)$^{\sim}$	7.27e+03(5.3e+03)$^{\sim}$	4.42e-02(6.0e-02)$^{+}$
U-MOBO ($\beta = 10$)	5.56e-03(1.1e-02)$^{+}$	2.90e+03(9.9e+02)$^{\sim}$	3.84e-03(8.6e-03)$^{+}$	2.48e-03(5.9e-03)$^{+}$	2.97e-01(9.7e-02)$^{\sim}$	**7.55e-01(4.7e-03)**$^{\sim}$	**1.43e+04(3.8e+03)**$^{-}$	1.18e-02(1.5e-02)$^{+}$
PMOBO	**4.14e-01(1.3e-03)**	**3.29e+03(3.9e+03)**	**4.15e-01(8.1e-04)**	**4.14e-01(9.8e-04)**	2.95e-01(9.2e-02)	6.82e-01(1.3e-01)	1.09e+04(1.4e+04)	**4.25e-01(1.0e-01)**

Method	DTLZ3 ($m = 3$)	DTLZ4 ($m = 3$)	DTLZ5 ($m = 3$)	DTLZ6 ($m = 3$)	DTLZ7 ($m = 3$)	RE21	RE23	RE24
U-MOBO ($\beta = 0.1$)	9.54e+06(4.1e+06)$^{\sim}$	6.66e-01(1.5e-01)$^{+}$	1.29e-01(2.9e-03)$^{\sim}$	1.16e+03(5.1e+00)$^{\sim}$	1.04e+00(1.7e-01)$^{+}$	5.44e+01(4.4e-02)$^{\sim}$	1.12e+10(7.6e+07)$^{\sim}$	2.17e+04(7.9e+01)$^{\sim}$
U-MOBO ($\beta = 1.0$)	1.03e+07(3.9e+06)$^{\sim}$	1.96e-01(1.4e-01)$^{+}$	7.78e-02(2.5e-02)$^{+}$	**1.16e+03(9.1e-01)**$^{\sim}$	**1.15e+00(8.1e-02)**$^{\sim}$	5.43e+01(4.3e-02)$^{+}$	1.11e+10(7.1e+07)$^{+}$	2.13e+04(2.6e+02)$^{+}$
U-MOBO ($\beta = 10$)	9.59e+06(4.4e+06)$^{\sim}$	3.84e-02(2.8e-02)$^{+}$	5.10e-04(1.6e-03)$^{+}$	1.16e+03(2.9e+00)$^{\sim}$	6.88e-01(1.1e-01)$^{+}$	5.38e+01(8.9e-02)$^{+}$	1.11e+10(8.5e+07)$^{+}$	2.05e+04(4.1e+02)$^{+}$
PMOBO	**1.05e+07(3.9e+06)**	**7.82e-01(1.4e-01)**	**1.30e-01(2.5e-03)**	1.16e+03(1.0e+00)	1.14e+00(1.4e-01)	**5.44e+01(3.3e-02)**	**1.12e+10(5.5e+07)**	**2.17e+04(6.0e+01)**

Method	RE31	RE32	RE33	RE34	RE37	RE41	RE42	Summary
U-MOBO ($\beta = 0.1$)	1.73e+17(7.3e+14)$^{\sim}$	3.62e+14(1.9e+12)$^{\sim}$	8.84e+04(1.1e+02)$^{\sim}$	3.60e+01(5.9e-01)$^{\sim}$	1.54e+00(1.9e-03)$^{\sim}$	4.41e+02(5.0e+00)$^{\sim}$	9.18e+11(7.0e+09)$^{\sim}$	0/71/8
U-MOBO ($\beta = 1.0$)	**1.73e+17(4.2e+14)**$^{\sim}$	**3.63e+14(7.5e+11)**$^{\sim}$	8.83e+04(1.8e+02)$^{+}$	**3.62e+01(2.7e-02)**$^{\sim}$	1.54e+00(2.0e-03)$^{\sim}$	4.39e+02(3.8e+00)$^{+}$	9.12e+11(7.5e+09)$^{+}$	1/29/49
U-MOBO ($\beta = 10$)	1.72e+17(1.2e+15)$^{+}$	3.63e+14(4.5e+11)$^{\sim}$	8.81e+04(2.0e+02)$^{+}$	3.60e+01(1.3e-02)$^{+}$	1.53e+00(1.6e-03)$^{+}$	4.22e+02(4.1e+00)$^{+}$	8.39e+11(1.2e+10)$^{+}$	1/6/72
PMOBO	1.73e+17(9.2e+14)	3.61e+14(3.4e+12)	**8.85e+04(1.1e+02)**	3.61e+01(4.6e-01)	**1.54e+00(2.1e-03)**	**4.42e+02(4.1e+00)**	**9.20e+11(7.9e+09)**	–/–/–

C.4 USING THE SCALARISATION-BASED INSTANTIATION

Table 5: The HV results (mean and standard deviation) of the two methods on the 79 benchmark and real-world problems. The method with the best mean HV is highlighted in bold. The symbols "$-$", "$\sim$" and "$+$" indicate that our method PMOBO$_{PSL}$ is statistically worse than, equivalent to, and better than PSL, respectively.

Method	BBOB1	BBOB2	BBOB3	BBOB4	BBOB5	BBOB6	BBOB7	BBOB8
PSL	5.89e+04(2.4e+03)$^+$	1.53e+09(1.8e+07)$^+$	**5.76e+08(6.6e+06)**$^\sim$	1.87e+08(1.7e+06)$^+$	9.37e+05(3.2e+03)$^\sim$	1.15e+05(4.6e+02)$^+$	**3.04e+06(5.0e+03)**$^\sim$	**6.80e+03(2.8e+02)**$^\sim$
PMOBO$_{PSL}$	**5.96e+04(5.8e+00)**	**1.54e+09(1.6e+07)**	5.72e+08(4.8e+06)	**1.89e+08(5.7e+05)**	**9.38e+05(2.2e+03)**	**1.15e+05(3.0e+02)**	3.04e+06(5.3e+03)	6.73e+03(3.7e+02)
Method	**BBOB9**	**BBOB10**	**BBOB11**	**BBOB12**	**BBOB13**	**BBOB14**	**BBOB15**	**BBOB16**
PSL	1.41e+07(1.1e+05)$^+$	1.27e+04(5.9e+02)$^+$	**3.06e+14(1.8e+12)**$^\sim$	**1.33e+13(9.8e+10)**$^\sim$	7.25e+12(4.2e+10)$^+$	7.20e+09(1.2e+09)$^+$	7.30e+07(6.8e+06)$^+$	5.22e+09(1.2e+08)$^\sim$
PMOBO$_{PSL}$	**1.42e+07(3.9e+04)**	**1.30e+04(4.0e+02)**	3.05e+14(2.1e+12)	1.33e+13(1.8e+11)	**7.28e+12(1.6e+10)**	**8.00e+09(4.2e+08)**	**8.02e+07(3.1e+06)**	**5.26e+09(1.2e+08)**
Method	**BBOB17**	**BBOB18**	**BBOB19**	**BBOB20**	**BBOB21**	**BBOB22**	**BBOB23**	**BBOB24**
PSL	1.01e+08(1.1e+07)$^\sim$	7.20e+11(1.3e+10)$^+$	1.31e+09(1.6e+08)$^+$	2.11e+11(2.4e+09)$^\sim$	1.94e+11(1.6e+09)$^\sim$	**1.73e+09(1.3e+08)**$^\sim$	6.91e+07(4.0e+06)$^+$	1.31e+09(2.3e+07)$^\sim$
PMOBO$_{PSL}$	**1.04e+08(1.1e+07)**	**7.27e+11(6.1e+09)**	**1.39e+09(1.7e+08)**	**2.12e+11(1.6e+08)**	**1.95e+11(1.3e+09)**	1.69e+09(1.5e+08)	**7.17e+07(2.7e+06)**	**1.32e+09(2.2e+07)**
Method	**BBOB25**	**BBOB26**	**BBOB27**	**BBOB28**	**BBOB29**	**BBOB30**	**BBOB31**	**BBOB32**
PSL	2.08e+07(1.9e+06)$^\sim$	1.15e+08(2.6e+06)$^\sim$	3.21e+07(4.3e+06)$^\sim$	4.03e+09(4.3e+07)$^\sim$	9.85e+08(1.1e+08)$^+$	**7.23e+06(7.5e+05)**$^\sim$	2.98e+08(6.5e+06)$^\sim$	**2.33e+04(7.2e+03)**$^\sim$
PMOBO$_{PSL}$	**2.14e+07(1.7e+06)**	**1.15e+08(4.1e+06)**	**3.22e+07(5.7e+06)**	**4.04e+09(3.0e+07)**	**1.03e+09(8.0e+07)**	7.03e+06(5.6e+05)	**3.01e+08(6.1e+06)**	2.29e+04(6.8e+03)
Method	**BBOB33**	**BBOB34**	**BBOB35**	**BBOB36**	**BBOB37**	**BBOB38**	**BBOB39**	**BBOB40**
PSL	9.93e+09(7.0e+07)$^+$	3.13e+07(5.6e+06)$^\sim$	6.41e+06(4.9e+05)$^\sim$	**3.56e+05(5.0e+03)**$^\sim$	**1.12e+07(2.0e+05)**$^\sim$	6.30e+04(3.8e+03)$^\sim$	**2.79e+08(1.3e+07)**$^\sim$	**2.02e+05(2.4e+04)**$^\sim$
PMOBO$_{PSL}$	**9.96e+09(4.1e+07)**	**3.14e+07(5.8e+06)**	**6.45e+06(3.0e+05)**	3.51e+05(1.5e+04)	1.11e+07(2.5e+05)	**6.34e+04(3.3e+03)**	2.78e+08(1.4e+07)	1.91e+05(3.3e+04)
Method	**BBOB41**	**BBOB42**	**BBOB43**	**BBOB44**	**BBOB45**	**BBOB46**	**BBOB47**	**BBOB48**
PSL	**2.49e+04(5.6e+02)**$^\sim$	**1.84e+05(3.8e+03)**$^\sim$	9.70e+02(1.2e+02)$^\sim$	1.42e+06(8.7e+04)$^+$	**3.70e+03(4.1e+02)**$^\sim$	1.10e+06(2.9e+04)$^\sim$	5.79e+03(4.3e+02)$^\sim$	**1.46e+08(5.0e+06)**$^\sim$
PMOBO$_{PSL}$	2.49e+04(5.4e+02)	1.83e+05(3.6e+03)	**9.80e+02(1.2e+02)**	**1.48e+06(4.0e+04)**	3.63e+03(5.0e+02)	**1.12e+06(2.5e+04)**	**5.81e+03(4.3e+02)**	1.46e+08(4.8e+06)
Method	**BBOB49**	**BBOB50**	**BBOB51**	**BBOB52**	**BBOB53**	**BBOB54**	**BBOB55**	**DTLZ1** ($m = 2$)
PSL	**7.18e+04(1.1e+04)**$^\sim$	**1.19e+03(6.4e+01)**$^\sim$	1.37e+06(1.2e+05)$^\sim$	**6.74e+02(1.5e+02)**$^-$	**1.35e+10(9.0e+07)**$^\sim$	4.49e+06(6.2e+05)$^\sim$	**3.60e+03(7.6e+02)**$^\sim$	9.03e+01(1.4e+02)$^+$
PMOBO$_{PSL}$	6.96e+04(9.2e+03)	1.18e+03(5.5e+01)	**1.39e+06(8.4e+04)**	4.99e+02(2.1e+02)	1.35e+10(1.1e+08)	**4.56e+06(4.4e+05)**	3.48e+03(7.2e+02)	**2.97e+02(1.8e+02)**
Method	**DTLZ2** ($m = 2$)	**DTLZ3** ($m = 2$)	**DTLZ4** ($m = 2$)	**DTLZ5** ($m = 2$)	**DTLZ6** ($m = 2$)	**DTLZ7** ($m = 2$)	**DTLZ1** ($m = 3$)	**DTLZ2** ($m = 3$)
PSL	3.93e-01(1.1e-02)$^+$	**1.05e+03(1.6e+03)**$^\sim$	**1.17e-01(2.1e-02)**$^\sim$	3.94e-01(1.1e-02)$^+$	0.00e+00(0.0e+00)$^\sim$	4.92e-01(1.6e-01)$^+$	1.05e+04(9.0e+03)$^\sim$	2.74e-01(1.2e-01)$^\sim$
PMOBO$_{PSL}$	**4.05e-01(5.7e-03)**	3.61e+02(1.1e+03)	1.12e-01(9.5e-03)	**4.05e-01(4.7e-03)**	**0.00e+00(0.0e+00)**	**5.14e-01(2.8e-05)**	**1.34e+04(1.3e+04)**	**3.14e-01(1.3e-01)**
Method	**DTLZ3** ($m = 3$)	**DTLZ4** ($m = 3$)	**DTLZ5** ($m = 3$)	**DTLZ6** ($m = 3$)	**DTLZ7** ($m = 3$)	**RE21**	**RE23**	**RE24**
PSL	**1.43e+07(1.6e+06)**$^-$	3.19e-01(8.8e-02)$^\sim$	1.26e-01(5.3e-03)$^\sim$	**1.10e+03(1.3e+01)**$^\sim$	2.69e-01(2.1e-01)$^+$	5.40e+01(3.7e-02)$^+$	1.12e+10(1.6e+08)$^\sim$	2.18e+04(7.4e+00)$^+$
PMOBO$_{PSL}$	1.26e+07(2.1e+06)	**3.27e-01(8.2e-02)**	**1.28e-01(4.1e-03)**	1.10e+03(1.5e+01)	**4.44e-01(2.7e-01)**	**5.43e+01(1.6e-02)**	**1.12e+10(1.3e+08)**	**2.18e+04(1.2e+01)**
Method	**RE31**	**RE32**	**RE33**	**RE34**	**RE37**	**RE41**	**RE42**	**Summary**
PSL	1.73e+17(5.5e+12)$^\sim$	3.63e+14(4.7e+11)$^\sim$	8.84e+04(1.9e+02)$^+$	3.10e+01(1.8e+00)$^+$	1.51e+00(1.6e-03)$^+$	4.21e+02(1.1e+01)$^\sim$	8.78e+11(3.0e+10)$^\sim$	3/51/25
PMOBO$_{PSL}$	**1.73e+17(8.1e+11)**	**3.64e+14(6.5e+11)**	**8.86e+04(8.1e+01)**	**3.25e+01(1.6e-01)**	**1.51e+00(1.7e-03)**	**4.22e+02(1.3e+01)**	**8.78e+11(1.9e+10)**	–/–/–