THE GATING MECHANISM OF KV CACHE EVICTION

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ABSTRACT

KV cache eviction must decide which entries carry information that the rest of the cache can no longer supply. A natural way to estimate this is a learnable gate: give each cached entry a gate on its attention weight and train the gate on the model's own objective. The gate belongs inside the normalizing softmax, where suppressing one entry hands its mass to the others rather than discarding it. That same normalizer absorbs any per-query constant, so a constant gate cancels and reproduces the frozen model exactly. Trained on the language-modeling loss the gate stays near a constant score while its loss falls throughout: optimization succeeds, but a frozen backbone was fit for dense attention, so its loss prefers the no-op, and a near-constant score ranks nothing. The no-op is not a floor the gate improves from but what its objective asks for. A second obstacle survives once the gate moves, since the same gradient charges the gate for errors the frozen model already makes, which no reweighting of positions separates out. Much of the machinery in learnable eviction, from sparsity penalties to relaxations of the discrete decision, works around the first obstacle rather than removing it; both are removed by putting dense attention beyond the gate's reach and making the dense model's predictions its target. Under these two conditions capacity is never the missing ingredient: neither a larger scorer, nor explicit access to the past, nor a trainable backbone helps. The same holds for what eviction unavoidably discards, which is recovered by parameter-free centroids read inside the same softmax far better than by any memory read outside it, since the mass they must return is a share of that normalizer and available nowhere else. At 75% KV cache compression, this gate reaches 90.14 on RULER at 8K and 46.03 on LongBench with Qwen3-8B, 3.72 and 1.36 points below full-cache inference.

1 INTRODUCTION

Large language models are evolving from chatbot assistants (Touvron et al., 2023) into long-horizon agents (Yao et al., 2023). These agentic workloads naturally induce long interaction histories, making efficient long-context inference essential. During autoregressive decoding, the KV cache is increasingly the main bottleneck. As the cache grows linearly with sequence length, KV-cache storage and data movement become the dominant systems bottlenecks (Kwon et al., 2023). Reducing this footprint has therefore become a central objective in efficient LLM inference. Existing approaches compress KV cache along several axes: quantization reduces precision (Liu et al., 2024), cross-layer sharing reduces number of layer-specific KV (Sun et al., 2024b), and latent representations reduce KV dimension (DeepSeek-AI, 2024). These approaches are complementary and can be combined. In this work, we focus on KV eviction because it not only reduces memory consumption but also shortens the effective sequence length of attention, thereby reducing attention computation. KV eviction is possible because information becomes redundant across the cache. Through causal self-attention, each hidden state aggregates preceding context, allowing later KV entries to carry information originally introduced by earlier tokens. This layer-wise propagation explains why sliding-window attention can access information beyond its explicit window (Fu et al., 2025).

However, this propagation is incomplete: isolated facts such as passwords or retrieval keys may remain unique to their original entries. Effective eviction must therefore distinguish two kinds of entries: those whose information can be recovered from the rest of the cache, and those that carry the only copy of what they contribute. Sparse attention and KV eviction appear similar because both select a subset of KV entries, but their objectives differ. Sparse attention selects entries relevant

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to the current query; eviction must identify entries that are redundant regardless of future queries. Existing eviction methods however often treat relevance as redundancy. H_2O and SnapKV use attention from previously observed queries (Zhang et al., 2023; Li et al., 2024). Relevance can serve as a partial proxy for redundancy, but does not measure it directly.

We instead define redundancy relative to the retained cache: an entry is redundant if its information can be recovered from the other KV entries that remain. Hand-crafted measures such as cosine similarity (Park et al., 2026; Cai et al., 2025) capture only limited aspects of this relationship. We therefore learn a lightweight, query-independent gate that maps each contextualized hidden state to a retention score for each KV head. And where that gate is attached is not a free choice. Attention weights are shares of one softmax normalizer, so only a gate inside that normalizer hands a suppressed entry's mass to the remaining entries, which is what removal means.

Naively training this gate does not work: it learns nearly identical retention scores for all tokens, reproducing dense attention without learning which entries can be evicted. The reason is structural and it comes from the same normalizer that gives the gate its meaning. Because a softmax absorbs any per-query constant, a constant score cancels inside it and is a no-op, and a frozen backbone's loss prefers exactly that. The gate must therefore be denied dense attention and asked to match the dense model's predictions instead of the next token. The same invariance also bounds what a score can say: it ranks entries within a head but cannot compare heads, so the division of budget among them must be settled outside the score.

Even accurate retention scores cannot make hard eviction lossless: retained entries may preserve part of an evicted entry's information, but not all of it. We find that discarded entries still receive substantial attention mass, motivating a compact mechanism for recovering their residual contribution. Our key finding is that the readout of this compressed information matters more than the capacity of the memory that stores it, and the reason is the normalizer again. Recovering the missing contribution requires both its direction and the fraction of attention mass it holds, and that fraction is a ratio of softmax denominators: it is available exactly where the gate was, and only estimable indirectly anywhere else. A memory read outside attention must learn that calibration in output space and does not improve as it grows, whereas parameter-free centroids of the evicted keys and values, inserted into the cache and weighted by the same softmax as the retained entries, recover it far more closely. Removal and restoration are thus the same statement about where the normalizer applies. We instantiate these findings in IndexMem++, which follows a two-branch design (Yang et al., 2026), summarized in Figure 1: a selection branch evicts redundant KV entries, and a recovery branch summarizes what was evicted, both reading through one softmax. Experiments on Qwen3-4B and Qwen3-8B show consistent gains across long-context understanding and mathematical reasoning. At 75% compression, IndexMem++ reaches 90.14 on RULER with Qwen3-8B and 46.03 on LongBench, within 1.36 points of Full-KV, and it retains 0.807 pass@1 on Math500 with a 512-entry cache. Section 4 reports the full comparison.

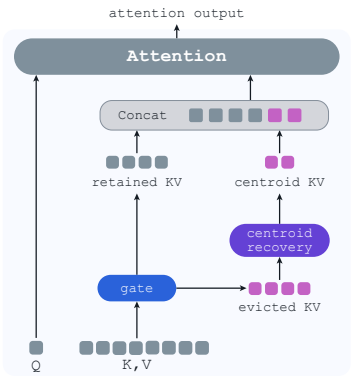

Figure 1: The two branches of IndexMem++: evict and recovery both inside one softmax.

Contributions. Our contributions are threefold:

- We distinguish redundancy from query relevance, formulate KV eviction as query-independent redundancy estimation, and unify learnable gate methods under a common design space.
- We show that the same normalizer is why naive gate training fails: it absorbs a per-query constant, so dense attention is always reachable and is what a frozen backbone's loss prefers. Learning a usable score requires denying the gate that solution, by exempting the entries that must be retained anyway and making the rest compete for a fixed mass, and supervising it against the dense model rather than the next token. The one thing the score cannot express, the division of budget across heads, follows from the same invariance and is set by retained attention mass.
- We show that readout matters more than memory capacity for recovering evicted information, because the missing mass fraction is a ratio of softmax denominators, and introduce content-clustered centroid KV tokens that re-enter the attention softmax at a matched cache budget.

2 ATTENTION MASS AND THE EVICTION GATE

Attention and KV eviction. Writing h_t for the hidden state of token t and d for the head dimension, self-attention projects h_t into a query, key, and value, and token t attends to all preceding tokens:

$$o_t = \sum_{j \le t} a_{tj} v_j, \qquad a_{tj} = \frac{\exp(\ell_{tj})}{Z_t}, \qquad \ell_{tj} = \frac{q_t k_j^\top}{\sqrt{d}}, \qquad Z_t = \sum_{\ell \le t} \exp(\ell_{t\ell}). \tag{1}$$

The weights a_{tj} are shares of the single normalizer Z_t and sum to one; we call a_{tj} the *attention mass* that query t assigns to entry j, and it is the quantity the rest of this paper is about. The keys and values are cached and reused at every decoding step, so the cache grows linearly with context length. Under a budget of B entries, an eviction policy scores each entry and keeps the current top-B, selecting at every step among the surviving entries and the new one; everything outside the top-B is physically removed and never reconsidered.

From hard eviction to differentiable gating. Training a scorer requires a target. Sparse attention scores query relevance, which dense attention already measures: its attention weights are a per-key label to distill against. Eviction scores redundancy, whether the rest of the cache can still supply an entry's information, which no attention weight reveals. Lacking a direct label, we learn the scores end to end from the model's own training objective. This raises a first requirement, differentiability: hard top-B selection has zero gradient almost everywhere, so if the scores only pick which entries to keep, no gradient reaches the scorer. The gate must instead sit on a path the loss can differentiate through. This is necessary but not sufficient: a gate on the query, key, value, or attention output is equally differentiable, so the placement follows from what the score must mean rather than from where a gradient exists. A retention score is a claim about one entry relative to the others, since under a shared budget an entry is kept only in place of another. The gate must therefore make the entries compete for a quantity that is conserved among them, and attention has exactly one such quantity, the mass distributed by the softmax normalizer (Appendix A.3).

To do so, we relax the binary keep-or-drop decision into a continuous one. Every cached key–value pair (k_j, v_j) receives a retention gate $g_j \ge 0$, a soft mask on that entry: $g_j = 0$ removes it, a small g_j marks it as redundant and attenuates its contribution, and a large g_j marks it as worth retaining. The gate can be applied either outside or inside the softmax. An outer gate scales each cached value independently,

$$o_i = \sum_j a_{ij} g_j v_j,$$

so each gate acts on its corresponding pair (k_j, v_j) through the value v_j. Although $g_j = 0$ suppresses that entry, the original attention weights a_{ij} are not renormalized over the remaining KV entries: entry j still consumes its share of the normalizer, and that share is discarded rather than transferred to the entries that remain. The gates therefore never compete. Each one attenuates its own entry in isolation, so a low score means only that this entry contributes little, not that another deserves its place, which is what a shared budget requires. Outer gating also changes the total attention mass and hence the scale of the output.

We instead attach the gate inside the softmax, so that the gated weights are renormalized over the cached entries,

$$a_{ij}^{(g)} = \frac{a_{ij}\, g_j}{\sum_m a_{im}\, g_m}, \qquad o_i = \sum_{j=1}^{i} a_{ij}^{(g)} v_j. \tag{2}$$

The only difference from the outer gate is the denominator, and it is decisive: the mass withdrawn from one entry is now handed to the others, so lowering one score raises every remaining weight and the entries compete exactly as a shared budget requires. Thus, $g_j = 0$ excludes the corresponding KV entry, a uniform gate recovers standard attention. Since $\exp(s + \log g_j) = g_j \exp(s)$, this gate is equivalently an additive bias $\log g_j$ on the attention logit of k_j, which is the form we use throughout. This continuous relaxation allows the scorer to receive gradients from the model loss. At inference, we convert the learnable gates into hard eviction by retaining only the top-B entries. The shared normalizer is what makes a retention score meaningful, and it also settles how the discarded entries are put back: they hold a share of the same mass, so restoring their contribution means recovering that share relative to the entries that remain, which only a joint softmax supplies (Section 3.3).

The paradigm. Our learned eviction framework is summarized as

$$\mathbf{a}_i = \mathrm{softmax}\left(\frac{q_i K_{\le i}^\top}{\sqrt{d}} + \log \mathbf{g}_{\le i}\right), \qquad s_j = f_\theta(h_j), \qquad \log g_j = s_j, \tag{3}$$

where the scorer f_θ maps each contextualized hidden state h_j to the retention score s_j of the pair (k_j, v_j), and the score is the gate in logit space. During training, θ is optimized through the model objective $\mathcal{L}$ using soft eviction. During inference, s_j is used only for cache selection: the model retains the top-B KV and computes standard attention over them without applying the soft gates.

3 METHOD

Equation 3 fixes the mechanism but not the design. We begin from its most direct instantiation and find that it does not work at all, for a reason that is structural rather than a matter of tuning. The shared normalizer that gives the gate its meaning has a second consequence: it absorbs any per-query constant, so dense attention is always within the gate's reach. This section follows that consequence through, to what has to be taken away from the gate, what has to be given to it, and the one question a retention score is unable to answer. A pilot ablation on Qwen3-0.6B isolates each step.

Pilot setup. All ablations use Qwen3-0.6B on RULER at 8K with a cache budget of 2048 entries, 25% of the context. Every arm trains for 100 steps and differs from the reference in a single field. Appendix F gives the full protocol, the per-task scores.

3.1 LEARNING A RELIABLE EVICTION SCORER

The most direct instantiation does not work. We first consider the most direct design of Equation 3: an MLP maps each hidden state h_j to a score s_j and is trained with the language-modeling loss. This setup collapses to nearly constant scores, reaching 4.33 where the dense model reaches 70.01. If all g_j take the same positive value, $\log g_j$ adds the same constant to every key logit, leaving the attention weights unchanged. The constant score is thus a no-op, and the objective prefers it: the frozen backbone was fit for dense attention, so its minimizer is a small perturbation of the constant score. Nearly constant scores are useless for eviction: the top-B cut falls among near-ties, so the retained set is arbitrary (Appendix E).

Removing the no-op solution. The scores should not be able to reproduce dense attention. Eviction already keeps a few attention sinks and a causal local window under any policy, and these entries give the gate a reference frame: holding their gates at one fixes the scale, so the level of the remaining gates is no longer free. We then require the evictable entries to share a fixed total mass β, which they cannot do at the value that would recover dense attention. Writing P_i for the protected set and E_i for the evictable one,

$$g_{ij} = \begin{cases} 1, & j \in P_i, \\ \beta \dfrac{\exp(s_j)}{\sum_{\ell \in E_i} \exp(s_\ell)}, & j \in E_i, \end{cases} \qquad a_{ij} \propto \exp\left(\frac{q_i k_j^\top}{\sqrt{d}}\right) g_{ij}. \tag{4}$$

Dense attention is now out of the question: it needs $g_{ij} = 1$ throughout E_i, hence $\beta = |E_i|$, whereas we set $\beta \ll |E_i|$. The scorer has to rank them (Appendix E). Because $\sum_{j \in E_i} g_{ij} = \beta$ holds exactly at every step, β is also the sparsity used during training rather than a target approached through a penalty, which is what makes a sparsity regularizer or schedule unnecessary (Bui et al., 2025; Łańcucki et al., 2025). This mechanism raises pilot RULER from 4.33 to 44.86 (Table 1).

Distillation isolates the error caused by eviction. With the language-modeling loss, the design above reaches only 44.86 because the cross-entropy gradient mixes errors caused by eviction with errors already made by the frozen backbone. Let p_t^{gated} and p_t^{dense} denote the gated and dense next-token probability distributions, and let e_{y_t} be the one-hot target. The gradient with respect to the output logits decomposes as

$$p_t^{\text{gated}} - e_{y_t} = \underbrace{\left(p_t^{\text{gated}} - p_t^{\text{dense}}\right)}_{\text{caused by eviction}} + \underbrace{\left(p_t^{\text{dense}} - e_{y_t}\right)}_{\text{frozen-backbone error}}. \tag{5}$$

Table 1: **Each component of the paradigm, added one at a time.** Qwen3-0.6B, RULER 8K, cache budget 2048 of 8192 entries (75% compress ratio). Every arm trains for 100 steps and differs from the row above in the single field marked in bold.

	Component			RULER 8K	
Configuration	Always-attended	Loss	Head budget allocation	Avg.	NIAH
Direct instantiation	none	LM	uniform	4.33	2.00
+ always-attended entries	**sinks + window**	LM	uniform	44.86 (+40.53)	53.73
+ distillation	sinks + window	**KL**	uniform	58.86 (+14.00)	72.78
+ head budget	sinks + window	KL	**mass**	67.60 (+8.74)	86.17
Dense backbone (no eviction)	—	—	—	70.01	86.95

Table 2: **Which choice, within each component.** Every arm is the reference recipe (†) with a single field changed, and Δ is its Avg. against that reference. Two seeds of the reference itself differ by 0.28, so a Δ of that size is not a difference. Setup and per-task scores are in Appendix F.

Component	Choice	Avg.	NIAH	Δ
Always-attended	none	4.72	2.00	−62.87
	sinks only	0.57	0.00	−67.03
	window only	57.81	72.71	−9.79
	sinks + window†	67.60	86.17	—
Loss	LM loss (CE)	51.50	64.65	−16.09
	LongCE	53.01	61.99	−14.59
	forward KL	66.84	83.88	−0.76
	reverse KL + 0.1 CE	67.33	85.00	−0.27
	reverse KL†	67.60	86.17	—
Scorer	h_j only, linear (0.5M)	53.53	63.21	−14.07
	h_j only, MLP (7.5M)†	67.60	86.17	—
	+ 8 preceding state (16.8M)	66.82	85.67	−0.78
	+ all preceding recurrent state (24.1M)	68.76	87.20	+1.16
	+ all preceding attention (19.46M)	66.63	85.48	−0.97
	fastkvzip-style (11.36M)	67.25	84.76	−0.35
Head budget allocation	uniform	58.86	72.78	−8.74
	permuted	57.14	68.92	−10.46
	offline table	62.33	75.25	−5.26
	retained mass†	67.60	86.17	—
Retention rate	off	69.27	88.15	+1.67
	on†	67.60	86.17	—

The gate can reduce only the first term, while the second is irreducible with a frozen backbone and can dominate at difficult, high-entropy positions. Token reweighting cannot remove this interference. LongCE (Fang et al., 2024) usefully emphasizes tokens that benefit from long context, but still scales both terms in Equation 5 together.

Distillation removes the second term by supervising the gate against the dense distribution. The gradient then depends only on p_t^{gated} and p_t^{dense}, and vanishes once the two agree, so the frozen-backbone error never enters. Either direction of the KL has this property: the forward direction gives exactly the first term of Equation 5, while the reverse direction we adopt matches it to first order and differs only in how it treats the teacher's low-probability region. We choose between them empirically (Appendix E). Supervision also becomes dense, since each position contributes a full distribution over the vocabulary instead of a single target token. This is the +14.00 from 44.86 to 58.86 in Table 1.

Scores are not comparable across heads. Attention heads specialize in different local and long-range patterns, so they need different cache capacities, and a fixed offline budget table already improves over a uniform split, confirming that reallocating capacity is beneficial. But the retention score cannot decide that allocation. Adding a constant to every score of one head shifts its normalizer by the same amount, leaving both Equation 4 and the entries retained within that head

Table 3: Train the backbone with gate decalibrates the gate rather than damage the model.

			RULER 8K		
Loss	Trained	Tokens	Evict 75%	Full KV	Compression gap
reverse KL	gate only†	7M	**67.60**	70.01	−2.41
reverse KL	gate + backbone	7M	62.83 (−4.77)	71.65	−8.82
LM loss	gate + backbone	7M	60.45 (−7.15)	69.07	−8.62
LM loss	gate + backbone	79M	59.68 (−7.92)	69.26	−9.57
LM loss	gate + backbone	197M	57.42 (−10.18)	69.24	−11.82

unchanged, so the absolute level of a head's scores is not identifiable and scores from different heads are not comparable. The allocation must therefore be read off a quantity that this shift does not move, and the attention mass a head retains is such a quantity. It also depends on the current context, which the offline table cannot follow. We therefore allocate the budget online from the final prefill query. For each head, we order the evictable entries by the retention score and compute the cumulative attention mass retained by its top k entries, including the always-attended sinks and window. We then give every head the smallest k_h that reaches a shared mass target $\rho^\star$, adjusting $\rho^\star$ and any integer remainder so that $\sum_h k_h$ exactly equals the uniform baseline's total budget. This assigns more slots to heads whose attention is distributed over more entries and raises the average from 58.86 to 67.60. Permuting the same set of per-head budgets reduces performance to 57.14, below the uniform split, confirming that the gain comes from matching each budget to the head and context that need it, rather than from using unequal budgets alone (Table 2). This allocator can be viewed as budget-constrained per-head top-p, with the shared mass threshold chosen to satisfy the global cache budget.

3.2 Scorer Architecture and Training Stability

The scorer needs capacity, but not a wider view of the past. We next study the scorer architecture. The scorer requires sufficient capacity: replacing the 7.5M-parameter MLP with a 0.52M linear readout costs 14.07 points (Table 2). Because identifying redundancy may require comparing an entry with information already stored in the past, we augment the MLP with three forms of explicit history: an eight-token causal convolution, a recurrent state, and full-prefix attention. Despite increasing the scorer to as many as 24.1M parameters, all three variants remain within 1.2 points of the reference. This suggests that the contextualized hidden state h_j already contains the historical information needed to estimate redundancy. A Fast-KVzip-style adaptive reference head (Kim et al., 2026) also provides no benefit. We therefore use a two-layer MLP applied directly to h_j.

A per-key retention rate stabilizes the retention scores. A score that depends only on content has nothing anchoring its overall level, so scores can drift upward together as training proceeds and the choice of retained entries becomes sensitive to small differences among large values. Following TrimKV (Bui et al., 2025), we give each entry a learned lifetime alongside its magnitude,

$$s_j(i) = m_j + \log \beta_j \, \frac{i-j}{\tau}, \qquad \log \beta_j \le 0, \tag{6}$$

Query independence here refers to independence from future query content; the score may still vary with token age. The policy remains top-B at every step, but because the ranking now drifts with i, an entry once evicted is never reconsidered even if a retained entry later decays below it (Appendix A.3). Disabling the retention rate reaches 69.27 against 67.60 (Table 2); we keep it for a stable, explicitly parameterized age scale rather than for a gain.

Training the backbone jointly decalibrates the scorer. SP-KV (Szilvasy et al., 2026) jointly trains the language model and utility predictor, allowing sparsity to emerge during training. We do not observe the same benefit: unfreezing the attention projections and norms consistently reduces eviction performance. To distinguish backbone degradation from scorer failure, we also evaluate each checkpoint with Full-KV. Under reverse KL, Full-KV improves from 70.01 to 71.65 (Table 3), while the eviction performance falls from 67.60 to 62.83. The backbone remains intact, but its changed representations decalibrate the scorer. We therefore keep the backbone frozen.

3.3 Recovering Evicted Information

Compensating for what eviction discards. Even accurate retention scores cannot make eviction lossless, since the retained entries may preserve part of an evicted entry's information but not all of it. With o_S and o_E the normalized outputs over the retained and evicted sets and $\rho = Z_E/(Z_S+Z_E)$ the mass on the evicted ones, dense attention gives $o = (1-\rho)o_S + \rho o_E$ while eviction returns o_S, an error of $\rho(o_E - o_S)$. It therefore grows with both how much mass is removed and how different the discarded content is from what remains, and we measure $\rho \approx 0.2$, so the missing contribution is not negligible. Recovery needs both quantities, and they are not equally accessible: ρ is a ratio of softmax normalizers, available where the gate was and only indirectly elsewhere. Since o_E pools many entries, a fixed-size memory is the natural way to summarize it. We test a linear associative state $W : k \mapsto v$ and a TTT-style MLP fitted to the evicted pairs at inference, both read afterwards as $\alpha(q)o_S + \beta(q)m_E(q)$ (Figure 4). Measured by cosine similarity to the exact o_E, they reach only -0.09 to 0.44, and enlarging them barely helps. The reason is not what they store but where they are read: a memory recovering o_E exactly would still owe $\alpha = 1-\rho$ and $\beta = \rho$, and a fusion placed after the softmax cannot obtain either from Z_S and Z_E, so it has to learn the calibration indirectly in output space (Appendix A.4).

The key is therefore to read the summary through attention's softmax. We cluster the evicted entries, average each cluster into a centroid key and value, and append the R centroids to the retained cache, where $q_i k_c^\top/\sqrt{d} + \log n_r$ sets a centroid's weight and v_c its output. The values supply o_E, the logits and cluster-size corrections supply ρ, and one softmax settles both. This parameter-free construction reaches cosine similarity of 0.85–0.96 at most layers and improves Math500 and RULER 8K at matched budgets. The Math500 gain is the larger one, consistent with compensation preserving information accumulated over a long CoT, whereas NIAH depends on retaining the correct evidence entry, which an average cannot substitute for.

Model & Benchmark	K	No slots	64 slots
Qwen3-0.6B Math500	512	48.00	**52.00** (+4.00)
Qwen3-8B RULER 8K	2048	76.65	**77.15** (+0.49)

4 Experiments

4.1 Experimental Setup

Models and baselines. We evaluate Qwen3-4B and Qwen3-8B (Yang et al., 2025) on RULER (Hsieh et al., 2024), LongBench (Bai et al., 2024), Math500 (Lightman et al., 2023), and AIME25, against uncompressed Full-KV and against SnapKV (Li et al., 2024), KeyDiff (Park et al., 2026), DMA (Shi et al., 2025), TrimKV (Bui et al., 2025), KVpop (Hauzenberger et al., 2026), RKV (Cai et al., 2025), and TriAttention (Mao et al., 2026). DMA produces a query-independent score from value representations; in our adaptation, the score is added to attention logits before softmax during training and used for hard eviction at inference (Appendix G).

Tasks. RULER and LongBench use greedy decoding at 50% and 75% compression; LongBench reports 15 tasks with 3,550 examples, excluding TREC, whose score is sensitive to prompt format and does not vary with the cache budget. For math reasoning we estimate pass@1 from two rollouts per Math500 problem and eight per AIME25 problem, with thinking enabled, temperature 0.6, top-p 0.95, top-k 20, and a 32k generation limit; budgets are 512 and 1,024 entries for Math500 and 1,024 and 2,048 for AIME25. Throughout, a budget is the average per-KV-head total attention support and is inclusive: every method funds its sinks, local window (Appendix H).

Training IndexMem++. IndexMem++ is the configuration Section 3 arrives at: a MLP of width 256 with a per-key retention rate, and reverse-KL distillation against full KV model. We train on long documents from LongMino (Sagels, 2026) under an 8K→16K curriculum; the head allocator and the centroid slots act at inference and add no trained parameters (Appendix G).

4.2 Long-Context Understanding

RULER highlights the importance of identifying redundancy. SnapKV scores relevance from observed queries, but the target question is unavailable when the prefill cache is compressed, so recent-context attention is a weak signal for what a future query will need, and it degrades substantially on the retrieval tasks. Methods scoring redundancy directly, including KeyDiff, TrimKV, and

Table 4: **RULER overall results.** The Qwen3-4B checkpoint for KVpop is not open source.

	Qwen3-4B (8K)		**Qwen3-4B** (16K)		**Qwen3-8B** (8K)		**Qwen3-8B** (16K)	
Method	50%	75%	50%	75%	50%	75%	50%	75%
Full-KV	92.41	92.41	91.64	91.64	93.86	93.86	92.96	92.96
SnapKV	54.24	30.85	61.61	35.80	54.32	35.13	62.74	41.54
KeyDiff	73.72	62.31	70.00	61.99	77.35	67.14	74.44	66.87
DMA	68.18	36.34	64.69	33.30	75.22	41.76	71.78	36.19
TrimKV	86.38	75.42	86.40	76.29	87.23	80.26	85.91	79.09
KVpop	—	—	—	—	88.03	72.71	85.49	71.81
IndexMem++	**91.75**	**83.89**	**89.18**	**76.56**	**93.79**	**90.14**	**90.87**	**80.93**

KVpop, do better. A single needle is easy to tell from filler or ordinary text; MultiKey-2/3 are not, holding 425 and 125 distractor records at 8K that share the target's template and key–value type. Because compression precedes the question, every record is potentially relevant and no distractor can be identified in advance, which is where similarity-based criteria degrade sharply and IndexMem++ remains robust. DMA also degrades sharply at high compression. Its score is query-independent, but it is value-derived and trained against the language-modeling loss rather than the dense model, and the resulting ranking transfers poorly to hard eviction on the retrieval-heavy tasks.

Table 5: **LongBench results on Qwen3-8B.** Average over 15 tasks.

Method	**Compression ratio**	**Single QA**	**Multi QA**	**Summ.**	**Few-shot**	**Synth.**	**Code**	**Avg.**
Full-KV		41.96	48.58	27.61	65.40	50.30	62.47	47.39
SnapKV	50%	35.60	46.52	25.93	65.26	50.00	**63.10**	45.39
KeyDiff	50%	28.87	28.91	24.46	62.27	45.61	45.61	36.91
DMA	50%	38.87	42.72	26.53	64.33	35.25	55.13	42.25
TrimKV	50%	**41.93**	46.29	27.20	**67.81**	51.75	56.35	46.54
IndexMem++	50%	40.82	**49.06**	**27.64**	65.27	**52.61**	62.32	**47.53**
SnapKV	75%	28.59	39.69	24.12	65.04	47.92	**63.62**	42.02
KeyDiff	75%	20.14	19.35	19.12	58.91	26.58	38.03	28.19
DMA	75%	29.17	34.39	23.54	59.69	5.00	50.09	32.72
TrimKV	75%	**39.64**	44.58	**27.00**	**66.45**	**51.50**	55.84	45.41
IndexMem++	75%	38.69	**47.43**	26.22	64.47	50.96	61.26	**46.03**

LongBench highlights the importance of information recoverability. IndexMem++ achieves the highest overall average at both compression ratios and consistently leads on multi-document QA. SnapKV relies on previously observed query attention, while KeyDiff and TrimKV use query-independent criteria based on key geometry and token age; none of these signals determines whether an entry's information can be recovered from the retained cache. The distinction matters most in multi-document QA, where evidence is distributed across documents and positions and may be neither geometrically distinctive nor recently encountered. Appendix H.2 provides a task-level analysis, covering the document-QA categories, the complementary requirements of the synthetic tasks, and DMA's behavior under hard eviction.

4.3 Preserving Reasoning State under KV-Cache Compression

The strong performance of Random Attention suggests that, once the problem prompt is protected, many intermediate chain-of-thought states are redundant. At the lower compression setting on Math500 ($K = 1024$), Random Attention is slightly more accurate than IndexMem++, but requires substantially longer generation (7,342 versus 5,251 tokens). As compression increases to $K = 512$, IndexMem++ becomes both more accurate (0.807 versus 0.784) and more efficient (5,350 versus 9,714 tokens). The same trend is clearer on AIME25: its advantage over Random Attention

Table 6: **Math reasoning with Qwen3-8B with pass@1.** The mean number of generated tokens shown in gray text.

	Math500		**AIME25**	
Method	K=1024	K=512	K=2048	K=1024
Full-KV	0.832 (5,258)		0.700 (17,845)	
KVzap	0.792 (6,326)	0.548 (9,850)	0.446 (22,019)	0.183 (29,013)
Fast-kvzip	0.736 (6,974)	0.480 (11,146)	0.375 (23,904)	0.154 (29,462)
R-KV	0.811 (9,741)	0.605 (12,757)	0.229 (27,044)	0.121 (29,625)
TriAttention	0.863 (8,563)	0.758 (12,056)	0.383 (23,109)	0.192 (28,507)
Random Attn.	**0.869** (7,342)	0.784 (9,714)	0.388 (23,199)	0.221 (27,538)
IndexMem++	0.832 (5,251)	**0.807** (5,350)	**0.554** (19,568)	**0.479** (19,878)

grows from 16.6 points at $K = 2048$ to 25.8 points at $K = 1024$. Moreover, IndexMem++ produces 5,251 and 5,350 tokens on Math500, closely matching Full-KV's 5,258 tokens; on AIME25, its 19,568–19,878 tokens also remain much closer to Full-KV's 17,845 than competing methods. These results suggest that IndexMem++ preserves the reasoning states needed at high compression without forcing the model to compensate through substantially longer chains of thought.

4.4 Decoding Efficiency under a Fixed Budget

Figure 2 reports batch throughput for 256 decode steps on a single H20 GPU at an average per-KV-head budget of 2,048 entries, excluding prefill and setup costs. The missing Full-KV* points are out-of-memory runs. Appendix D gives the full protocol. IndexMem++ largely decouples decoding throughput from the input sequence length. Continuous eviction bounds the historical KV cache of each sequence by a fixed budget, so the per-step KV reads and attention computation are determined by that budget rather than by the sequence length, whereas Full-KV must read a cache that keeps growing.

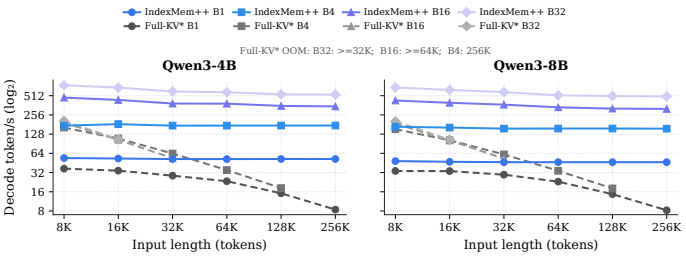

Figure 2: Batch decoding throughput versus input length.

5 Related work

Learnable eviction gates. Learnable eviction replaces fixed heuristics with trainable, content-dependent scores. TrimKV (Bui et al., 2025) predicts per-key retention scores that decay with age, while DMC (Nawrot et al., 2024) and DMS (Łańcucki et al., 2025) learn discrete token-level compression decisions through continuous relaxations and explicit budget objectives. SP-KV (Szilvasy et al., 2026) predicts future utility to gate long-range attention, whereas Locret (Huang et al., 2024) and Attention-Gate (Zeng et al., 2024) train auxiliary modules to score entries during prefill. Although motivated differently, these methods share the same paradigm: a lightweight scorer assigns retention scores to KV entries. We study the scorer, gate normalization and budget, supervision, and allocation across heads in Sections 2 and 3.

Recovering evicted information. Summarizing a discarded past is a long-standing idea: compressive memories retain a coarse record of activations that leave the attention window (Rae et al., 2020), gist tokens compress a prompt into a few learnable slots (Mu et al., 2023), and recurrent or test-time-trained states have been used as memory alongside attention (Sun et al., 2024a; Behrouz et al., 2024). Such a summary can be read back either as a residual added to the attention output, whose scale is set by a gain learned outside the softmax, or as additional participants in attention, which compete for mass in the same softmax and are thus normalized by construction. We find this readout choice matters more than the capacity of the summary (Section 3).

6 Conclusion

Asked which of its own KV entries are expendable, a gate trained on the model's own loss answers that keeping everything is preferable, and returns constant scores. The question becomes answerable once that answer is unavailable: the evictable entries compete for a fixed gate mass, and the gate matches the dense model rather than the next token. What the score cannot say, how much budget each head deserves, is read off the mass each head retains. Some information is lost even so, and there the lesson is not to build a larger memory but to read the small one through the same softmax as the cache it supplements.

7 Future work

Our gate is trained on dense or teacher-forced prefixes but applied to a cache its own earlier decisions have compressed. On-policy distillation, with prefixes generated under hard eviction, would expose it to the state distribution it actually meets at inference.

AI use statement

We used generative AI tools in two roles. First, as coding assistants during the development of our training and evaluation code, including the fused training-time attention kernel of Appendix B and the paged decode path of Appendix C. All such code was reviewed by the authors, and every reported number comes from runs of code we have inspected; correctness of the kernels was additionally checked against reference implementations, as described in the corresponding appendices. Second, as a writing aid for improving the clarity and grammar of the manuscript. We did not use generative AI tools to generate research ideas, to design the method, to produce or modify experimental results, or to write the analysis of those results. We take responsibility for the final content of this work, including all text, claims, and artifacts.

Ethics statement

This work studies KV-cache eviction for long-context inference. Our experiments use publicly released model checkpoints and public benchmarks, and involve no human subjects, no personally identifying information, and no newly collected data. We release no new dataset or model. The method reduces the memory and bandwidth cost of serving long-context models, which lowers the cost of deployment; it does not change what those models can be asked to do, and so inherits the capabilities and the risks of the underlying checkpoints rather than introducing new ones. One consideration specific to eviction deserves mention: because compression discards part of the cache, a compressed model can lose evidence that the uncompressed model would have used, and the resulting errors are not always visible to the user. Deployments in settings where faithfulness to the provided context matters should therefore validate accuracy at the compression ratio they intend to use, rather than assuming the ratios reported here transfer.

Reproducibility statement

Section 4.1 states the models, baselines, benchmarks, and compression ratios; Appendix G gives the complete training recipe, including the training corpus and its subsets, the length curriculum, the optimizer schedule, the gate configuration, the distillation objective, and the inference-time components that add no trained parameters. Appendix H reports per-task results for all RULER and LongBench tasks, together with the budget accounting, retention criterion, and protected region used for each method, so that the budget each baseline receives is explicit. Appendix F gives the protocol for the pilot ablations of Section 3, including the seeds, and states which tasks carry no usable signal at that model scale. Appendices B and C describe the training kernel and the conversion from soft gating to hard eviction, with the core routines listed as code. Appendix D gives the measurement protocol for the decoding-throughput comparison, including the decode implementation used for each curve.

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A Other related work

This appendix discusses other works that Section 5 did not claim in detail.

A.1 Training-free eviction.

Training-free policies differ mainly in the signal they use to score entries. The largest group derives importance from attention already observed. H_2O retains heavy hitters by accumulating attention over past queries (Zhang et al., 2023; Wan et al., 2025), SnapKV pools attention within a trailing observation window to decide what a prompt will need (Li et al., 2024), and R-KV adds a redundancy term so that repetitive reasoning traces are not retained many times over (Cai et al., 2025). Because these scores depend on which queries have been seen, the induced ranking is revised as decoding proceeds, while the deletion it triggers is permanent.

A second group avoids queries altogether and scores entries from the statistics of the keys themselves. TriAttention compares keys against calibrated pre-RoPE query centers in the frequency domain (Mao et al., 2026), while the L_2 key norm (Devoto et al., 2024) and KeyDiff's angular diversity criterion (Park et al., 2026) use the magnitude and the mutual similarity of the key vectors. These criteria are query-independent, which suits the irreversibility of eviction, but the statistic itself is fixed by hand. A third group uses position alone: StreamingLLM retains attention sinks together with a sliding window (Xiao et al., 2024), and uniformly random retention serves as the natural control for how much of the benefit is attributable to the scoring at all (Wang et al., 2026). None of these methods requires training, which is their principal appeal; the cost is that a hand-chosen signal cannot adapt to what a particular context happens to make redundant.

A.2 Sparse attention.

Sparse attention also computes attention over a subset of entries, but it retains the full cache and chooses a new subset for every query, so no memory is reclaimed and no decision is irreversible. Training-free variants select blocks by cheap surrogates of the attention score (Tang et al., 2024; Jiang et al., 2024), and a growing body of work trains the selector jointly with the model (Yuan et al., 2025; Lu et al., 2025; DeepSeek-AI, 2025; Shi et al., 2025; Hu et al., 2026). The trained selectors are close to our setting in machinery: they too must pass gradients through a discrete choice, and some study explicitly how a block-level gate should be injected into the attention computation and normalized (Li et al., 2026). They are far from our setting in objective, however, because a selection that a query gets wrong there can be revised at the next query, whereas an eviction cannot.

A.3 Gating in attention.

The term *gate* is used for several mechanisms at different stages of attention, which have different physical meanings. Let

$$s_{ij} = \frac{q_i^\top k_j}{\sqrt{d}}, \qquad a_{ij} = \frac{\exp(s_{ij})}{\sum_\ell \exp(s_{i\ell})}, \qquad o_i = \sum_j a_{ij} v_j. \tag{7}$$

A gate may act on the query, key, value, normalized attention output, or directly on the logits:

query gate:	$q'_i = g_i^Q \odot q_i,$	controls how the current token retrieves;
key gate:	$k'_j = g_j^K \odot k_j,$	controls how a memory is addressed;
value gate:	$v'_j = g_j^V \odot v_j,$	controls how much payload a memory returns;
output gate:	$o'_i = g_i^O \odot \sum_j a_{ij} v_j,$	controls whether the retrieved message is used;
logit gate:	$a'_{ij} = \mathrm{softmax}_j(s_{ij} + b_{ij}),$	controls whether query i may read key j;
temperature gate:	$a'_{ij} = \mathrm{softmax}_j(\gamma_j s_{ij}),$	controls how sharply key j competes;
denominator gate:	$a'_{ij} = \frac{\exp(s_{ij})}{Z_i + c_i},$	controls how much mass leaves the row.

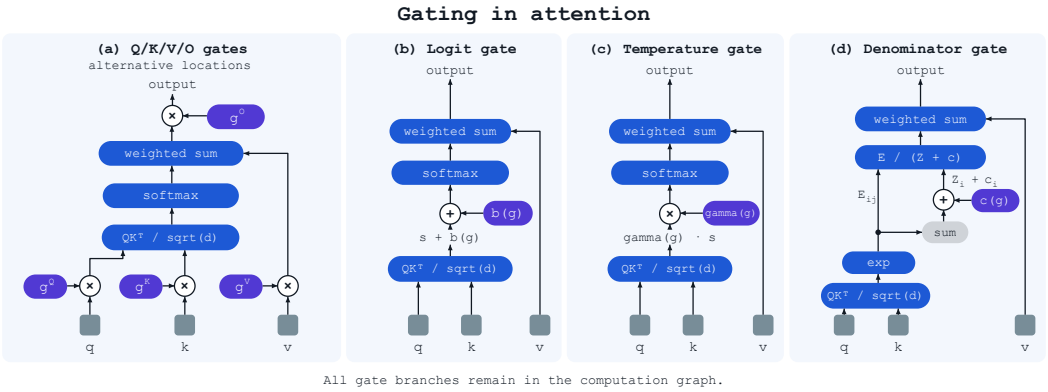

Figure 3: **Where a gate can enter the attention computation.** All four placements keep the gate branch differentiable, so the scorer receives a gradient in every case; they differ in what the gate does to the attention weights. (a) Q/K gates modify retrieval logits, whereas V/O gates rescale values or outputs without renormalizing attention. (b) An additive logit gate shifts a key's logit before the softmax, which scales its unnormalized weight and hands the removed mass to the remaining entries; this is the placement we use. (c) A multiplicative logit gate is a per-key temperature: it pulls the logit towards zero, attenuating the entry where its logit is positive and amplifying it where the logit is negative, so its effect on retention depends on the current query. (d) A denominator gate adds mass c_i to the normalizer alone, so attention mass leaves the row without being assigned to any cached entry.

These locations are not interchangeable. In particular, a gate outside the softmax changes the magnitude of an already-normalized message but does not reallocate probability mass. For example, value gating gives

$$o_i^V = \sum_j \underbrace{\frac{\exp(s_{ij})}{\sum_\ell \exp(s_{i\ell})}}_{a_{ij}} g_j^V v_j.$$

Even if $g_j^V \to 0$, key j may still consume substantial attention probability; its mass is simply discarded rather than reassigned. The same distinction applies to the gated-attention mechanism of (Qiu et al., 2025), which multiplies the output of scaled dot-product attention by a learned sigmoid,

$$o_i^{\text{gate}} = g_i^O \odot \left(\sum_j a_{ij} v_j \right).$$

Besides providing a non-linearity between W_V and W_O, this gate has been reported to suppress massive activations and attention sinks. Its role is nevertheless an *output gate*: it can decide that the result of a retrieval should contribute little, but it does not decide which memory receives the released attention mass. Related analyses of attention and residual sinks show that a small number of positions can absorb a disproportionate fraction of the normalized mass (Gu et al., 2025); this is also why entries that must remain permanently accessible require explicit treatment in an eviction mechanism.

By contrast, a gate *inside* the softmax acts directly on competition between memories. Writing

$$b_{ij} = \log g_{ij}, \quad g_{ij} > 0, \quad a'_{ij} = \frac{\exp(s_{ij} + b_{ij})}{\sum_\ell \exp(s_{i\ell} + b_{i\ell})} = \frac{\exp(s_{ij})\, g_{ij}}{\sum_\ell \exp(s_{i\ell})\, g_{i\ell}}.$$

Thus an additive bias in logit space is exactly a multiplicative gate on the *unnormalized* attention weights, followed by renormalization. Suppressing one entry therefore transfers its probability mass to the remaining entries. In the hard limit $g_{ij} \to 0$, $b_{ij} \to -\infty$, recovering an attention mask. This makes the in-softmax location qualitatively different from query, key, value, or output gating: it is the natural location for a *read gate* and hence for eviction.

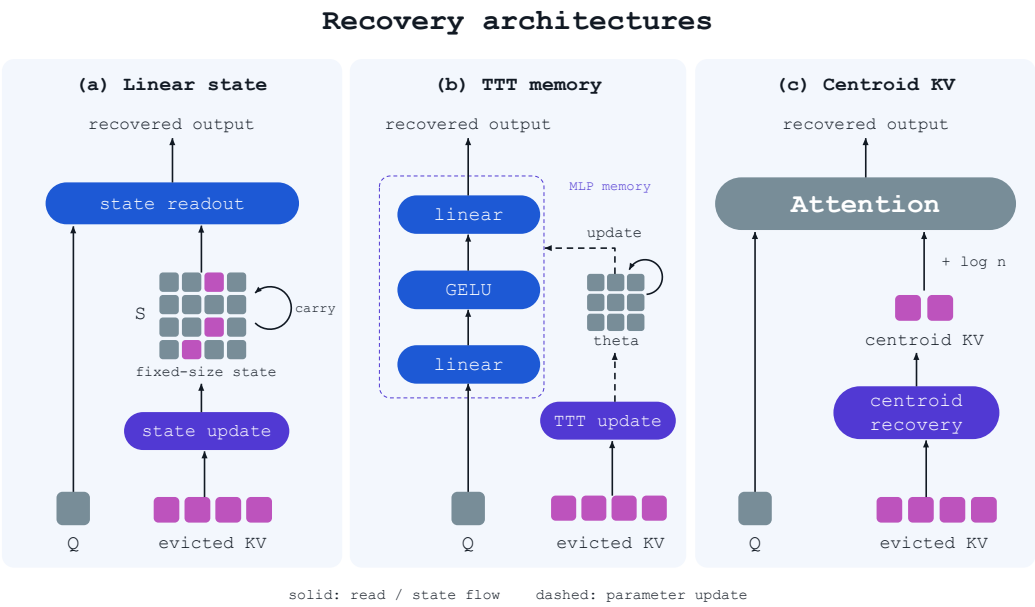

Figure 4: **Recovery architectures.** Three ways to read back information that is no longer stored as individual KV entries. (a) A fixed-size linear state and (b) a test-time-trained MLP memory absorb the evicted KV pairs and are queried outside attention, so their output is fused with the attention output and its scale is set independently of the retained cache. (c) Centroid KV instead inserts the centroids of the evicted entries into the cache, with a $\log n_r$ correction for cluster size, so they and the retained entries are weighted by one softmax. Solid arrows are reads and state flow; dashed arrows are parameter updates.

Two other mechanisms also act inside the softmax but are unsuitable as retention scores. A multiplicative gate on the logit, $s_{ij} \leftarrow \gamma_j s_{ij}$, is a per-key temperature, and a scalar key gate is the same operation because $g_j^K q_i^\top k_j$ rescales the logit rather than shifting it. Shrinking γ_j pulls the logit towards zero, which attenuates the entry where $s_{ij} > 0$ but amplifies it where $s_{ij} < 0$. Its effect therefore depends on the sign of the logit under the current query, so a single γ_j does not order entries by how much they should be retained, whereas b_j shifts every logit of key j in the same direction regardless of the query. Key-Value Means uses such temperatures, but to control the sharpness of already-chosen regions rather than to select among them (Goldstein et al., 2026).

A second mechanism modifies only the normalizer, $a_{ij} = \exp(s_{ij})/(Z_i + c_i)$, as in sigmoid attention, a null or register slot, and the sink construction analyzed by (Gu et al., 2025). The mass $c_i/(Z_i + c_i)$ leaves the row without being assigned to any cached entry, so unlike an additive gate this suppresses an entry without making any other entry more visible. This is useful for absorbing mass that no entry should receive, which is why sinks need explicit treatment, but it provides no per-key quantity to rank.

Additive in-softmax biases beyond eviction. An additive bias inside the softmax is not specific to eviction. The same mechanism is used to encode a broad range of architectural inductive biases:

$$a_{ij} = \mathrm{softmax}_j(s_{ij} + b_{ij})\,. \tag{8}$$

Examples include fixed positional slopes (Press et al., 2022), learned token lifetimes (Sukhbaatar et al., 2021), cumulative forget gates (Lin et al., 2025), pairwise cumulative suppression over intervening tokens (Leviathan et al., 2025; Anagnostidis et al., 2023), and query-conditioned allocations of attention mass (Tan et al., 2024). These mechanisms differ primarily in what information b_{ij} is allowed to depend on.

Softmax is invariant to a row-wise offset:

$$\mathrm{softmax}_j(s_{ij} + b_{ij}) = \mathrm{softmax}_j(s_{ij} + b_{ij} + c_i). \tag{9}$$

Gate location	Form	Physical meaning	Redistributes mass?
Query	$q_i \leftarrow g_i^Q \odot q_i$	retrieval mode	Yes
Key	$k_j \leftarrow g_j^K \odot k_j$	address / retrievability	Yes, non-monotonically
Value	$v_j \leftarrow g_j^V \odot v_j$	returned content	No
SDPA output	$o_i \leftarrow g_i^O \odot o_i$	whether retrieval is used	No
Logit, multiplicative	$s_{ij} \leftarrow \gamma_j s_{ij}$	competition sharpness	Non-monotonically
Denominator only	$Z_i \leftarrow Z_i + c_i$	mass leaves the row	No
Logit, additive	$s_{ij} \leftarrow s_{ij} + b_{ij}$	read visibility	**Yes**

Table 7: Gates at different locations in attention have different semantics. Among per-entry retention mechanisms, an additive logit gate provides monotonic control over normalized attention mass: lowering one entry's bias lowers its weight and transfers mass to the others. A scalar key gate is the multiplicative logit gate, since $g_j^K q_i^\top k_j$ rescales the logit, and its effect on an entry's weight depends on the sign of that logit. A denominator gate removes mass from the row without assigning it to any entry.

Hence b_{ij} is only defined up to a query-dependent constant. Removing this irrelevant degree of freedom gives a useful *canonical form*. For eviction, the key distinction is whether that canonical form still depends on the query index i. If not, the gate assigns every cached entry a persistent scalar ordering; if it does, the ordering may evolve over time and an entry that is weak now can become competitive again later.

We summarize the relevant bias families in Table 8. We additionally define the *fold width* as the smallest r such that, up to a row constant,

$$b_{ij} = u_i^\top v_j, \qquad u_i, v_j \in \mathbb{R}^r. \tag{10}$$

A small fold width allows the bias to be evaluated from compact per-query and per-key state without materializing an $n \times n$ matrix.

This distinction has three consequences for eviction. Our own gate belongs to the third row of Table 8 rather than the first: the magnitude term m_j alone would be a per-key scalar, but the retention rate of Equation 6 makes the canonical form depend on the query index. We state each consequence in general and then say where it places our gate.

Persistent ordering. A canonical bias that does not depend on i provides a persistent ordering of cached entries. Once an entry falls below a fixed keep budget according to this gate, advancing the sequence cannot by itself make its gate score overtake another already-existing entry. Under this condition the greedy online policy of Section 2, which selects from the surviving entries at every step, retains exactly the set that global top-B selection over the full context would retain, so irreversible deletion loses nothing. For example, both

$$b_{ij} = c_i - c_j \tag{11}$$

and a fixed positional slope

$$b_{ij} = -\alpha(i - j) = -\alpha i + \alpha j \tag{12}$$

reduce, after removal of the row constant, to a key-only score. By contrast, a key-specific decay rate,

$$b_{ij} = s_j + \frac{i - j}{\text{ref}} \log \beta_j, \tag{13}$$

cannot remove its i dependence because the coefficient of i itself depends on j. Its ranking therefore evolves with time, which is useful for forgetting but conflicts with irreversible eviction: an entry removed at one step may have become competitive at a later step had it remained available. This is the case of our Equation 6, so the equivalence above does not hold for it.

The policy itself is unchanged. At every step we still retain the current top-B of the surviving entries, exactly as in Section 2; what changes is that this is no longer the same set as top-B over the full context. An entry that leaves the retained set is deleted and never restored, so if the entries kept in its place decay faster and later fall below it, the cache keeps them and not it. Because Equation 6 is affine in i, the scores of any two entries cross at most once, so no pair can exchange places twice and

Bias family	b_{ij}	**Canonical form**	**Frozen gate ranking?**	**Fold width**
Per-key scalar	$\phi(j)$	$\phi(j)$	Yes	1
Separable lifetime	$c_i - c_j$	$-c_j$	Yes	1
Key-specific decay	$s_j + \frac{i-j}{\text{ref}} \log \beta_j$	depends on i	No	2
Pairwise cumulative	$\sum_{j<k<i} S_{kj}$	depends on i	No	high
Query-conditioned	$f(q_i, k_j)$	depends on i	No	–

Table 8: Additive in-softmax biases viewed through their canonical form. The second row includes forms such as the linear part of ALiBi and cumulative forgetting that can be written as a difference of prefix potentials; the third captures token-specific lifetime decay as used by methods such as TrimKV and by the per-key retention rate of Equation 6; the fourth and fifth capture richer pairwise or query-conditioned biases. Only biases whose canonical form is key-only induce a persistent ordering; the remaining rows can change the ordering of memories over time.

an entry cannot lose to a given rival, regain the lead, and lose again. This constrains the individual comparisons but does not prevent an evicted entry from re-entering the true top-B through crossings with different rivals, since that boundary is an order statistic over all surviving entries rather than a single comparison. Irreversible deletion is therefore a genuine approximation under Equation 6, and Section 3.2 reports what it costs.

Computational foldability. A second requirement is that the bias remain inexpensive to evaluate. A key-only scalar can be stored once per cached entry. A key-specific linear decay still has constant fold width:

$$b_{ij} = \underbrace{\begin{bmatrix} 1 & i \end{bmatrix}}_{u_i^\top} \underbrace{\begin{bmatrix} s_j - \frac{j}{\text{ref}} \log \beta_j \\ \frac{1}{\text{ref}} \log \beta_j \end{bmatrix}}_{v_j}. \tag{14}$$

In contrast, cumulative pairwise suppression of the form $\sum_{j<k<i} S_{kj}$ depends on interactions between a candidate memory j and the intervening sequence. In general this object does not admit a low-width factorization, and therefore requires substantially richer state or explicit pairwise bias computation. Our gate sits at the second of these three levels. Width two is what keeps the cost of the retention rate confined to bookkeeping: each cached entry stores m_j and $\log \beta_j$ instead of a single scalar, and a score at any later position follows from the pair without rescoring the hidden state, which is what allows one scorer call to serve a batch whose sequences sit at different absolute positions.

Forced horizons. Finally, an age-dependent bias determines whether sufficiently old memories must eventually disappear regardless of their content. Suppose the content-dependent score difference is bounded by $\Delta_{\max}$. If the relative forgetting penalty between an old and a new entry grows without bound with their age difference, then beyond some distance it necessarily dominates $\Delta_{\max}$:

$$|b_{ij_{\text{old}}} - b_{ij_{\text{new}}}| > \Delta_{\max}. \tag{15}$$

The old entry is then excluded irrespective of its semantic importance, implicitly imposing a finite horizon. A per-key gate has no such forced horizon, while a per-key lifetime model can avoid one only if it is allowed to assign a zero decay rate to entries that should persist. Our parameterization is of the second kind: the constraint $\log \beta_j \leq 0$ in Equation 6 admits zero decay, so no horizon is imposed by construction and an entry the scorer judges important can be given an unbounded lifetime. Our initialization, however, deliberately starts the lifetime branch away from zero (Appendix E), and we do not measure whether the trained rates of the entries that survive longest return to it.

Where this leaves our gate. Our gate therefore does not occupy the simplest cell of this family. It is query-independent in the sense that matters for eviction, namely that the score never depends on the content of a future query, but it is not query-independent after canonicalization: the retention rate makes it a key-specific decay of fold width two. It keeps the two properties that bear on cost, a compact per-entry state and no forced horizon, and gives up the third, the frozen ranking that would make online eviction exactly equivalent to global top-B_h selection. The pilot ablation is consistent with paying something for that: disabling the retention rate raises the average from 67.60 to 69.27

(Table 2), against a two-seed spread of 0.28 on the same 13-task average. We keep it for the reason given in Section 3.2, a stable and explicitly parameterized age scale, and note that the frozen-ranking variant $b_{ij} = \phi(j)$ is the structurally cleaner choice when irreversibility is the binding constraint.

Why we relax the gate rather than estimate a gradient through a discrete decision. Softening the gate is not the only way to keep the scorer in the computation graph. A per-entry keep-or-drop decision can be sampled and differentiated with a straight-through or Gumbel-softmax estimator, which is the route taken by DMC (Nawrot et al., 2024) and DMS (Łańcucki et al., 2025). We relax the gate instead, for two reasons. First, a straight-through estimator substitutes the gradient of the relaxed variable for the gradient of the decision that was actually executed, so the signal it delivers is biased in a way that does not vanish as training proceeds, whereas the soft gate is differentiated exactly at the value used in the forward pass. Second, the decisions in eviction are coupled: under a cache budget, retaining one entry means dropping another, and independent per-entry Bernoulli variables cannot express this without an added penalty that pushes their expected number towards the budget. Equation 4 enforces the same constraint by construction, since the gates of the evictable set sum to β exactly at every step. The relaxation also gives every entry a gradient at every query that can see it, rather than only at the positions where a sampled decision happened to differ.

Why we do not regress an external per-key label. A related line supervises the score against a label produced by a separate procedure rather than through the model's own objective. KVzip (Kim et al., 2025) scores an entry by the attention it receives while the model is prompted to reconstruct its context, which is query-agnostic but costs a second prefill pass and cannot be evaluated during decoding; Fast KVzip (Kim et al., 2026) and KVzap (Jegou & Jeblick, 2026) regress that score from hidden states, removing the extra pass. This is cheaper and more stable to train than the objective we use, and we do not claim it performs worse at a matched budget. We do claim it optimizes a different quantity.

A.4 Recovering evicted information.

From compression to recovery. Summarizing a past that is no longer represented token by token is a long-standing idea. Compressive memories preserve a coarse record of activations after they leave the exact attention window (Rae et al., 2020); gist tokens bottleneck a prompt into a small number of learned token states (Mu et al., 2023); and recurrent or test-time-trained states store history in the activations or parameters of a bounded memory module (Sun et al., 2024a; Behrouz et al., 2024). These approaches primarily differ in what is stored and how the state is updated. For KV eviction, however, there is a separate question: *how should the compressed history be read relative to the exact entries that remain?*

To make this distinction precise, consider a query q and partition its original KV cache into retained entries S and evicted entries E. Let

$$Z_A = \sum_{j \in A} \exp(\ell_j), \qquad o_A = \frac{1}{Z_A} \sum_{j \in A} \exp(\ell_j) v_j, \qquad A \in \{S, E\}, \tag{16}$$

$$\rho = \frac{Z_E}{Z_S + Z_E}, \qquad \ell_j = q k_j^\top / \sqrt{d}. \tag{17}$$

The dense-attention output can then be written as

$$o_{\text{full}} = (1 - \rho) o_S + \rho o_E, \qquad o_{\text{evict}} = o_S, \tag{18}$$

and hence

$$o_{\text{full}} - o_{\text{evict}} = \rho \, (o_E - o_S) \, . \tag{19}$$

Recovering evicted information therefore requires two distinct quantities: the *content* or direction o_E contributed by the evicted entries, and their *mass* ρ relative to the retained cache. A memory with high nominal capacity can still fail if its readout does not recover this relative mass.

Output-space residual recovery. A common design reads a compressed state independently of attention and fuses the resulting vector with the exact-attention output:

$$m_E(q) = \text{Read}(q, M_E), \qquad o_{\text{outer}} = \alpha(q) o_S + \beta(q) m_E(q). \tag{20}$$

Infini-attention accumulates completed segments in a normalized associative state and combines its readout with segment-local softmax attention through a learned scalar gate (Munkhdalai et al., 2024). RAttention maintains exact SWA over recent tokens and a delayed recurrent linear state over tokens outside the window, but independently RMS-normalizes and adds the two outputs (Wang et al., 2025):

$$o_t = \mathrm{RMS}(o_t^{\mathrm{swa}}) + \mathrm{RMS}(o_t^{\mathrm{rla}}). \tag{21}$$

Tensor Cache writes each FIFO-evicted KV pair into an outer-product fast-weight state,

$$A \leftarrow \lambda A + \eta(k_{\mathrm{old}} \otimes v_{\mathrm{old}}), \qquad m_t = q_t A, \tag{22}$$

and fuses it with local attention using $o_t = o_t^{\mathrm{local}} + \sigma(g) m_t$ (Swain et al., 2026). The gated-memory variant of Titans and related TTT memories provide a more expressive state, but use the same general output-space interface (Sun et al., 2024a; Behrouz et al., 2024).

These constructions can learn useful long-term corrections, especially when trained end to end. Nevertheless, the normalization required by Eq. 18 is not enforced by their architecture. Even if $m_E(q) = o_E$ exactly, recovering dense attention requires $\alpha(q) = 1 - \rho(q)$ and $\beta(q) = \rho(q)$. A fixed learned gate, an independently normalized branch, or an additive residual does not obtain these coefficients from Z_S and Z_E; it must instead learn their calibration indirectly in output space. This distinction is particularly consequential when retrofitting a frozen backbone, whose original attention layers already assign a precise probabilistic meaning to their softmax mass.

Attention-space recovery. An alternative is to expose the compressed history as additional KV entries. Figure 4 contrasts this readout with the output-space memories above. Native Hybrid Attention maintains recurrent long-term KV slots and concatenates them with a local window before a single softmax (Du et al., 2025). Key-Value Means merges overflow blocks into a fixed or growing set of compressed KV rows and likewise attends jointly to the compressed state and the uncompressed block window, with learned regional temperatures (Goldstein et al., 2026). CentroidKV periodically merges similar cache entries into degree-weighted centroids and corrects each centroid logit by the logarithm of its cluster degree (Hu et al., 2025). Unlike output-space fusion, these methods allow compressed and exact entries to compete directly for attention mass.

Concretely, let $\mathcal{C}_r \subseteq E$ be a content cluster, $n_r = |\mathcal{C}_r|$, and $(\bar{k}_r, \bar{v}_r)$ its centroid. The contribution of E can be approximated by

$$\widehat{Z}_E(q) = \sum_{r=1}^{R} \exp\left(q\bar{k}_r^\top/\sqrt{d} + \log n_r\right), \tag{23}$$

$$\widehat{N}_E(q) = \sum_{r=1}^{R} \exp\left(q\bar{k}_r^\top/\sqrt{d} + \log n_r\right)\bar{v}_r. \tag{24}$$

Appending these centroids to the retained cache gives

$$o_{\mathrm{joint}} = \frac{Z_S o_S + \widehat{N}_E}{Z_S + \widehat{Z}_E} \tag{25}$$

$$= (1-\widehat{\rho})o_S + \widehat{\rho}\,\widehat{o}_E, \qquad \widehat{\rho} = \frac{\widehat{Z}_E}{Z_S + \widehat{Z}_E}, \quad \widehat{o}_E = \frac{\widehat{N}_E}{\widehat{Z}_E}. \tag{26}$$

Thus, the centroid values approximate the missing direction o_E, while their logits and cluster-size corrections estimate the missing mass ρ. Both are reconciled by the same softmax that weights the retained entries. When a query is unrelated to the compressed history, the centroids receive negligible mass; when it matches that history, they take probability mass away from the retained entries rather than adding an independently scaled residual.

IndexMem++ belongs to the attention-space family, but differs in the source of the exact and compressed sets. Local-memory architectures partition history by age, while cache-merging methods determine the compressed set primarily from geometric similarity. We first learn a query-independent redundancy score that determines which entries must remain individually addressable, and only then summarize the rejected set E by content. Selection and recovery are therefore decoupled: the learned gate decides *what must remain exact*, whereas the centroids approximate the residual contribution of everything else under a fixed total support budget $|S| + R$. Section 3 shows that enlarging

Table 9: Representative mechanisms for retaining information that is no longer stored as individual exact KV entries. "Joint" denotes a single softmax over the exact and compressed supports.

Method	Exact support	Compressed history and write policy	Readout	Joint
Infini-attention	Current segment	Associative state over completed segments	Learned convex gate between two outputs	No
RAttention	Recent SWA window	Linear state read only for out-of-window tokens	Independently RMS-normalized sum	No
Tensor Cache	Recent SWA window	Fast-weight state updated on FIFO eviction	Scalar-gated additive residual	No
NHA	Recent SWA window	Fixed recurrent KV slots	Concatenate slots and local KV	Yes
Key-Value Means	Recent block window	Overflow tokens merged into compressed KV rows	Single softmax with regional temperatures	Yes
CentroidKV	Sink, recent, and unmerged KV	Similar KV entries merged into degree-weighted centroids	Single softmax with $\log n_r$ bias	Yes
IndexMem++	**Learned retained set**	**Content centroids formed only from learned-evicted entries**	**Single softmax with $\log n_r$ bias**	**Yes**

an externally read linear or TTT memory provides little benefit, while moving a comparable summary into the shared softmax substantially improves recovery. This indicates that, in this setting, the limiting factor is not merely memory capacity but the normalization through which that capacity is accessed.

B Training-time kernel for gated attention

For each layer and KV head, the scorer maps the contextualized hidden state of key j directly to a retention score $s_j = f_\theta(h_j)$. We suppress the head index for readability. If the retention-rate term in Equation 6 is enabled, s_j is replaced by its age-dependent value for query i. Let P_i denote the always-attended sinks and causal local window, and let E_i contain the remaining visible keys. The gate added to the attention logits is

$$\bar{g}_{ij} = \begin{cases} 0, & j \in P_i, \\ s_j - \log \sum_{\ell \in E_i} \exp(s_\ell) + \log \beta, & j \in E_i, \end{cases} \qquad o_i = \sum_j \mathrm{softmax}_j \left(\frac{q_i k_j^\top}{\sqrt{d}} + \bar{g}_{ij} \right) v_j. \quad (27)$$

This is the training-time form of Equation 4, written additively in logit space: $\bar{g}_{ij} = \log g_{ij}$, so protected entries remain unchanged, while the other entries compete for total gate mass β. The full causal key axis is used during training so that every score receives a gradient.

Why a standard implementation runs out of memory. Although s_j is computed only once per key, the effective gate $\bar{g}_{ij}$ depends on the query row because each query has a different causal local window P_i. A direct implementation therefore broadcasts the per-key scores into a pairwise gate matrix and constructs gated logits of shape (B, H, S_q, S_k). At $S_q = S_k = 8192$ with 32 query heads, one bf16 matrix occupies about 4 GiB per layer; retaining these activations for backward across 36 layers requires approximately 144 GiB and causes OOM.

Using a standard attention call with an explicit pairwise gate does not avoid this cost. The always-attended mask prevents the gate from being represented as one static per-key bias, and the resulting SDPA call falls back to the math backend, which materializes the full attention matrix. We therefore implement the operation directly as a fused training kernel.

Fused forward and backward. Code 1 shows the core implementation. The MLP computes one s_j per key outside the kernel, and a streaming reduction computes the row-wise gate normalizer. The forward kernel loads these values with each key tile, constructs the sink/local mask arithmetically,

and combines the normalized gate with an online attention softmax. The backward kernel recomputes the same tiled logits and accumulates the gradient of each s_j over the queries that attended to it; ordinary autograd then propagates this gradient through the MLP.

```
g = MLP(h)                              # [batch, H_kv, length]
lse = streaming_logsumexp(g, evictable, beta)

# One Triton program owns a query tile and streams key tiles.
def forward(Q, K, V, g, lse):
    row_max = -inf
    row_sum = 0
    out_acc = 0

    for key_tile in key_tiles:
        q = load(Q, query_tile)
        k = load(K, key_tile)
        v = load(V, key_tile)
        g_key = load(g, key_tile)

        logits = q @ k.T / sqrt(d)
        protected = is_sink(key_tile) | is_local(query_tile, key_tile)
        gate = where(protected, 0, g_key - lse[:, None])
        logits = where(causal(query_tile, key_tile), logits + gate, -inf)

        # Online softmax; no pairwise gate or attention matrix is stored.
        new_max = max(row_max, rowmax(logits))
        rescale = exp(row_max - new_max)
        p = exp(logits - new_max[:, None])
        out_acc = out_acc * rescale[:, None] + p @ v
        row_sum = row_sum * rescale + rowsum(p)
        row_max = new_max

    out = out_acc / row_sum[:, None]
    row_lse = row_max + log(row_sum) # saved for backward
    return out, row_lse


def backward(Q, K, V, g, lse, out, row_lse, d_out):
    delta = rowsum(out * d_out)
    dQ, dK, dV, dg, d_lse = zeros()

    for key_tile in key_tiles:
        # Recompute the same gated logits instead of saving O(L^2) activations.
        logits, protected = recompute_logits(Q, K, g, lse, key_tile)
        p = exp(logits - row_lse[:, None])

        atomic_add(dV[key_tile], p.T @ d_out)
        dP = d_out @ V[key_tile].T
        dZ = p * (dP - delta[:, None])

        dQ += dZ @ K[key_tile] / sqrt(d)
        atomic_add(dK[key_tile], dZ.T @ Q / sqrt(d))
        d_gate = where(protected, 0, dZ)
        atomic_add(dg[key_tile], rowsum_over_queries(d_gate))
        d_lse += -rowsum_over_keys(d_gate)

    return dQ, dK, dV, dg, d_lse
```

Listing 1: Core training-time gated-attention kernel.

Neither pass materializes the pairwise gate, logits, or attention probabilities. The saved activation memory is therefore $O(S_q+S_k)$ rather than $O(S_qS_k)$, although the arithmetic remains quadratic because every visible key receives an independent training signal. Softmax and gradient accumulation use fp32 for numerical stability.

A causal row can initially contain only always-attended entries, leaving E_i empty and its log-sum-exp undefined. Such rows use an inert gate, which preserves dense attention and prevents NaNs without suppressing any meaningful retention signal.

C From the soft gate to hard eviction

At inference, the learned gate is used only to score cache entries, while attention is computed normally over the retained entries. Our implementation uses sparse attention to simulate eviction during prefill, then converts the final prefill state into a physically compressed cache for streaming decode.

Sparse prefill. During prefill, each query attends only to its sinks, local window, and highest-scoring visible entries. We evaluate the retention score once for the whole prefill and hold that score vector fixed while the mask is built. Against a fixed score vector the candidate pool of successive queries only grows, so the number of entries outranking a given key is non-decreasing and each key is selected by one contiguous interval of query rows. Each key's support is then summarized by the last position at which it survives, and the mask is expressed as a FlexAttention block mask, avoiding a per-query top-B_h support tensor and indirect KV gathers. This reproduces per-row top-B_h selection over that score vector exactly; with the per-key retention rate enabled, holding the vector fixed is an additional prefill approximation, since a query midway through the prefill would otherwise rank entries by their scores at its own position. The full KV cache is retained until prefill finishes, so this stage reduces attention computation but not cache storage.

Streaming eviction. After prefill, we copy the final row's retained entries into the compressed cache and release the dense cache. During decoding, each new token first enters the local window; the entry leaving the window then competes with the weakest entry in the retained pool, so at most one cache slot is replaced per head and step and the cache size remains fixed. This update is exactly equivalent to global top-B_h selection when the ordering is fixed; with learned per-key decay, it is an irreversible online approximation because deleted entries cannot be reconsidered after their scores cross.

Ragged paged cache. The head allocator assigns a different budget B_h to each KV head, so padding all heads to the largest budget would waste much of the saved memory. We instead store each $(\text{layer}, \text{sequence}, \text{KV head})$ row in its own pages. Decode then performs dense paged GQA directly over the retained entries, without a mask or gather, and both cache storage and historical KV reads scale with $\sum_h B_h$ rather than the original context length, as reflected in Figure 2.

D Efficiency

Measurement setup. Figure 2, summarized in Section 4.4, reports aggregate batch decode throughput (token/s) on one NVIDIA H20 GPU for Qwen3-4B and Qwen3-8B. Input lengths are 8K, 16K, 32K, 64K, 128K, and 256K (where 1K denotes 1,024 tokens), with batch sizes $B \in \{1, 4, 16, 32\}$, giving 48 configurations. Every configuration decodes 256 steps in BF16, and prefill, initial compression, graph capture, and one-time setup are excluded from the timing. IndexMem++ decodes through a captured CUDA Graph on deterministic pseudo-random token inputs and we report the fastest of three repetitions; its average per-KV-head budget is 2,048 entries, inclusive of the 64 CMP slots, the four sink tokens, and the 128-token local window, under the mass-based head allocation of Table 10. The Full-KV* reference decodes eagerly with FlashAttention-2 on GovReport inputs, aggregated over 15 measurements. The star marks this difference in decode implementation: part of any gap between the two curves reflects graph replay rather than the retained-cache budget, so we read the figure for how each curve scales with input length and treat cross-curve ratios as indicative only.

Context-length scaling. A fixed retained KV budget largely decouples decode throughput from the original input length. From 8K to 256K, IndexMem++ throughput at $B = 1$ changes from 53.95 to 52.18 token/s for Qwen3-4B and from 48.38 to 46.51 token/s for Qwen3-8B, while the Full-KV* reference measurements fall from 36.85 to 8.42 and from 33.88 to 8.21, respectively. At $B = 4$, IndexMem++ changes from 173.69 to 174.69 and from 166.96 to 155.95 token/s for the two models. The $B = 16$ and $B = 32$ curves decline more, but IndexMem++ remains substantially less sensitive to the original context length than the Full-KV* reference curves. This difference is consistent with the execution model in Appendix C: continuous eviction bounds the historical KV read by IndexMem++ to the retained cache budget, whereas Full-KV reads a historical cache that grows with sequence length.

Batch-size scaling. Within the IndexMem++ measurements, throughput increases substantially with batch size while each sequence retains the same bounded historical-cache budget: at 16K it rises from 53.13 to 685.72 token/s for Qwen3-4B and from 47.19 to 630.07 token/s for Qwen3-8B as B goes from 1 to 32. The Full-KV* reference is comparatively flat in batch size at this length, since each added sequence brings its own growing history to read. The two effects compound with the implementation difference noted above, so we do not attribute a specific ratio to the budget alone. In our implementation, scoring and cache maintenance are also batched across sequences and layers.

Benchmark coverage and cache footprint. Figure 2 reports IndexMem++ decode throughput for all 48 model–length–batch configurations. The Full-KV* reference contains 32 measured configurations and 16 OOM cases. For both models, Full-KV* fails from 32K onward at $B = 32$, from 64K onward at $B = 16$, and at 256K for $B = 4$, while $B = 1$ succeeds through 256K. The ragged paged cache in Appendix C allocates physical KV storage according to fixed per-head retained budgets, so retained-cache storage scales with $\sum_h B_h$ per layer and sequence rather than with the original context length. Inputs of 64K–256K exceed the 40,960-position configuration used for quality evaluation and serve as throughput and memory stress tests, not long-context accuracy results.

E Details deferred from the method

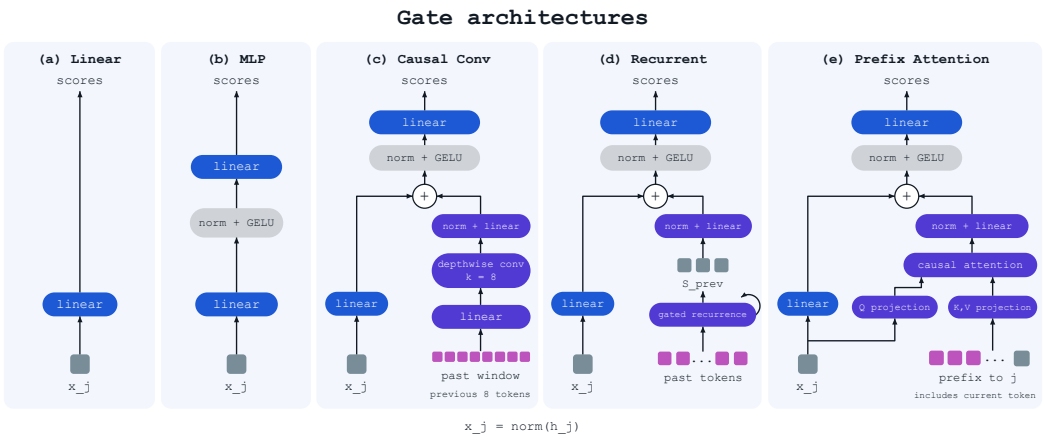

Figure 5: **Scorer architectures.** All variants map $x_j = \mathrm{Norm}(h_j)$ to per-head retention scores and differ only in what they read besides x_j. (a) A linear readout and (b) the token-local MLP used as the reference scorer see the current hidden state alone. The three history variants add one branch that is summed into the MLP's pre-activation: (c) a depthwise causal convolution over the previous eight tokens, (d) a gated recurrence carrying a fixed-size state S_{j-1}, and (e) a single-head causal attention over the prefix up to j. The added branch is initialized to zero, so each variant starts exactly from (b).

E.1 Both halves of the always-attended construction are required.

Normalizing all gates without exempting any entry is ineffective because the row-wise normalizer adds the same constant to every attention logit and therefore cancels inside the softmax. Exempting sinks and the local window alone is also insufficient: the remaining gates can all approach 1 and exactly recover dense attention. Equation 4 combines the two requirements. Protected entries remain fixed at one, while the evictable entries must share a total gate mass β; they therefore cannot all equal one and must compete with one another. This removes the dense no-op solution and turns β into an exact training-time budget rather than a target imposed through a penalty.

What the scorer learns at the constant point. The construction does not forbid a constant score, and it does not need to. Substituting $s_j \equiv c$ into Equation 4 gives $g_{ij} = \beta/|E_i|$ regardless of c: a single policy that attenuates every evictable entry equally and is dense only if $\beta = |E_i|$. Because that

policy is not dense, the loss gradient there is nonzero. Writing $r_{ij} = \langle \partial \mathcal{L}/\partial o_i,\, v_j \rangle$ for the sensitivity of the loss to entry j through the output of query i, and $\bar{r}_i = \sum_k a_{ik} r_{ik}$ for its attention-weighted mean, the gradient with respect to a score is

$$\frac{\partial \mathcal{L}}{\partial s_m} = \sum_{i:\, m \in E_i} \Big(D_{im} - p^{(s)}_{im} \sum_{j \in E_i} D_{ij} \Big), \qquad D_{ij} = a_{ij}\big(r_{ij} - \bar{r}_i\big), \qquad p^{(s)}_{im} = \frac{\exp(s_m)}{\sum_{\ell \in E_i} \exp(s_\ell)}. \tag{28}$$

Each score thus receives the advantage of its own entry over the current evictable average: an entry contributing more than average gains gate mass, and it gains it at the expense of the others. Competition for the limited mass, rather than the unavailability of a constant score, is what drives the scorer to rank entries.

Two further consequences are worth stating. First, the separation from dense attention is graded rather than binary: the constant solution attenuates evictable entries by $\beta/|E_i|$ relative to the protected ones, so the pressure away from the no-op grows with $\log(|E_i|/\beta)$ and disappears as $\beta \to |E_i|$. The requirement is therefore $\beta \ll |E_i|$ rather than merely $\beta < |E_i|$; in the pilot setting $\beta/|E_i| \approx 256/8064$, roughly 3%. This is also why β is held fixed rather than scaled with the sequence length, since a β growing with $|E_i|$ would weaken the mechanism at exactly the lengths that need it. Second, summing Equation 28 over a uniform shift of all scores within a head gives exactly zero, because $\sum_{m \in E_i} (\delta_{jm} - p^{(s)}_{im}) = 0$. The absolute score level is therefore unidentifiable, which is the same degeneracy that prevents raw scores from being compared across heads and motivates the retained-mass allocator.

E.2 Loss design.

Forward $\mathrm{KL}(p^{\mathrm{dense}} \| p^{\mathrm{gated}})$ is mass-covering, or mean-seeking: it encourages the gated model to cover the full teacher distribution, including its lower-probability alternatives. Reverse $\mathrm{KL}(p^{\mathrm{gated}} \| p^{\mathrm{dense}})$ is mode-seeking: it concentrates the student on the teacher's highest-probability modes and penalizes probability assigned outside them.

The two directions agree to first order. Writing $q = p^{\mathrm{gated}}$ and $p = p^{\mathrm{dense}}$ for one position and differentiating with respect to the gated logits z,

$$\frac{\partial\, \mathrm{KL}(p\|q)}{\partial z_k} = q_k - p_k, \qquad \frac{\partial\, \mathrm{KL}(q\|p)}{\partial z_k} = q_k\big(r_k - \bar{r}\big), \qquad r_k = \log \frac{q_k}{p_k}, \quad \bar{r} = \sum_j q_j r_j. \tag{29}$$

The forward direction reproduces the first term of Equation 5 exactly. The reverse direction is not a reweighting of it, since components with $0 < r_k < \bar{r}$ receive a gradient of the opposite sign, but the two coincide near the matching point: substituting $q = p + \epsilon$ gives $r_k = \epsilon_k/p_k + O(\|\epsilon\|^2)$ and $\bar{r} = O(\|\epsilon\|^2)$, because $\sum_k \epsilon_k = 0$, so $q_k(r_k - \bar{r}) = \epsilon_k + O(\|\epsilon\|^2)$. Both directions therefore share the same leading-order gradient and differ only in higher-order terms, which is where the mass-covering and mode-seeking behaviors originate. Neither direction contains the frozen-backbone term of Equation 5, so the choice between them is not a question of which error is isolated and we make it empirically.

We also add cross-entropy with weight 0.1, chosen to place its contribution on approximately the same numerical scale as the KL term; this changes the average from 67.60 to 67.33. Adding this term reintroduces the frozen-backbone error $p^{\mathrm{dense}}_t - e_{y_t}$ into the gradient, and the result does not improve, so the final recipe uses no cross-entropy term and the language-modeling target provides no additional benefit in this setting.

We try LongCE because it emphasizes tokens whose prediction benefits from long context, which should focus training on positions most affected by KV eviction. However, LongCE only reweights the cross-entropy gradient: it scales the error caused by eviction and the irreducible error of the frozen dense model together, rather than removing the latter as distillation does. It therefore improves CE only from 51.50 to 53.01, remaining far below reverse KL at 67.60 (Table 2).

In future work we plan to do on-policy KL distillation, where training prefixes are generated by the current hard-eviction policy rather than by dense or teacher-forced trajectories. This would expose the gate to the state distribution and accumulated errors it encounters at inference, thereby reducing the train–inference mismatch. It is also attractive for reinforcement-learning pipelines,

whose rollouts maintain many long-lived KV caches: applying eviction during policy generation bounds the rollout cache by the retained budget and can reduce the memory and bandwidth cost of sampling, although dense teacher or reference scoring may still require a separate full-cache pass.

E.3 Heads budget design.

Adding a constant c to all retention scores of one head shifts its normalizer by the same amount, leaving Equation 4 and the within-head top-k ordering unchanged. The absolute score level is therefore not identifiable across heads, so pooling raw scores across heads cannot determine their relative budgets. Retained attention mass is invariant to this shift and provides a comparable measure of demand. For the final prefill query, we order each head's evictable entries by its retention score and define $\rho_h(k)$ as the attention mass retained by the first k entries together with the protected sinks and window; query heads sharing one KV head are averaged. We choose the smallest k_h for which $\rho_h(k_h) \geq \rho^\star$ and adjust the shared target $\rho^\star$ until the total budget is met. Any integer remainder is assigned by marginal mass: an extra slot goes to the head whose next entry adds the most attention mass, or is removed from the head whose last entry contributes the least. This is a discrete water-filling allocation that preserves the layer budget exactly. Under a uniform split, the retained-mass range across heads is 0.268 on average and reaches 0.606 in layer 4, leaving substantial room for reallocation.

Both controls preserve exactly the same total number of KV entries. The permuted control shuffles the online allocator's budget vector across heads, preserving its ragged shape while breaking the correspondence between each head and its measured demand. Its score falls to 57.14, below the uniform allocation at 58.86, showing that unequal budgets alone do not explain the improvement. The offline control averages per-head allocations over a calibration set and uses the resulting fixed layer–head table for every input. It reaches 62.33, recovering part of the gain without online measurement and showing that some head specialization is stable across inputs. Its remaining gap to the context-dependent allocator at 67.60 shows that the appropriate allocation also depends on the current context (Table 2).

Code 2 summarizes the online allocation.

```
# Scores rank entries within a head; attention mass allocates across heads.
def mass_curve(Q, K, score, ref_row, sinks, window):
    logits = Q[:, ref_row] @ K.T / sqrt(d)
    prob = softmax(mask_future(logits, ref_row), dim=-1)
    prob = mean_over_gqa_group(prob)       # [H_kv, length]

    pinned = is_sink(sinks) | is_local(ref_row, window)
    pool = is_causal(ref_row) & ~pinned
    pool_size = sum(pool)
    order = argsort(where(pool, score, -inf), descending=True, stable=True)

    pinned_mass = rowsum(where(pinned, prob, 0))
    ranked_mass = gather(prob, order[:, :pool_size])
    return pinned_mass[:, None] + cumsum(ranked_mass, dim=-1)


def allocate_by_mass(curve, total_slots):
    H_kv, pool_size = curve.shape
    # Fewest slots with retained mass at least rho.
    def counts(rho):
        reached = curve >= rho
        return where(any(reached, dim=-1), first_true(reached) + 1, pool_size)

    # Find a common retained-mass target.
    lo, hi = 0.0, 1.0
    for _ in range(60):
        rho = (lo + hi) / 2
        if sum(counts(rho)) > total_slots: hi = rho
        else: lo = rho
    k = counts(lo)

    # Integer correction keeps the layer budget exact.
    while sum(k) < total_slots: gain = mass_of_next_entry(curve, k), k[argmax(gain)] +=
     1
    while sum(k) > total_slots: loss = mass_of_last_entry(curve, k), k[argmin(loss)] -=
     1
```

```
    return k


def head_budgets(Q, K, score, topk, sinks, window):
    curve = mass_curve(Q, K, score, last_prefill_row, sinks, window)
    H_kv = curve.shape[0]
    evictable = topk - sinks - window
    k = allocate_by_mass(curve, H_kv * evictable)
    return k + sinks + window
```

Listing 2: Core retained-mass head-budget allocation.

E.4 The three history scorers, and why each is a single-variable test.

All three variants retain the token-local MLP and add one history branch before its output nonlinearity. Writing $x_j = \mathrm{Norm}(h_j)$ and a_j for the history feature, the shared form is $z_j = \mathrm{GELU}(\mathrm{Norm}(W_{\text{in}}x_j + W_a\,\mathrm{Norm}(a_j)))$, followed by the same per-head score projection as the reference scorer. We initialize W_a to zero, so every variant begins exactly from the token-local MLP and differs only in the construction of a_j. Figure 5 shows the five scorers side by side, including the linear readout and the token-local MLP used as references.

Causal convolution. We project the normalized hidden states to $u_j = W_c x_j$ and apply a width-eight depthwise causal convolution over $u_{j-8}, \ldots, u_{j-1}$. The current token is excluded because its information is already carried by the MLP branch. Each channel has its own signed temporal filter, allowing the scorer to compare a token with its recent neighborhood while requiring only the last eight projected states during decoding.

Gated recurrent state. We maintain a per-channel state $S_j = g_j \odot S_{j-1} + (1 - g_j) \odot u_j$, where $u_j = W_u x_j$ and $g_j = \sigma(W_g x_j)$. The score of token j reads S_{j-1} rather than S_j, so this branch contains only information from preceding tokens. The input-dependent gate lets different channels retain or replace information over different time scales. During training the recurrence is evaluated in fp32 with an associative scan, while decoding carries one fixed-size state vector per layer.

Prefix attention. We give the scorer a separate single-head causal attention with $q_j = W_q x_j$, $k_i = W_k x_i$, $v_i = W_v x_i$, and $a_j = \mathrm{softmax}(q_j K_{\le j}^{\top}/\sqrt{D})V_{\le j}$. This is still query-independent for eviction: q_j is formed from the hidden state of key token j, rather than from a later model query, so the score is fixed when the key is created. It provides access to the entire prefix, but incremental scoring requires retaining the scorer's own K/V cache and attending over a history that grows with the sequence.

The convolution, recurrent, and prefix-attention scorers use 16.8M, 24.1M, and 19.5M parameters, respectively, compared with 7.5M for the token-local MLP, yet change the RULER average by only -0.78, $+1.16$, and -0.97 points (Table 2). A Fast-KVzip-style adaptive-reference head is likewise within 0.35 points. We therefore conclude that sufficient scorer capacity is necessary at this model scale, but explicitly re-reading the history provides little benefit because h_j is already contextualized.

E.5 Retention-rate

Equation 6 uses a fixed age scale τ rather than the current sequence length. Without normalization, $\partial s_j(i)/\partial \log \beta_j = i - j$ can be 10^4 of magnitude larger than $\partial s_j(i)/\partial m_j = 1$ at long context lengths, making the lifetime head effectively train at a much larger learning rate than the magnitude head. Dividing by a fixed τ keeps the two derivatives comparable and gives $\log \beta_j$ a consistent interpretation as the score decrease over τ tokens. Using the live sequence length instead would continually rescale the age contribution of every cached key as the context grows, introducing changes in which entries are retained that are caused by the changing denominator rather than by elapsed age; in a positional-score diagnostic, the fixed scale produced no re-entries over 1,500 steps, whereas live-length normalization produced 27.

Retention-rate initialization. We initialize $\log \beta_j$ so that its change over one reference interval is comparable to the standard deviation of the magnitude score. This differs from TrimKV, where de-

cay starts near zero and is driven by a separate retention hinge. Our fixed-mass normalizer enforces a budget but does not itself encourage shorter lifetimes, so a near-zero initialization could leave the lifetime branch inactive and reduce the model to the content-only scorer. The retention rate does not provide a consistent accuracy gain: its effect is approximately neutral under a uniform budget and reverses under the head-budget allocator. We therefore retain it for a stable, explicitly parameterized age scale rather than as an accuracy-improving component.

F Pilot ablation protocol

Protocol. Every arm in Tables 1, 3 uses Qwen3-0.6B and RULER at an 8K context length. Eviction evaluation retains 25% of the context. Unless explicitly stated otherwise, evaluation uses seed 42. Each gate is trained for 100 steps at 8K with a gate-mass budget of 256, peak learning rate 10^{-3}, and data seed 1000.

Two tasks carry no signal at this model size. At this scale, `cwe` remains between 0.00 and 0.65 for every arm, indicating that Qwen3-0.6B cannot solve the task under this protocol. `multikey3` ranges from 0.00 to 47.83 across arms, but two seeds of the same reference configuration score 26.09 and 25.40; its apparent between-arm variation is therefore not cleanly separable from sampling variation. Both tasks remain in Avg. because they belong to the standard 13-task RULER suite, but no claim in Section 3 relies on either individual column.

G IndexMem++ implementation settings

This section gives the complete training recipe and the inference-time configuration promised by Section 4.1.

Training data. We train on LongMino, using its 256K-filtered split in pre-tokenized form and drawing from the `2e15`, `2e16`, `synth_cwe`, and `synth_rex` subsets. The DMA baseline uses the same mixture; its value-derived query-independent score is trained with LM loss as an in-softmax gate and used for hard eviction. Documents are drawn at random with a shuffle buffer of 64 and a data seed of 1000.

Length curriculum and optimization. The gate is trained under a length curriculum of 300 steps at 8K, 300 steps at 16K, with global batch of 8. We use AdamW with a peak learning rate of 10^{-3} decaying to 5×10^{-6}, warming up over the first 10% of steps and holding the peak for the following 60%. Only the gate parameters are placed in the optimizer; the backbone, the embeddings, and the LM head remain frozen throughout, and Section 3 reports what unfreezing them costs.

Gate configuration. The scorer is a two-layer MLP applied to the hidden state h_j with a middle width of 256, emitting one score per KV head. The per-key retention rate of Equation 6 is enabled. Four attention sinks and a 128-token causal window are always attended; these entries are exempt from eviction and from the gate normalizer. The gate mass in Equation 4 is $\beta = 256$.

Inference-time components. Two components are applied only at inference and introduce no trained parameters. The head-budget allocator redistributes a fixed layer-wide support budget across KV heads by retained attention mass. Let B_h denote the total support assigned to KV head h. Sink entries, local-window entries, and the centroid slots are funded within B_h, rather than added on top. Under mass allocation, the layer-wide budget satisfies $\sum_h B_h = H_{\mathrm{KV}} K$, where K is the average per-KV-head support used for the reported compression setting.

The allocator is subject to a per-head floor, which we set to $K/4$ rather than to an absolute number of entries. A floor fixed in absolute terms would interact with the budget: since the layer total is exactly $H_{\mathrm{KV}} K$ and every head must reach the floor, a floor equal to K would force every head to exactly K and reduce the allocator to a uniform split, and a floor close to K would leave it little room to reallocate. Tying the floor to K keeps the same fraction of the budget available for redistribution at every compression setting, so the allocator behaves consistently across the budgets reported in this paper. At the RULER setting $K = 2048$ this gives a floor of 512 entries.

Per-benchmark inference configuration. Table 10 lists the inference-time configuration of IndexMem++ on each benchmark. The trained gate, the always-attended regions, and the budget accounting are identical everywhere; the benchmarks differ only in the budget K that the reported compression setting implies.

Table 10: **IndexMem++ inference configuration per benchmark.** One trained gate is used throughout; the budget K is the average per-KV-head total support and is inclusive of every protected entry and centroid slot listed here.

Benchmark	Budget K	Allocator	Floor	Sink / local	CMP
RULER	$\lfloor(1-c)N\rfloor$	mass	$K/4$	4 / 128	64
LongBench	$\lfloor(1-c)N\rfloor$	mass	$K/4$	4 / 128	64
Math500	512, 1,024	mass	$K/4$	4 / 128	64
AIME25	1,024, 2,048	mass	$K/4$	4 / 128	64

H Additional Long-Context Results

H.1 Complete RULER results

The tables below report Full-KV and the displayed methods at compression ratios 50% and 75%, across all 13 RULER tasks. The average includes all 13 tasks.

Table 11: **Complete RULER scores: Qwen3-4B, 8K.** Compression ratios 50% and 75%.

Method	S1	S2	S3	MK1	MK2	MK3	MV	MQ	VT	CWE	FWE	QA1	QA2	Avg.
Full-KV	100.00	100.00	100.00	99.80	99.80	99.80	99.85	99.70	99.96	84.56	86.27	75.20	56.40	92.41
Compression ratio: 50%														
SnapKV	85.20	89.00	5.60	43.60	31.60	28.80	43.00	41.05	78.12	77.10	**93.27**	45.80	43.00	54.24
KeyDiff	**100.00**	**100.00**	98.80	91.80	43.00	11.00	98.85	95.80	**99.96**	**80.98**	82.80	34.00	21.40	73.72
DMA	99.80	84.80	29.80	80.60	61.60	0.60	81.85	88.60	96.20	56.44	85.80	70.80	49.40	68.18
TrimKV	**100.00**	99.60	99.80	**99.80**	99.00	42.40	99.65	**99.95**	**99.96**	80.88	77.73	67.20	57.00	86.38
IndexMem++	**100.00**	**100.00**	**100.00**	99.60	**99.60**	**99.40**	**99.80**	99.90	**99.96**	75.20	86.27	**75.40**	**57.60**	**91.75**
Compression ratio: 75%														
SnapKV	70.20	22.20	2.40	15.40	8.00	3.40	14.05	13.85	38.36	62.18	**89.00**	29.60	32.40	30.85
KeyDiff	**100.00**	88.60	92.00	74.60	4.60	0.00	89.90	89.50	**100.00**	69.14	74.07	15.00	12.60	62.31
DMA	99.00	24.20	2.40	27.40	5.40	0.00	14.30	20.70	79.60	48.16	71.47	47.80	32.00	36.34
TrimKV	**100.00**	98.80	**92.60**	95.00	48.40	10.80	91.55	95.60	99.96	**77.00**	71.60	49.60	49.60	75.42
IndexMem++	**100.00**	**99.20**	91.20	**97.80**	**94.80**	**60.40**	**98.40**	**99.20**	**100.00**	44.28	84.07	**68.00**	**53.20**	**83.89**

Task abbreviations follow Table 15.

H.2 Complete LongBench results

Table 16 reports the same 15 tasks and methods as Table 5, with 3,550 examples in the displayed task set. Panel (a) contains document QA and summarization; panel (b) contains the remaining tasks and the overall average. The budget accounting is given below.

Task-level analysis. The document-QA results further separate information recoverability from proxy importance. Low recent attention in SnapKV does not imply that an entry has become redundant, and the geometric distinctiveness used by KeyDiff does not determine whether its information is represented elsewhere in the cache. TrimKV additionally biases retention by token age, although evidence required by a later question may occur anywhere in the context. These mismatches become more consequential in multi-document QA, where a correct answer may depend on multiple pieces of evidence distributed across documents and positions, and where no single piece need be recent

Table 12: **Complete RULER scores: Qwen3-4B, 16K.** Compression ratios 50% and 75%.

Method	S1	S2	S3	MK1	MK2	MK3	MV	MQ	VT	CWE	FWE	QA1	QA2	Avg.
Full-KV	100.00	100.00	100.00	99.40	99.00	98.80	99.95	99.95	99.80	69.82	97.00	72.80	54.80	91.64
Compression ratio: 50%														
SnapKV	92.40	93.80	13.40	69.20	31.40	21.40	60.10	74.35	94.36	66.28	98.27	43.00	43.00	61.61
KeyDiff	**100.00**	99.80	99.40	75.00	27.80	3.80	93.60	95.80	**99.88**	**70.16**	90.73	29.00	25.00	70.00
DMA	**100.00**	83.40	32.60	77.20	51.60	0.20	79.35	83.75	92.56	33.70	94.07	65.60	47.00	64.69
TrimKV	**100.00**	**100.00**	**100.00**	98.60	97.40	40.40	99.15	**99.90**	99.80	63.98	**99.13**	71.60	53.20	86.40
IndexMem++	**100.00**	**100.00**	**100.00**	**99.00**	**98.40**	**85.20**	**99.85**	**99.90**	99.68	52.98	97.47	**72.60**	**54.20**	**89.18**
Compression ratio: 75%														
SnapKV	72.60	39.00	4.40	19.60	5.60	5.20	14.60	20.45	73.40	51.92	97.00	30.80	30.80	35.80
KeyDiff	**100.00**	87.00	92.60	65.20	2.20	0.00	91.30	89.10	99.48	**65.74**	83.87	14.00	15.40	61.99
DMA	98.60	20.40	2.40	17.60	2.20	0.00	14.00	15.30	76.96	30.26	77.20	45.00	33.00	33.30
TrimKV	**100.00**	97.40	92.80	88.40	**42.40**	13.80	93.60	95.55	98.60	60.34	96.67	65.20	47.00	76.29
IndexMem++	**100.00**	**100.00**	**98.80**	**97.00**	41.80	**18.40**	**98.20**	**99.40**	**99.52**	22.60	**98.13**	**72.80**	**48.60**	**76.56**

Task abbreviations follow Table 15.

Table 13: **Complete RULER scores: Qwen3-8B, 8K.** Compression ratios 50% and 75%.

Method	S1	S2	S3	MK1	MK2	MK3	MV	MQ	VT	CWE	FWE	QA1	QA2	Avg.
Full-KV	100.00	100.00	100.00	99.80	99.60	99.60	100.00	100.00	100.00	94.24	91.00	75.80	60.20	93.86
Compression ratio: 50%														
SnapKV	88.40	73.20	3.00	42.40	29.80	24.40	37.95	37.70	88.80	91.74	**92.33**	49.00	47.40	54.32
KeyDiff	**100.00**	99.80	99.00	95.20	53.40	17.00	99.70	99.55	**100.00**	93.20	87.67	34.60	26.40	77.35
DMA	99.00	82.40	80.80	83.60	75.60	1.60	75.60	89.25	99.16	77.28	88.93	74.00	50.60	75.22
TrimKV	**100.00**	**100.00**	99.80	99.40	**99.00**	21.00	99.55	99.90	**100.00**	**94.30**	90.40	72.00	58.60	87.23
KVpop	**100.00**	**100.00**	99.60	98.00	96.80	48.20	97.95	99.45	**100.00**	93.00	88.53	67.60	55.20	88.03
IndexMem++	**100.00**	**100.00**	**100.00**	**100.00**	**99.00**	**99.20**	**100.00**	**100.00**	**100.00**	94.10	90.80	**75.00**	**61.20**	**93.79**
Compression ratio: 75%														
SnapKV	76.00	21.20	2.40	16.00	5.80	4.40	10.70	12.40	66.00	78.32	89.33	36.20	38.00	35.13
KeyDiff	**100.00**	95.20	94.80	86.60	6.80	1.00	98.20	95.90	**100.00**	83.62	74.87	20.20	15.60	67.14
DMA	91.20	26.00	13.00	34.40	8.20	0.00	18.85	29.05	92.72	58.24	80.67	55.00	35.60	41.76
TrimKV	**100.00**	**100.00**	**99.60**	97.80	51.00	4.40	97.80	99.35	**100.00**	91.34	91.27	58.00	52.80	80.26
KVpop	**100.00**	94.20	83.20	79.20	37.80	2.00	89.55	89.70	**100.00**	**92.06**	86.93	44.20	46.40	72.71
IndexMem++	**100.00**	**100.00**	96.40	**99.20**	**97.80**	**73.00**	**98.80**	**100.00**	**100.00**	83.18	**91.40**	**71.40**	**60.60**	**90.14**

Task abbreviations follow Table 15.

Table 14: **Complete RULER scores: Qwen3-8B, 16K.** Compression ratios 50% and 75%.

Method	S1	S2	S3	MK1	MK2	MK3	MV	MQ	VT	CWE	FWE	QA1	QA2	Avg.
Full-KV	100.00	100.00	99.80	99.60	100.00	99.60	99.60	100.00	100.00	82.54	93.80	75.20	58.40	92.96
Compression ratio: 50%														
SnapKV	98.20	94.80	9.20	67.60	33.40	18.40	63.80	67.95	99.00	77.86	95.60	44.60	45.20	62.74
KeyDiff	**100.00**	**100.00**	99.00	82.20	31.60	13.60	98.70	98.00	**100.00**	**86.40**	94.67	33.00	30.60	74.44
DMA	99.60	80.40	79.80	79.40	61.80	0.80	73.95	90.25	98.56	55.06	94.47	68.40	50.60	71.78
TrimKV	**100.00**	**100.00**	99.60	97.80	98.80	17.80	97.65	99.70	**100.00**	82.04	**97.07**	70.40	56.00	85.91
KVpop	**100.00**	**100.00**	99.60	98.80	84.80	40.00	**99.30**	99.25	**100.00**	74.40	92.47	70.20	52.60	85.49
IndexMem++	**100.00**	**100.00**	**99.80**	**99.80**	**99.40**	**79.40**	**99.30**	**99.90**	**100.00**	76.30	95.40	**75.20**	**56.80**	**90.87**
Compression ratio: 75%														
SnapKV	91.40	58.60	2.40	24.80	7.80	3.80	17.10	20.45	94.96	58.02	93.87	33.00	33.80	41.54
KeyDiff	**100.00**	97.00	95.40	70.60	5.00	0.80	96.95	94.35	**100.00**	**82.92**	89.13	19.80	17.40	66.87
DMA	94.80	25.00	7.80	20.00	2.40	0.00	13.05	15.90	90.48	34.20	83.00	48.20	35.60	36.19
TrimKV	**100.00**	**100.00**	**99.20**	96.40	45.60	4.60	95.80	98.50	**100.00**	77.66	**98.27**	62.20	50.00	79.09
KVpop	**100.00**	93.20	80.20	83.40	31.80	1.60	91.35	89.95	**100.00**	67.84	88.13	63.80	42.20	71.81
IndexMem++	**100.00**	**100.00**	98.40	**98.80**	**51.20**	**22.40**	**98.95**	**99.70**	99.96	56.04	96.47	**73.80**	**56.40**	**80.93**

Task abbreviations follow Table 15.

Table 15: **RULER task-level results at 8K and a compression ratio of 75%.** Comparison of Qwen3-4B and Qwen3-8B across all 13 RULER tasks.

Model	**Method**	**RULER Tasks (8K)**												
		S1	S2	S3	MK1	MK2	MK3	MV	MQ	VT	CWE	FWE	QA1	QA2	Avg.
Qwen3-4B	Full-KV	100.00	100.00	100.00	99.80	99.80	99.80	99.85	99.70	99.96	84.56	86.27	75.20	56.40	92.41
	SnapKV	70.20	22.20	2.40	15.40	8.00	3.40	14.05	13.85	38.36	62.18	**89.00**	29.60	32.40	30.85
	KeyDiff	**100.00**	88.60	92.00	74.60	4.60	0.00	89.90	89.50	**100.00**	69.14	74.07	15.00	12.60	62.31
	DMA	99.00	24.20	2.40	27.40	5.40	0.00	14.30	20.70	79.60	48.16	71.47	47.80	32.00	36.34
	TrimKV	**100.00**	98.80	**92.60**	95.00	48.40	10.80	91.55	95.60	99.96	**77.00**	71.60	49.60	49.60	75.42
	IndexMem++	**100.00**	**99.20**	91.20	**97.80**	**94.80**	**60.40**	**98.40**	**99.20**	**100.00**	44.28	84.07	**68.00**	**53.20**	**83.89**
Qwen3-8B	Full-KV	100.00	100.00	100.00	99.80	99.60	99.60	100.00	100.00	100.00	94.24	91.00	75.80	60.20	93.86
	SnapKV	76.00	21.20	2.40	16.00	5.80	4.40	10.70	12.40	66.00	78.32	89.33	36.20	38.00	35.13
	KeyDiff	**100.00**	95.20	94.80	86.60	6.80	1.00	98.20	95.90	**100.00**	83.62	74.87	20.20	15.60	67.14
	DMA	91.20	26.00	13.00	34.40	8.20	0.00	18.85	29.05	92.72	58.24	80.67	55.00	35.60	41.76
	TrimKV	**100.00**	**100.00**	**99.60**	97.80	51.00	4.40	97.80	99.35	**100.00**	91.34	91.27	58.00	52.80	80.26
	KVpop	**100.00**	94.20	83.20	79.20	37.80	2.00	89.55	89.70	**100.00**	**92.06**	86.93	44.20	46.40	72.71
	IndexMem++	**100.00**	**100.00**	96.40	**99.20**	**97.80**	**73.00**	**98.80**	**100.00**	**100.00**	83.18	**91.40**	**71.40**	**60.60**	**90.14**

S1–S3 = single-needle tasks; MK1–MK3 = multi-key tasks; MV = multi-value; MQ = multi-query; VT = variable tracking; CWE/FWE = common/frequent word extraction; QA1/QA2 = question answering.

Table 16: **Complete LongBench results on Qwen3-8B.** 15 tasks (3,550 examples), at compression ratios 50% and 75%.

(a) Single-document QA, multi-document QA, and summarization

Method	NQA	Qasper	MFQA	Hotpot	2Wiki	MuSiQue	GovR	QMSum	MNews
Full-KV	26.31	43.98	55.59	62.30	48.22	35.23	33.55	24.50	24.77
Compression ratio: 50%									
SnapKV	23.50	37.36	45.94	59.48	47.17	32.91	31.91	22.63	23.26
KeyDiff	16.81	29.46	40.33	36.23	34.46	16.03	29.34	22.37	21.66
DMA	22.40	42.19	52.01	55.56	39.37	33.24	32.21	23.22	24.16
TrimKV	**26.25**	**47.66**	51.89	59.33	43.08	**36.45**	33.19	24.09	24.31
IndexMem++	25.87	43.38	**53.20**	**62.84**	**48.82**	35.51	**33.64**	**24.50**	**24.79**
Compression ratio: 75%									
SnapKV	22.09	28.96	34.71	52.64	39.71	26.71	29.52	21.37	21.46
KeyDiff	12.11	19.56	28.76	20.70	27.82	9.54	21.91	21.52	13.94
DMA	17.30	33.19	37.01	40.60	38.11	24.45	28.05	21.26	21.31
TrimKV	**24.75**	**44.34**	49.82	59.53	42.40	31.81	**32.87**	**23.72**	**24.40**
IndexMem++	24.66	40.61	**50.81**	**61.57**	**44.82**	**35.89**	32.07	23.25	23.35

(b) Few-shot, synthetic, and code tasks; overall average

Method	TriviaQA	SAMSum	PCount	PRetr.	LCC	Repo-P	Avg.
Full-KV	90.56	40.23	10.00	90.60	64.74	60.20	47.39
Compression ratio: 50%							
SnapKV	89.74	40.77	9.00	91.00	**66.20**	**60.00**	45.39
KeyDiff	85.86	38.68	4.89	86.33	40.74	50.47	36.91
DMA	85.16	43.50	3.50	67.00	57.45	52.80	42.25
TrimKV	**90.85**	**44.76**	3.50	**100.00**	57.13	55.56	46.54
IndexMem++	90.29	40.25	**10.00**	95.22	65.05	59.59	**47.53**
Compression ratio: 75%							
SnapKV	**89.59**	40.49	**7.55**	88.29	**65.78**	**61.46**	42.02
KeyDiff	81.63	36.19	4.82	48.33	25.63	50.43	28.19
DMA	77.46	41.91	1.50	8.50	53.87	46.30	32.72
TrimKV	89.42	**43.47**	3.00	**100.00**	56.50	55.17	45.41
IndexMem++	89.50	39.44	7.50	94.42	63.79	58.72	**46.03**

NQA = NarrativeQA; MFQA = MultiFieldQA-en; Hotpot = HotpotQA; 2Wiki = 2WikiMQA; GovR = GovReport; MNews = MultiNews; PCount = PassageCount; PRetr. = PassageRetrieval-en; Repo-P = RepoBench-P. Avg. is the equal-weight mean of all 15 displayed tasks. Budget accounting and protected regions are given in Appendix H.

or geometrically distinctive. At 75% compression, IndexMem++ scores 61.57, 44.82, and 35.89 on HotpotQA, 2WikiMQA, and MuSiQue, respectively, compared with 59.53, 42.40, and 31.81 for TrimKV, while KeyDiff degrades more substantially; IndexMem++ leads this category at both compression ratios. These results are consistent with IndexMem++ better preserving evidence that remains irreplaceable despite being old, recently unattended, or geometrically unremarkable.

The synthetic tasks stress two complementary forms of information preservation. PassageRetrieval requires precise discrimination among candidate passages, that is, maintaining enough semantic detail to distinguish the target from competing ones, whereas PassageCount requires global coverage of the unique passages despite repeated content. DMA also degrades on both tasks under hard eviction: its query-independent score is derived from value representations and trained with the language-modeling objective rather than full-KV distillation. The effect is particularly pronounced on PassageRetrieval at 75% compression, where DMA drops to 8.50 while IndexMem++ retains 94.42; on PassageCount, the corresponding scores are 1.50 and 7.50. IndexMem++ instead trains its scorer against the full-KV prediction distribution, so attenuating information that changes the model's predictions provides a direct signal to increase its retention priority.

RULER budget accounting. The input is processed as a context followed by a question and generation suffix. RULER is evaluated at nominal context lengths of 8K and 16K. Let N denote the actual tokenized context length used for compression. For each input and compression ratio c, every method receives the same average per-KV-head budget $K = \lfloor (1-c)N \rfloor$. The budget is inclusive: the attention sinks, local window, and any auxiliary slots a method maintains are funded within K rather than added on top, so the total support budget is matched across methods for each input. Table 17 lists the retention score and the protected region of each method under this shared budget.

For IndexMem++, K is the average per-KV-head budget. The mass allocator redistributes the fixed layer-wide total $\sum_h B_h = H_{\mathrm{KV}} K$ across KV heads, so an individual head may hold more or fewer than K entries while the layer total is preserved.

Table 17: **RULER retention criteria and protected regions.** All methods use the same inclusive average per-KV-head budget $K = \lfloor (1-c)N \rfloor$ for each input; the protected region of each method is funded within K rather than added on top.

Method	Retention score	Protected region (within K)
SnapKV	Observed-attention mean with width-5 average pooling	Final 64 context positions
KeyDiff	Cosine-based post-RoPE key scoring	Final context position
DMA	Query-independent value-derived gate score	Sink 4, local 128
TrimKV	Released gate deltas, magnitude and lifetime	Sink 4, local 128
KVpop	Recurrent scorer state	Sink 4, local 256
IndexMem++	Learned gate on h_j; mass allocation across heads	Sink 4, local 128, CMP 64

LongBench. All methods use a maximum context length of 32,768 tokens and the task-specific generation limits defined by the benchmark. Prompts longer than this limit are truncated before prefill. For compression ratio c, the average per-KV-head budget is $K = \lfloor (1-c)N \rfloor$, where N is the prompt length after truncation; the budget is fixed once at the end of prefill and held during decoding. As on RULER, the budget is inclusive of each method's protected region, and the retention criteria and protected regions are those of Table 17.

Averaging. RULER and LongBench scores are shown on the benchmark's 0–100 scale. RULER uses the arithmetic mean of the 13 scorer-returned task scores. LongBench uses the equal-weight mean of the 15 displayed task scores; its category sizes are 3, 3, 3, 2, 2, and 2. Few-shot averages only TriviaQA and SAMSum. We calculate category and overall means from task scores before rounding the displayed result to two decimals.