BIDIRECTIONAL SINGLE-CLASS LEARNING FOR PARTIAL MULTI-LABEL LEARNING

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ABSTRACT

Partial multi-label learning (PML) is a special case of multi-label learning, in which label noise is confined to the observed positive labels. Although numerous PML methods have been developed to handle noisy label spaces, many of them rely on general noise disambiguation mechanisms without explicitly exploiting the asymmetric noise structure of PML. To better leverage this characteristic, we propose Bidirectional Single-Class Learning for Partial Multi-Label Learning (BSCPML), which treats positive and negative labels separately. Specifically, clean negative labels can be directly fitted through single-class learning, whereas positive labels may require purification before learning. To theoretically justify this asymmetric treatment, we develop a unified loss estimation framework that compares several representative PML losses in terms of bias, variance, and their combined estimation error. The analysis demonstrates that an appropriately weighted BSCPML loss can minimize the gap to the clean-label loss. Extensive experiments on 12 benchmark datasets demonstrate that BSCPML achieves competitive or superior performance compared with state-of-the-art PML methods, particularly under severe label noise.

1 INTRODUCTION

Multi-label learning (MLL) addresses supervised learning problems where each instance may be associated with multiple labels Zhang & Zhou (2013). It has been widely applied to real-world tasks involving multiple semantics, such as text classification Ostapuk et al. (2024), object detection Zhao et al. (2020), medical diagnosis Fernandez et al. (2024), and bioinformatics Li et al. (2025a). However, the expansion of the label space substantially increases the cost and difficulty of data annotation. In addition, annotators may struggle to distinguish semantically similar labels, which can further degrade annotation quality Gao et al. (2016).

Partial multi-label learning (PML) Xie & Huang (2018) alleviates this problem by allowing uncertain labels to be included in the candidate positive label set, thereby reducing annotation effort at the cost of introducing false-positive labels. Consequently, PML can be regarded as a special form of MLL with an asymmetric noise structure, where observed negative labels are assumed to be reliable while observed positive labels may contain noise.

Existing PML methods can be broadly categorized into two strategies: end-to-end and two-stage approaches. End-to-end methods jointly optimize label disambiguation and predictive model learning within a unified objective Chen et al. (2025); Lyu et al. (2025); Wang et al. (2025); Hang & Zhang (2025); Xu et al. (2026), whereas two-stage methods first estimate the confidence of candidate positive labels and subsequently train a predictive model using the refined labels Li et al. (2025b); Lin et al. (2025); Wang et al. (2025). Both strategies mainly focus on identifying or suppressing false-positive labels, frequently relying on general-purpose noise disambiguation techniques such as robust principal component analysis and non-negative matrix factorization. However, these techniques are not specifically designed for the asymmetric noise structure of PML and can also be applied to more general noisy MLL problems Xu et al. (2020); Li et al. (2021; 2022a); Ding et al. (2024).

To develop an objective tailored specifically to PML, we decompose the multi-label learning problem into single-class subproblems and propose Bidirectional Single-Class Learning for PML (BSCPML). The key idea is to explicitly exploit the asymmetric noise structure of PML at the loss

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level. Since observed negative labels are assumed to be reliable, they can be directly fitted through single-class learning. In contrast, observed positive labels may contain false positives and therefore require more cautious treatment. Accordingly, BSCPML assigns asymmetric roles to the two label classes: reliable negative labels are directly emphasized in model fitting, while potentially corrupted positive labels are either down-weighted or purified before positive-class learning.

Recent studies have also recognized the asymmetric noise structure of PML, particularly the reliability of negative labels. However, negative labels in these methods are mainly used indirectly for label confidence refinement Zhong et al. (2024), positive label enhancement Lin et al. (2025), or adversarial learning Yang et al. (2024). In contrast, BSCPML directly incorporates the asymmetry between positive and negative labels into the loss function and explicitly models reliable negative labels as an independent source of supervision. Based on this principle, we first develop a naive formulation that strengthens negative-label fitting, and further introduce a robust formulation with positive-label purification for more severely corrupted label spaces.

Beyond the methodological design, we provide a theoretical analysis of the proposed asymmetric loss. Specifically, we establish a unified loss estimation framework under one-sided false-positive noise and compare representative PML losses in terms of bias, variance, and their trade-off. The analysis shows that unbiasedness alone does not necessarily yield a better approximation to the clean-label loss, due to the increased variance introduced by noise correction. More importantly, it demonstrates that, with appropriate asymmetric weighting, the proposed BSCPML loss can more closely approximate the loss computed from clean labels, providing theoretical support for assigning greater emphasis to reliable negative labels.

2 PRELIMINARIES

Formally, let $\mathcal{X} \in \mathbb{R}^d$ be feature space and $\mathcal{Y} = \{0, 1\}^c$ be label space, where $0, 1$ represent negative and positive labels, respectively. A multi-label dataset is defined as $\mathcal{D} = (X_{n\times d}, Y_{n\times c})$, where n, d, c denote the numbers of instances, features and labels, respectively. In partial multi-label learning (PML), the label matrix Y is assumed to contain local noise, where 0 is considered correct, and 1 may be erroneous.

3 METHOD

3.1 NAIVE BSCPML

Under the false positive assumption in PML, we choose to fit the negative labels and formulate the optimization problem:

$$\min_W \|\mathcal{R}_0(XW) - \mathcal{R}_0(Y)\|_F^2 \tag{1}$$

where $\mathcal{R}_\theta(A), \theta \in \{0, 1\}$ is an observation operator defined on the top of label matrix Y:

$$\mathcal{R}_\theta(A_{ij}) = \begin{cases} A_{ij}, & Y_{ij} = \theta \\ 0, & Y_{ij} \neq \theta \end{cases} \tag{2}$$

In the PML settings, $Y_{ij} \in \{0, 1\}$, so that Eq. (1) can be equivalently rewritten as:

$$\min_W \|(1 - Y) \odot (XW)\|_F^2 \tag{3}$$

where $\odot$ denotes Hadamard product.

Dually, fitting the positive labels can be formulated as:

$$\min_W \|Y \odot (XW - 1)\|_F^2 \tag{4}$$

When label noise is relatively rare, the harmful impact may be mitigated by reducing the weight assigned to positive-label fitting. In this case, we refer to the loss function as a naive scheme, with the following objective:

$$\min_W \frac{\lambda_1}{2}\|(1 - Y) \odot (XW)\|_F^2 + \frac{\lambda_2}{2}\|Y \odot (XW - 1)\|_F^2 \tag{5}$$

However, Eq. (5) involves two Hadamard product terms, resulting in a complex gradient structure. Factually, fitting both positive and negative labels can be viewed as fitting all labels. Consequently, the loss function can be simplified as:

$$\min_{W} \frac{\lambda_1 - \lambda_2}{2} \|(1 - Y) \odot (XW)\|_F^2 + \frac{\lambda_2}{2} \|XW - Y\|_F^2 \tag{6}$$

where the second term corresponds to a standard squared loss over all labels, combining the contributions of fitting both negative and positive labels with equal weight.

To further exploit the dependency among labels, the relationship between the predictions associated with two labels is encouraged to follow the corresponding relationship observed in the label space:

$$\min_{W} S_{ij} (XW)_i ((XW)_j)^\top \tag{7}$$

where S_{ij} represents the similarity between labels Y_i, Y_j. To account for all label pairs in a unified manner, a Laplacian regularizer can be introduced:

$$\min_{W} \mathrm{tr}(XWL(XW)^\top) \tag{8}$$

where L is the Laplacian matrix of Y and $\mathrm{tr}(\cdot)$ denotes its trace. Specifically, given $S = [S_{ij}]$ and diagonal degree matrix $D = \mathrm{diag}(D_i)$ with $D_i = \sum_j S_{ij}$, Laplacian is given by $L = D - S$. In this paper, S_{ij} is based on cosine similarity.

At the parameter level, we use $\mathrm{tr}(WLW^\top)$ as the corresponding label-graph regularizer. This term encourages the classifier parameters associated with strongly related labels to remain similar, thereby incorporating the global label dependency structure into model learning. Incorporating the weight-level Laplacian regularizer into loss function Eq. (6), we can eventually get BSCPMLn (naive BSCPML):

$$\min_{W} \frac{1}{2} \|XW - Y\|_F^2 + \frac{\lambda}{2} \|(1 - Y) \odot (XW)\|_F^2 + \frac{\gamma}{2} \mathrm{tr}(WLW^\top) \tag{9}$$

3.2 Robust BSCPML

In poorly labeled PML, the naive assumption that down-weight label noise becomes infeasible. To explicitly capture the noise, robust principal component analysis (RPCA) can be incorporated into the objective in Eq. (9):

$$\begin{aligned} \min_{W,Z,N} \ & \frac{1}{2} \|XW - Y\|_F^2 + \frac{\lambda_1}{2} \|(1 - Y) \odot (XW)\|_F^2 + \frac{\lambda_2}{2} \|Y \odot (XW - Z)\|_F^2 + \gamma_1 \|Z\|_* + \gamma_2 \|N\|_1 \\ \text{s.t.} \quad & Y = Z + N \end{aligned} \tag{10}$$

where the nuclear norm $\|Z\|_*$ serves as a differentiable surrogate of the matrix rank, computed as the sum of singular values of Z. The Laplacian regularizer in Eq. (9) is removed, as explicit label correlation modeling and overfitting control become unnecessary. First, label correlation is already captured implicitly by the nuclear norm. Second, the objective function encourages W to fit three complementary components, namely the full label matrix, negative labels, and purified positive labels, which prevents the model from overfitting to any single label source.

However, simple RPCA is not well suited for PML. The label matrices in PML are typically rectangular and exhibit asymmetric correlation, implying rank $r(Y) = c$ and making rank minimization or its surrogate of limited practical significance. Instead of assuming the clean label matrix itself to be low-rank, we only assume that the dependency structure among labels is low-rank. Concretely, we formulate the label matrix as $Y = YZ + N$, where the coefficient matrix Z captures label dependencies and is encouraged to be low-rank. To reduce the complexity of the objective function, we remove the equality constraint and relax the sparse ℓ_1 penalty to a Frobenius norm, and propose BSCPMLr (robust BSCPML):

$$\begin{aligned} \min_{W,Z} \ & \frac{1}{2} \|XW - Y\|_F^2 + \frac{\lambda_1}{2} \|(1 - Y) \odot (XW)\|_F^2 + \frac{\lambda_2}{2} \|Y \odot (XW - YZ)\|_F^2 \\ & + \gamma_1 \|Z\|_* + \frac{\gamma_2}{2} \|Y - YZ\|_F^2 \end{aligned} \tag{11}$$

3.3 Optimization

We optimize BSCPMLn using gradient descent and BSCPMLr using alternating optimization, where W is updated by gradient descent and Z by a proximal-gradient step with singular value thresholding. Detailed update rules and the complete algorithm are provided in Appendix B.2.

4 Bias-Variance Analysis of Loss Estimation

Assumption 1 (Uniform False-Positive Noise)**.** *We assume a one-sided class-conditional noise model, where negative labels are flipped to positive with a uniform noise rate $\rho \in [0, 1)$:*

$$P(y = 1 \mid y^* = 0) = \rho \tag{12}$$

while positive labels remain unchanged, i.e., $P(y = 1 \mid y^* = 1) = 1$.

The noise rate ρ is assumed to be independent of instances and labels.

4.1 Loss Estimators for Partial Multi-Label Learning

We first consider the clean squared loss

$$\mathcal{L}^*(XW, Y^*) = \|XW - Y^*\|_F^2 \tag{13}$$

where Y^* is noise-free label matrix. Since the Frobenius norm is separable over matrix entries, we formally denote $y = Y_{ij}, f = (XW)_{ij}, 1 \le i \le n, 1 \le j \le c$ as observed label and predicted value, and y^* is the corresponding clean label.

The entry-wise clean loss is:

$$\ell^*(f, y^*) = (1 - y^*)f^2 + y^*(f - 1)^2 = \begin{cases} (f-1)^2, & y^* = 1 \\ f^2, & y^* = 0 \end{cases} \tag{14}$$

We consider three estimators of the clean loss: the observed loss, the proposed bidirectional loss, and an unbiased loss estimator:

$$\ell_{\mathrm{OB}}(f, y) = (f - y)^2 = (1 - y)f^2 + y(f - 1)^2 \tag{15}$$

$$\ell_{\mathrm{BD}}(f, y) = \lambda_1(1 - y)f^2 + \lambda_2 y(f - 1)^2 \tag{16}$$

$$\ell_{\mathrm{UB}}(f, y) = \eta_1(1 - y)f^2 + \eta_2 y(f - 1)^2 - \eta_3(1 - y)(f - 1)^2 \tag{17}$$

Here, ℓ_{OB} corresponds to the standard squared loss commonly used in existing PML methods, while ℓ_{BD} is the bidirectional loss introduced in Eq. (5). ℓ_{UB} is an unbiased estimation of clean loss given coefficients by (detailed derivation can be seen in Appendix B.4):

$$\eta_1 = \frac{1}{1 - \rho}, \qquad \eta_2 = 1, \qquad \eta_3 = \frac{\rho}{1 - \rho} \tag{18}$$

4.2 Analysis on Bias, Variance, and Expected Squared Euclidean Gap

To characterize the statistical properties of the aforementioned loss estimators, we analyze their bias, variance, and a trade-off of them. For a controlled comparison, we set $\lambda_1 = \eta_1, \lambda_2 = \eta_2$ for ℓ_{BD}, matching the coefficients of its two primary fitting terms to those of ℓ_{UB}.

First, we quantify the systematic deviation of a loss estimator from the clean loss by its conditional bias:

$$\mathbb{B}[\ell] = \mathbb{E}[\ell] - \ell^* \tag{19}$$

According to Eq. (54), the biases of the aforementioned loss estimators are given by:

$$\mathbb{B}_{y|y^*=1}[\ell] = 0 \tag{20}$$

$$\mathbb{B}_{y|y^*=0}[\ell_{\mathrm{UB}}] = 0 \tag{21}$$

$$\mathbb{B}_{y|y^*=0}[\ell_{\mathrm{OB}}] = \rho(1 - 2f) \tag{22}$$

$$\mathbb{B}_{y|y^*=0}[\ell_{\mathrm{BD}}] = \rho(f - 1)^2 \tag{23}$$

Here, $\mathbb{B}_{y|y^*}[\ell_{\mathrm{UB}}] \equiv 0$ follows from the unbiasedness of ℓ_{UB}. Under the matched coefficient setting, all three estimators also have zero bias when $y^* = 1$, since their positive-label terms coincide with the clean loss.

Second, we characterize the variability induced by label noise through the conditional variance:

$$\mathbb{V}[\ell] = \mathbb{E}[\ell^2] - \mathbb{E}^2[\ell] = \begin{cases} 0, & y^* = 1 \\ \rho\ell^2(f,1) + (1-\rho)\ell^2(f,0) - \left(\rho\ell(f,1) + (1-\rho)\ell(f,0)\right)^2, & y^* = 0 \end{cases} \tag{24}$$

where zero variance for $y^* = 1$ follows from the one-sided noise assumption: positive labels remain uncorrupted, and hence $y = 1$ almost surely given $y^* = 1$.

The conditional variances of the three loss estimators are:

$$\mathbb{V}_{y|y^*=1}[\ell] = 0 \tag{25}$$

$$\mathbb{V}_{y|y^*=0}[\ell_{\mathrm{UB}}] = \frac{\rho}{1-\rho}(2f-1)^2 \tag{26}$$

$$\mathbb{V}_{y|y^*=0}[\ell_{\mathrm{OB}}] = \rho(1-\rho)(2f-1)^2 \tag{27}$$

$$\mathbb{V}_{y|y^*=0}[\ell_{\mathrm{BD}}] = \rho(1-\rho)\left((f-1)^2 - \frac{f^2}{1-\rho}\right)^2 \tag{28}$$

Third, we further introduce a unified measure based on both bias and variance, namely the expected squared Euclidean gap between noisy loss estimator $\ell(f,y)$ and the clean loss $\ell^*(f,y^*)$:

$$\mathbb{G}[\ell] = \mathbb{E}[(\ell - \ell^*)^2] = \mathbb{B}^2[\ell] + \mathbb{V}[\ell] \tag{29}$$

This decomposition follows from the standard bias-variance decomposition of an estimator and measures how accurately a noisy loss estimates its clean counterpart.

Accordingly, the expected squared gaps are:

$$\mathbb{G}_{y|y^*=1}[\ell] = 0 \tag{30}$$

$$\mathbb{G}_{y|y^*=0}[\ell_{\mathrm{UB}}] = \frac{\rho}{1-\rho}(2f-1)^2 \tag{31}$$

$$\mathbb{G}_{y|y^*=0}[\ell_{\mathrm{OB}}] = \rho(2f-1)^2 \tag{32}$$

$$\mathbb{G}_{y|y^*=0}[\ell_{\mathrm{BD}}] = \rho(2f-1)^2 + \frac{\rho^2}{1-\rho}f^4 \tag{33}$$

It follows immediately that:

$$\begin{cases} \mathbb{G}_{y|y^*}[\ell_{\mathrm{OB}}] \le \mathbb{G}_{y|y^*}[\ell_{\mathrm{BD}}] \le \mathbb{G}_{y|y^*}[\ell_{\mathrm{UB}}], & -\sqrt{2}-1 < f < \sqrt{2}-1 \\ \mathbb{G}_{y|y^*}[\ell_{\mathrm{OB}}] \le \mathbb{G}_{y|y^*}[\ell_{\mathrm{UB}}] \le \mathbb{G}_{y|y^*}[\ell_{\mathrm{BD}}], & \text{otherwise} \end{cases} \tag{34}$$

These results show that unbiasedness alone does not guarantee a smaller overall loss-estimation gap. In particular, although ℓ_{UB} completely eliminates bias, its increased variance leads to a larger expected squared gap than ℓ_{OB}. This observation motivates us to relax the matched coefficient setting and further investigate the weighting strategy of ℓ_{BD} from the perspective of minimizing the gap between itself and clean loss.

4.3 Implication for Asymmetric Loss Weighting

We further investigate how the weighting coefficients of the bidirectional loss affect its loss-estimation accuracy. Without imposing the matched coefficient setting used in the preceding comparison, the expected squared loss gap of ℓ_{BD} is:

$$\mathbb{G}_{y|y^*}[\ell_{\mathrm{BD}}] = \mathbb{E}_{y|y^*}[(\ell_{\mathrm{BD}} - \ell^*)^2] = \begin{cases} (\lambda_2 - 1)^2(f-1)^4, & y^* = 1 \\ \rho(\lambda_2(f-1)^2 - f^2)^2 + (1-\rho)(\lambda_1 - 1)^2 f^4, & y^* = 0 \end{cases} \tag{35}$$

To minimize $\mathbb{G}_{y|y^*}[\ell_{\mathrm{BD}}]$, we have $\lambda_1 = 1$, and

$$\lambda_2 = \begin{cases} 1, & y^* = 1 \\ \dfrac{f^2}{(f-1)^2}, & y^* = 0 \end{cases} \tag{36}$$

implying that λ_2 depends on both the clean label and the prediction.

Although the pointwise optimal λ_2 is data-dependent, its relative weighting provides a useful implication for the design of BSCPML. For the non-trivial case $y^* = 0$, a negative entry is expected to be predicted on the negative side of the decision boundary, i.e., $f < \frac{1}{2}$. We then have

$$\lambda_2 = \frac{f^2}{(f-1)^2} < 1 = \lambda_1 \tag{37}$$

Accordingly, after rewriting Eq. (5) into the normalized form of Eq. (9), the relative weight satisfies

$$\lambda = \frac{\lambda_1 - \lambda_2}{\lambda_2} > 0 \tag{38}$$

This result suggests that reliable negative-label fitting should receive a larger relative weight than potentially noisy positive-label fitting. Therefore, the objective in Eq. (9) can be interpreted as the observed loss augmented with an additional negative-label fitting term, which is consistent with the asymmetric design of BSCPML.

The pointwise solution above is not directly used as a training hyperparameter because it varies with individual predictions. In practice, we adopt a fixed positive λ, following the common hyperparameter setting of existing PML methods, which also enables a fair comparison without introducing an additional instance-dependent weighting mechanism.

5 Experiments

5.1 Experimental Setup

In the experiments, 12 public benchmark datasets are utilized, from 6 different domains. Details about the datasets are shown in Table 6 in Appendix C.1. To ensure a fair and reliable evaluation, each dataset is divided into training and testing sets using stratified sampling with a ratio of 2:1, following the recommendation of KDIS. False positive noise is randomly injected into the training set, while PML learners are evaluated on the clean test set. For each dataset, the noise generation process is repeated 10 times, and we report mean values and standard deviations.

For a comprehensive comparison, BSCPML is evaluated against 10 SOTA methods introduced in Appendix C.2, including NATAL Lyu et al. (2021), PAMB Liu et al. (2023), NLR Yang et al. (2024), LSNRLS Zou et al. (2024), PMLND Zhong et al. (2024), LENFN Chen et al. (2024), PMLFS Han et al. (2025), FSMIR Gao et al. (2025), PMLPLR Chen et al. (2025), and Schirn Si et al. (2026). Regarding parameter settings, we adopt the default configurations provided in the corresponding papers or public implementations. For the proposed method, we set $\lambda = 0.1, \gamma = 1$ for BSCPMLn and $\lambda_1 = 0.1, \lambda_2 = 0.01, \gamma_1 = 1, \gamma_2 = 1$ for BSCPMLr. All experiments are conducted on an i7-13650HX CPU (2.60 GHz).

For reproducibility, the optimization procedure is implemented with the following settings. Unless otherwise specified, both W and Z are initialized as zero matrices. The maximum number of iterations is set to 1000, and the optimization terminates when the relative change in the objective value between two consecutive iterations is smaller than 10^{-3}. For each noise level ρ, false-positive noise is injected by randomly selecting ρ proportion of the observed negative label entries and setting their values to 1, while the original positive entries remain unchanged.

The evaluation metrics include Accuracy, Hamming Loss, Ranking Loss, and Average Precision Madjarov et al. (2012), where the former two are used to measure classification performance and the latter two evaluate ranking quality. Among them, Hamming and Ranking Loss are minimized, whereas another two are maximized. The details can be seen in Appendix C.3.

5.2 Overall Performance Comparison

We conduct comprehensive comparisons on 12 datasets using 4 evaluation metrics, involving 10 SOTA PML methods and 2 variants of the proposed BSCPML. For each metric on each dataset, the best-performing result is highlighted in bold. In the tables, AVG means average values among 12 datasets, and RANK indicates an average ranking from 1 to 12.

To simulate scenarios with severe label noise, we set the noise rate to 30%, and the corresponding comparisons are reported in Table 1 to Table 4. To represent mild-noise scenarios, we further evaluate the methods with a 5% noise rate and provide comparison results in Table 7 to 10 in Appendix.

Table 1: Accuracy ↑ under 30% noise.

Dataset	NATAL	PAMB	NLR	LSNRLS	PMLND	LENFN	PMLFS	FSMIR	PMLPLR	Schirn	BSCPMLn	BSCPMLr
20NG	0.0701	0.3741	0.3025	0.4791	0.3904	0.3410	0.4429	0.4286	0.2180	0.3912	**0.5049**	0.5015
Birds	0.0865	0.0469	0.1528	0.5190	0.3796	0.2645	0.2709	0.4150	0.1956	0.2685	0.5220	**0.5264**
Emotions	0.3781	0.2125	0.2456	0.2490	0.1892	0.4899	0.4643	0.4373	0.4580	0.4761	0.4862	**0.4972**
Foodtruck	0.1746	0.1481	0.3883	**0.4275**	0.2213	0.1990	0.3347	0.4109	0.3445	0.3484	0.3501	0.3560
GpositiveGO	0.5371	0.6395	0.8954	0.6805	0.4650	0.6969	0.4815	0.8411	0.7359	0.6577	0.8911	**0.8967**
Langlog	0.1800	0.0581	0.1965	0.4145	0.1694	0.2204	0.3132	**0.4249**	0.1681	0.3169	0.4198	0.4200
Ohsumed	0.0355	0.1023	0.2129	0.4926	0.4739	0.2484	0.3356	0.3903	0.1191	0.1658	0.5097	**0.5104**
PlantGO	0.1087	0.1527	0.5765	0.4399	0.1346	0.2033	0.1709	0.4990	0.4322	0.2730	0.5805	**0.5876**
VirusGO	0.2513	0.3882	**0.8237**	0.6487	0.0822	0.5637	0.3995	0.7043	0.5779	0.5586	0.6722	0.6741
Water_quality	0.3301	0.0615	0.2088	0.4087	0.1286	0.3491	0.4126	0.3642	0.3286	0.4139	**0.4149**	0.4147
Yeast	0.3509	0.0792	0.2330	**0.5023**	0.1943	0.3469	0.4751	0.4340	0.4394	0.4836	0.4742	0.4704
Yelp	0.3277	0.2091	0.4944	0.4675	0.3821	0.4953	**0.5134**	0.3977	0.2855	0.4892	0.4919	0.4925
AVG	0.2359	0.2060	0.3942	0.4774	0.2676	0.3682	0.3846	0.4789	0.3586	0.4036	0.5265	**0.5290**
RANK	10.17	10.92	6.5	4.42	9.42	6.83	6	4.58	8.08	5.92	2.92	**2.25**

Table 2: Hamming Loss ↓ under 30% noise.

Dataset	NATAL	PAMB	NLR	LSNRLS	PMLND	LENFN	PMLFS	FSMIR	PMLPLR	Schirn	BSCPMLn	BSCPMLr
20NG	0.0701	0.0638	0.0926	0.0628	0.0717	0.2243	0.1297	0.1551	0.1285	0.1027	**0.0559**	0.0563
Birds	0.2466	0.2985	0.1183	0.1001	0.0918	0.3262	0.2299	0.1765	0.2364	0.1992	0.0978	**0.0763**
Emotions	0.4847	0.3426	0.3281	0.3728	0.3412	0.3127	0.3380	0.3629	**0.2551**	0.2767	0.3189	0.3224
Foodtruck	0.3496	0.2576	0.2571	0.2351	0.2997	0.4432	0.2605	**0.1983**	0.2298	0.2273	0.2709	0.2645
GpositiveGO	0.3869	0.1381	0.1138	0.1269	**0.0478**	0.1688	0.3453	0.1016	0.1323	0.1647	0.0568	0.0532
Langlog	0.3584	**0.1977**	0.2052	0.2513	0.2756	0.3435	0.3872	0.2819	0.2638	0.3362	0.2478	0.2477
Ohsumed	0.0355	0.1215	0.2153	0.0375	0.0355	0.4048	0.1474	0.0983	0.2535	0.1519	**0.0350**	0.0351
PlantGO	0.1536	0.0898	0.1297	0.1410	0.0965	0.3555	0.3511	0.2211	0.2193	0.1959	0.0584	**0.0583**
VirusGO	0.3044	0.1404	0.1314	0.1511	**0.0912**	0.1956	0.1803	0.1467	0.1732	0.1724	0.1447	0.1430
Water_quality	0.4826	0.3949	0.3862	0.4309	0.3845	0.4590	0.4133	0.4836	**0.3240**	0.3961	0.4150	0.4183
Yeast	0.4303	0.3218	0.2872	0.2922	0.3025	0.3971	0.2915	0.3289	**0.2373**	0.2578	0.2940	0.2980
Yelp	0.3277	0.3611	0.2814	0.3333	0.2878	**0.2596**	0.2812	0.4013	0.3383	0.2880	0.2826	0.2741
AVG	0.3025	0.2273	0.2122	0.2113	0.1938	0.3242	0.2796	0.2464	0.2326	0.2307	0.1898	**0.1873**
RANK	9.5	6.33	4.75	6.17	4.92	9.67	8.08	8	6.33	6.33	4.17	**3.75**

Table 3: Ranking Loss ↓ under 30% noise.

Dataset	NATAL	PAMB	NLR	LSNRLS	PMLND	LENFN	PMLFS	FSMIR	PMLPLR	Schirn	BSCPMLn	BSCPMLr
20NG	0.1519	0.8265	0.2913	0.1603	0.1563	0.2700	0.2062	0.1483	0.4511	0.2024	0.1321	**0.1320**
Birds	0.3038	0.8244	0.3058	0.2511	0.4100	0.3873	0.3251	0.3175	0.2331	0.3135	0.2371	**0.2277**
Emotions	0.3506	0.7771	0.2993	0.2384	0.3580	0.2014	0.2118	0.2281	**0.2007**	0.2153	0.2036	0.2026
Foodtruck	0.3531	0.8069	0.3066	0.2114	0.2295	0.3270	0.1993	**0.1740**	0.2434	0.2083	0.1895	0.1921
GpositiveGO	0.2364	0.3479	0.0986	0.0453	0.0483	0.0538	0.2306	0.0372	0.0843	0.1151	0.0372	**0.0355**
Langlog	0.4712	0.9639	0.3826	0.2289	0.3957	0.4224	0.3864	**0.1852**	0.4123	0.3833	0.2418	0.2405
Ohsumed	0.2661	0.8065	0.4438	0.2415	0.2589	0.3691	0.2967	0.2028	0.3751	0.3035	0.1963	**0.1890**
PlantGO	0.4158	0.8375	0.2103	0.1288	0.2103	0.1377	0.3377	**0.0627**	0.1070	0.2246	0.0797	0.0849
VirusGO	0.4600	0.6118	0.1123	0.0594	0.1333	0.0583	0.2010	0.0256	0.0296	0.1149	**0.0248**	0.0310
Water_quality	0.4007	0.9375	0.4773	0.2861	0.3867	0.4121	0.2788	0.3537	0.2868	0.2759	**0.2759**	0.2777
Yeast	0.3606	0.9208	0.3443	**0.1791**	0.4068	0.3455	0.1998	0.2094	0.2238	0.2088	0.2010	0.2001
Yelp	0.2490	0.7927	0.2477	0.1890	**0.1487**	0.2341	0.1675	0.2299	0.4536	0.1803	0.1708	0.1706
AVG	0.3349	0.7878	0.2933	0.1849	0.2619	0.2682	0.2534	0.1812	0.2584	0.2288	0.1658	**0.1653**
RANK	9.08	12	8.38	4.75	7.38	7.83	6.5	4.04	6.58	6.38	2.67	**2.42**

Table 4: Average Precision ↑ under 30% noise.

Dataset	NATAL	PAMB	NLR	LSNRLS	PMLND	LENFN	PMLFS	FSMIR	PMLPLR	Schirn	BSCPMLn	BSCPMLr
20NG	0.0982	0.3714	0.5748	0.6825	0.6703	0.4995	0.6190	0.6879	0.4165	0.6128	**0.7078**	0.7074
Birds	0.4629	0.2785	0.3990	0.5011	0.3130	0.3277	0.3784	0.3833	0.4711	0.4000	0.5117	**0.5245**
Emotions	0.6477	0.5623	0.6795	0.7224	0.6346	0.7596	0.7511	0.7301	**0.7717**	0.7483	0.7592	0.7607
Foodtruck	0.4993	0.5008	0.6281	0.6924	0.6838	0.4019	0.6873	**0.7320**	0.6321	0.6597	0.7044	0.7022
GpositiveGO	0.7860	0.8096	0.8859	0.9489	0.9472	0.9342	0.7521	0.9562	0.9205	0.8651	0.9564	**0.9572**
Langlog	0.4200	0.2338	0.4142	0.5854	0.3427	0.3916	0.4124	**0.6355**	0.4181	0.4138	0.5754	0.5756
Ohsumed	0.1057	0.2393	0.3014	0.4776	0.4126	0.2818	0.3960	0.5010	0.3067	0.3679	**0.5426**	0.5418
PlantGO	0.4441	0.3930	0.5668	0.7203	0.6050	0.6685	0.4283	**0.8237**	0.7695	0.5506	0.7851	0.7839
VirusGO	0.4183	0.5862	0.8196	0.8970	0.8141	0.8954	0.7231	0.9475	0.9496	0.8201	**0.9551**	0.9518
Water_quality	0.5703	0.4468	0.5030	0.6576	0.5751	0.5400	0.6659	0.5995	0.6670	0.6649	0.6671	**0.6680**
Yeast	0.5956	0.4852	0.6339	**0.7462**	0.5391	0.5774	0.7323	0.7068	0.7148	0.7183	0.7325	0.7340
Yelp	0.4011	0.6916	0.7881	0.7760	**0.8299**	0.7803	0.8219	0.7626	0.6224	0.8225	0.8151	0.8155
AVG	0.4541	0.4665	0.5995	0.7006	0.6140	0.5882	0.6140	0.7055	0.6383	0.6370	0.7260	**0.7269**
RANK	9.83	11.25	8.17	4.5	7.5	8.25	7	4.5	5.92	6.5	2.42	**2.17**

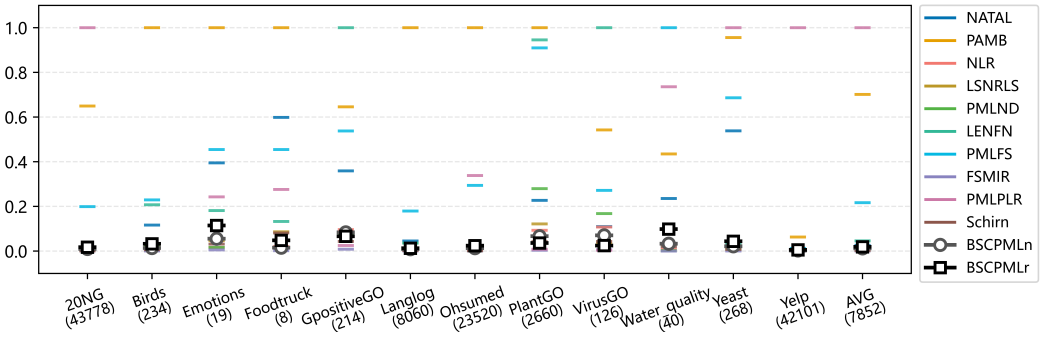

Figure 1: Runtime comparison of different PML methods on 12 datasets. For each dataset, the runtime of each method is normalized by the maximum runtime among all compared methods.

5.3 Runtime Comparison

To provide an intuitive comparison, Fig. 1 reports the runtime of all competing PML methods on the twelve datasets.

Since the absolute runtimes vary substantially across datasets, we normalize the runtime on each dataset by the maximum runtime among all compared methods on that dataset. Thus, the largest normalized runtime is 1.0, while smaller values indicate faster execution. The value in parentheses below each dataset name denotes the actual runtime corresponding to the normalized value of 1.0. The AVG column is obtained by first averaging the original runtimes of each method across all datasets and then normalizing these average runtimes in the same manner. Our methods, BSCPMLn and BSCPMLr, are highlighted using circle and square markers, respectively.

As shown in Fig. 1, BSCPMLn and BSCPMLr maintain relatively low runtime ratios on most datasets, although several competing methods, such as LSNRLS, PMLND, and FSMIR, are computationally faster in some cases. This is mainly because these methods involve fewer iterative operations or decompose the feature matrix into smaller matrices during optimization, thereby reducing computational cost. In contrast, BSCPML performs more comprehensive optimization to jointly model the underlying structures, resulting in additional computational overhead.

5.4 Ablation Study

To verify the contribution of BSCPML's each component, we compare the following baselines:

- B1: $\min_W \frac{1}{2}\|XW - Y\|_F^2$, the straightforward linear model.
- B2: B1 with negative-label fitness, i.e., BSCPMLn in Eq. (9) with $\gamma = 0$.
- B3: B2 further incorporating the regularization on label correlation and model complexity, i.e., BSCPMLn in Eq. (9).

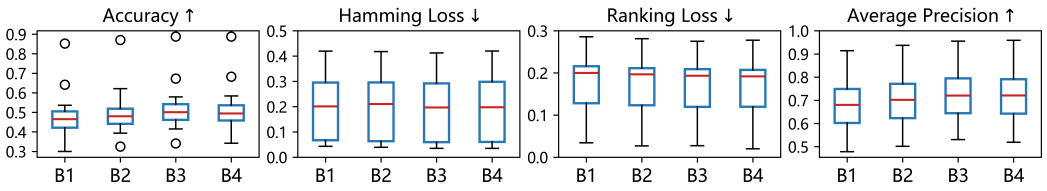

Figure 2: Ablation results.

- B4: B3 further incorporating positive-label purification, i.e., BSCPMLr in Eq. (11).

We use box plots to illustrate the performance differences among the above baselines in Fig. 2. In each sub-figure, the red line denotes the median, the blue box represents the interquartile range, and the black circles indicate outliers. According to the comparison results, the overall performance ranking is BSCPML $\succ$ B2 $\succ$ B1, especially in ranking-based metrics, demonstrating the effectiveness of each employed component.

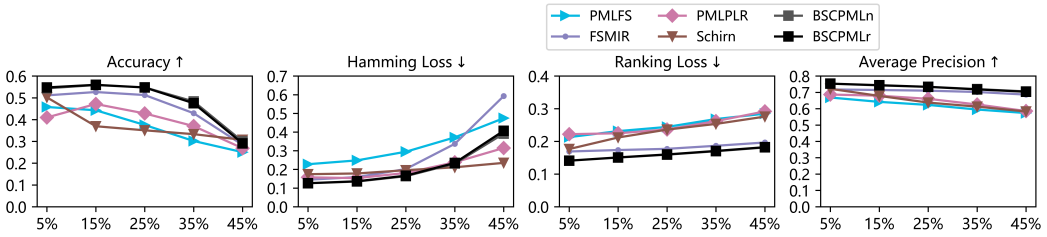

Figure 3: Model robustness to varying noise rates.

5.5 Robustness Test

To evaluate the robustness of BSCPML against noise, we increase the noise rate from 5% to 45% with a step size of 10%. For visual clarity, we select several competitive recent methods for comparison, since plotting all methods would make the curves difficult to distinguish. Each setting is repeated 5 times, and for the robustness stress test we report the worst-case performance of the compared methods in Fig. 3. As the noise rate increases, the performance of all PML learners degrades. BSCPML maintains leading Ranking Loss and Average Precision throughout the tested range and competitive Accuracy up to 35% noise. Compared with other methods, BSCPML demonstrates the highest stability and is less sensitive to noise.

6 Conclusions

In this paper, we propose BSCPML, a new perspective that explicitly exploits the asymmetric noise structure of PML by treating reliable negative labels and potentially corrupted positive labels differently. Based on this principle, we develop a naive formulation, BSCPMLn, that strengthens negative-label fitting, and a robust formulation, BSCPMLr, that further incorporates positive-label purification. Beyond the methodological design, we establish a unified loss-estimation framework to analyze representative PML objectives from the perspectives of bias, variance, and their combined estimation error. Our analysis reveals that unbiasedness alone does not necessarily lead to a better approximation of the clean-label loss, and further provides theoretical justification for assigning greater weight to reliable negative supervision. Extensive experiments on 12 benchmark datasets validate the effectiveness and robustness of BSCPML. Combined with our theoretical analysis, these results establish asymmetric label treatment as a principled approach to PML.

AI USE STATEMENT

Generative AI tools were used solely for language editing to improve the grammar, clarity, and readability of the manuscript. They were not used to develop the research methodology, formulate theoretical claims, design experiments, implement the proposed method, or interpret experimental results. All AI-assisted edits were reviewed and verified by the authors. The authors take full responsibility for the final content of the paper.

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A Related Work

Linear models for multi-label learning (MLL) Ren et al. (2019); Li et al. (2022b) typically define a mapping from $\mathcal{X}$ to $\mathcal{Y}$, with the objective function given by:

$$\min_{W} \quad \mathcal{L}(X, W, Y) + \lambda \mathcal{P}(W) + \gamma \mathcal{C}(X, W, Y) \tag{39}$$

where the decision variable $W \in \mathbb{R}^{d \times c}$ denotes the linear mapping, λ, γ are trade-off parameters, $\mathcal{L}(\cdot)$ is the loss function and typically an Euclidean distance $\|XW - Y\|_F^2$, $\mathcal{P}(\cdot)$ is a penalty term to prevent overfitting, e.g., ℓ_1, ℓ_2 or Frobenius norms, and $\mathcal{C}(\cdot)$ represents a soft constraint for label correlation, such as nuclear norm and Laplacian regularizer.

Based on Eq. (39), end-to-end PML methods generally introduce matrix factorization techniques to discriminate the noise. Assuming the original label matrix Y is factorized into a pure label matrix Z and a corruption component N, the objective function for PML can be formulated as:

$$\begin{aligned} \min_{W,Z,N} \quad & \mathcal{L}(X, W, Z) + \lambda_1 \mathcal{P}_1(Z) + \lambda_2 \mathcal{P}_2(N) + \lambda_3 \mathcal{P}_3(W) \\ \text{s.t.} \quad & \Phi(Y, Z, N) \end{aligned} \tag{40}$$

where $\mathcal{P}_i(\cdot), i = 1, 2, 3$ represent penalty terms for the matrices of pure label, corruption component and coefficient, respectively, and $\Phi(Y, Z, N)$ denotes the matrix factorization process. Typically, the pure label matrix Z is assumed to be low-rank, for which the nuclear norm is commonly used, i.e., $\mathcal{P}_1(Z) = \|Z\|_*$, while the assumption on corruption component N depends on distinct matrix factorization scheme, e.g., robust principal component analysis (RPCA) and non-negative matrix factorization (NMF).

RPCA performs a simple decomposition $\Phi(Y, Z, N) : Y = Z + N$, and correspondingly, $\mathcal{P}_2(N) = \|N\|_1$ Yu et al. (2018); Li et al. (2021). NMF purifies Y into a low-dimensional embedding $Z \in \mathbb{R}^{n \times k}, k \leq c$, that is, $\Phi(Y, Z, N) : Y = ZN^\top, Z \geq 0, N \geq 0$, and the penalty for this N is unnecessary, namely $\mathcal{P}_2(N) = 0$ Sun et al. (2019); Zhao et al. (2022). Additionally, some methods combine multiple matrix factorization techniques, such as $Y = Z^\top Q + N$ Sun et al. (2021).

In recent years, researchers have developed more sophisticated matrix factorization schemes, squeezing out the remaining gains in performance, but at the same time, leading to more hyperparameters, increased complexity, and a greater risk of overfitting Yang et al. (2024); Xu et al. (2026); Lyu et al. (2021). In parallel, another line of work departs from aforementioned discriminative pattern, and instead models disambiguation through probabilistic latent-variable frameworks, resolving label ambiguity via variational inference Lyu et al. (2020); Hang & Zhang (2023). However, these methods often suffer from high computational cost and restrictive modeling assumptions.

B Supplementary Description

B.1 An Overview for BSCPML

The observed positive labels in partial multi-label learning may contain false positives, whereas the observed negative labels are generally considered reliable. Instead of treating all observed labels uniformly, we exploit this asymmetric noise structure by decomposing the learning process into two complementary single-class learning directions. Specifically, reliable negative labels are directly utilized as an additional source of supervision, while potentially corrupted positive labels

are handled with a more cautious strategy through down-weighting or purification, corresponding to naive BSCPML (BSCPMLn) and robust BSCPML (BSCPMLr), respectively. The overall idea of bidirectional single-class learning is illustrated in Fig. 4.

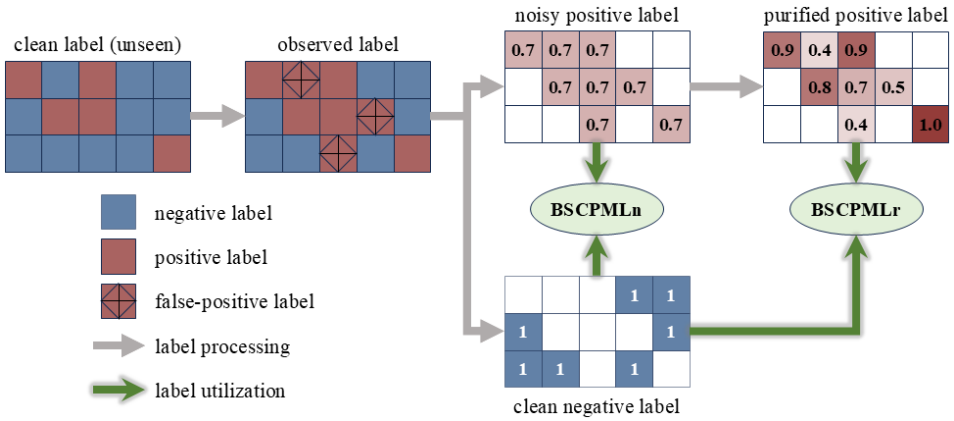

Figure 4: Runtime comparison results with each column representing log_{10}time.

B.2 Optimization

For BSCPMLn in Eq. (9), a closed-form solution is not available due to the trace of Laplacian regularizer. To facilitate optimization, we define objective as:

$$f(W) = \frac{1}{2}\|XW - Y\|_F^2 + \frac{\lambda}{2}\|(1-Y)\odot(XW)\|_F^2 + \frac{\gamma}{2}\mathrm{tr}(WLW^\top) \tag{41}$$

with gradient:

$$\nabla f = X^\top(XW - Y) + \lambda X^\top((1-Y)\odot(XW)) + \gamma WL \tag{42}$$

In summary, the update rule for BSCPMLn algorithm after the k-th iteration can be written as:

$$W^{k+1} = W^k - \alpha\nabla f \tag{43}$$

where α denotes learning rate. Since the Hessian matrices of the three terms in Eq. (42) can be approximated by $X^\top X, \lambda X^\top X, \gamma L$, respectively, we take a Lipschitz step with empirical approximation:

$$\alpha = \frac{1}{(1+\lambda)\|X^\top X\|_F + \gamma\|L\|_F} \tag{44}$$

For BSCPMLr in Eq. (11), we define

$$g(W) = \frac{\lambda_1}{2}\|(1-Y)\odot(XW)\|_F^2 + \frac{\lambda_2}{2}\|Y\odot(XW - YZ)\|_F^2 + \frac{1}{2}\|XW - Y\|_F^2 \tag{45}$$

with gradient:

$$\nabla g = \lambda_1 X^\top((1-Y)\odot(XW)) + \lambda_2 X^\top(Y\odot(XW - YZ)) + X^\top(XW - Y) \tag{46}$$

Similarly for Z, we define

$$h(Z) = \frac{\lambda_2}{2}\|Y\odot(YZ - XW)\|_F^2 + \frac{\gamma_2}{2}\|YZ - Y\|_F^2 \tag{47}$$

along with its gradient:

$$\nabla h = \lambda_2 Y^\top(Y\odot(YZ - XW)) + \gamma_2 Y^\top(YZ - Y) \tag{48}$$

BSCPMLr is optimized in an alternating manner, using a gradient step for W and a proximal-gradient step for Z. Specifically, at each iteration, we first fix Z and update W, and then fix W and update Z. At the $(k+1)$-th iteration, W is updated as:

$$W^{k+1} = W^k - \alpha_r\nabla g(W^k) \tag{49}$$

where α_r denotes the step size:

$$\alpha_r = \frac{1}{(1+\lambda_1+\lambda_2)\|X^\top X\|_F} \tag{50}$$

With W fixed, the optimization with respect to Z consists of the smooth term $h(Z)$ and the non-smooth nuclear-norm regularizer $\gamma_1\|Z\|_*$. For this composite form, singular value thresholding (SVT) provides a closed-form proximal update for the nuclear-norm term Cai et al. (2010). Accordingly, Z is updated as:

$$Z^{k+1} = \mathcal{S}_{\mu\gamma_1}(E^k), \tag{51}$$

where $E^k = Z^k - \mu\nabla h(Z^k)$, μ denotes the step size, and $\mathcal{S}_{\mu\gamma_1}(\cdot)$ is the SVT operator with threshold $\mu\gamma_1$:

$$\mathcal{S}_{\mu\gamma_1}(E) = U\mathcal{S}_{\mu\gamma_1}(\Sigma)V^\top = \sum_{i,\sigma_i\geq\mu\gamma_1}(\sigma_i - \mu\gamma_1)u_i v_i^\top \tag{52}$$

where $U\Sigma V^\top$ is the singular value decomposition (SVD) of E, and σ_i, u_i, v_i are the i-th singular value, left singular vector, right singular vector of E, respectively. The overall optimization procedure of BSCPMLr is summarized in Alg. 1.

Algorithm 1 Framework of BSCPMLr.

Input: Matrices X, Y, Hyper-parameters $\lambda_1, \lambda_2, \gamma_1, \gamma_2$.
Output: W.

1: **while** not converged **do**
2: $W^{k+1} \leftarrow W^k - \alpha_r\nabla g$
3: $E^k = Z^k - \mu\nabla h^k$
4: $Z^{k+1} \leftarrow \mathcal{S}_{\mu\gamma_1}(E^k)$;
5: $k \leftarrow k+1$;
6: **end while**

B.3 Complexity Analysis

For BSCPMLn, gradient descent requires the computation of ∇f as shown in Eq. (42). The time complexity of the matrix multiplication involved is $O(ndc + dc^2)$, while the space complexity corresponds to storing the matrices X, Y, W, L, i.e., $O(nd + nc + dc + c^2)$.

For BSCPMLr, gradient descent involves computing ∇g as shown in Eq. (46) and SVT operation requires performing SVD on the matrix $E \in \mathbb{R}^{c\times c}$. The time complexities of matrix multiplication and SVD are $O(ndc)$ and $O\min(c^3)$, respectively, and the space complexity for storing the matrices X, Y, W, Z is $O(nd + nc + dc + c^2)$.

Effective PML typically provides far more features or samples than labels. Denoting the number of iterations as T, the time and space complexities are summarized in Table 5.

Table 5: Complexities of BSCPML

	Time	Space
BSCPMLn	$O(Tndc)$	$O(nd)$
BSCPMLr	$O(T(ndc + c^3))$	$O(nd)$

B.4 Unbiased Loss Estimator for PML

Theorem 1 (Unbiased Loss Estimator for PML)**.** *Under Assumption 1, the loss $\ell_{\mathrm{UB}}(f, y)$ defined in Eq. (17) is an unbiased estimator of the clean loss $\ell^*(f, y^*)$, i.e.,*

$$\mathbb{E}_{y|y^*}[\ell_{\mathrm{UB}}(f, y)] = \ell^*(f, y^*) \tag{53}$$

for any prediction f and clean label y^*.

Proof. Under the one-sided false-positive noise model, the conditional expectation of any loss $\ell(f, y)$ is

$$\mathbb{E}_{y|y^*}[\ell(f,y)] = \begin{cases} \ell(f,1), & y^* = 1, \\ \rho\ell(f,1) + (1-\rho)\ell(f,0), & y^* = 0. \end{cases} \tag{54}$$

Therefore,

$$\mathbb{E}[\ell_{\mathrm{UB}}(f,y)] = \begin{cases} \eta_2(f-1)^2, & y^* = 1 \\ \rho\eta_2(f-1)^2 + (1-\rho)(\eta_1 f^2 - \eta_3(f-1)^2), & y^* = 0 \end{cases} \tag{55}$$

Substituting $\eta_1 = \frac{1}{1-\rho}$, $\eta_2 = 1$, and $\eta_3 = \frac{\rho}{1-\rho}$ into Eq. (55) gives $\mathbb{E}_{y|y^*}[\ell_{\mathrm{UB}}(f,y)] = \ell^*(f,y^*)$ exactly. □

C Detailed Experimental Analysis

C.1 Datasets

All datasets are available for download from the website of KDIS [1]. Details about the datasets are shown in Table 6, including number of instances (n), features (d), labels (c), and other three statistics. Card is the average number of labels associated with each instance, $\mathrm{Card} = \frac{sum(Y)}{n}$, where $sum(\cdot)$ represents the number of all positive labels. Dens describes the sparsity of the label space, $\mathrm{Dens} = \frac{Card}{c} = \frac{sum(Y)}{n \times c}$, and the smaller the Dens is, the sparser the dataset. DIV represents the diversity of multi-label, indicating the proportion of all 2^c possible label combinations that actually appear in the dataset. To measure class imbalance, the imbalance degree of a label is defined as the ratio between the majority label and the current label, e.g. for the j-th label, $IR_j = \frac{\max_k\{sum_k(Y)\}}{sum_j(Y)}$, where $sum_k(\cdot)$ represents the number of positive labels corresponding to the k-th column of the label matrix. MeanIR measures the average imbalance degree of all labels. It can be computed by $\mathrm{MeanIR} = \frac{\sum_{j=1}^{c} IR_j}{c}$. The greater MeanIR, the greater the imbalance of the dataset.

Table 6: Benchmark datasets.

Dataset	Domain	n	d	c	CARD	DENS	DIV	MeanIR
20NG	Text	19300	1006	20	1.029	0.051	0.003	1.007
Birds	Audio	645	260	19	1.014	0.053	0.206	5.407
Emotions	Music	593	72	6	1.868	0.311	0.422	1.478
Foodtruck	Recommend	407	21	12	2.29	0.191	0.285	7.095
GpositiveGO	Biology	519	912	4	1.008	0.252	0.438	3.861
Langlog	Text	1460	1004	75	1.18	0.016	0.208	39.267
Ohsumed	Text	13930	1002	23	1.663	0.072	0.082	7.869
PlantGO	Biology	978	3091	12	1.079	0.09	0.033	6.69
VirusGO	Biology	207	749	6	1.217	0.203	0.266	4.041
Water_quality	Chemistry	1060	16	14	5.073	0.362	0.778	1.767
Yeast	Biology	2417	103	14	4.237	0.303	0.082	7.197
Yelp	Text	10810	671	5	1.638	0.328	1	2.876

C.2 Comparison Methods

For a comprehensive comparison, BSCPML is evaluated against 10 SOTA methods, which are briefly described as follows:

- NATAL Lyu et al. (2021) reformulates PML as a feature completion problem and learns from the completed features together with candidate labels.
- PAMB Liu et al. (2023) uses error-correcting output codes to decompose PML into binary learning tasks.
- NLR Yang et al. (2024) removes noisy candidate labels through constrained regression and competitive learning with non-candidate labels.

[1] http://www.uco.es/kdis/mllresources/

- LSNRLS Zou et al. (2024) jointly performs label disambiguation and learns shared, non-redundant label-specific features.
- PMLND Zhong et al. (2024) uses negative-label correlations and noise information to guide label disambiguation and classifier learning.
- LENFN Chen et al. (2024) combines near-far neighborhood label enhancement with non-linear guidance for classifier learning.
- PMLFS Han et al. (2025) alternately estimates label confidence and selects label-specific features.
- FSMIR Gao et al. (2025) uses noise-resistant label reconstruction and representative-label information to guide feature selection.
- PMLPLR Chen et al. (2025) reconstructs pseudo-labels using instance correlations learned jointly from candidate labels and features.
- Schirn Si et al. (2026) models sparse label noise while encouraging a high-rank predicted label matrix.

Regarding parameter settings, we adopt the default configurations provided in the corresponding papers or public implementations. For the proposed method, we set $\lambda = 0.1, \gamma = 1$ for BSCPMLn and $\lambda_1 = 0.1, \lambda_2 = 0.01, \gamma_1 = 1, \gamma_2 = 1$ for BSCPMLr [2]. All experiments are conducted on an i7-13650HX CPU (2.60 GHz).

C.3 METRICS

Hamming Loss measures the average error proportion over all labels:

$$\mathrm{HL} = \frac{1}{nc} \sum_{i=1}^{n} \sum_{j=1}^{c} \mathbb{I}(Y_{ij} \neq \hat{Y}_{ij}) \tag{56}$$

where $Y_{ij}, \hat{Y}_{ij} \in \{0, 1\}$ denote the ground-truth and prediction for the j-th label of instance x_i.

Accuracy ignores true negative predictions and focuses more on positive labels, which is more consistent with PML setting.

$$\mathrm{ACC} = \frac{1}{n} \sum_{i=1}^{n} \frac{|Y_i \cap \hat{Y}_i|}{|Y_i \cup \hat{Y}_i|} \tag{57}$$

where $Y_i, \hat{Y}_i \subseteq \mathcal{Y}$ denote the ground-truth and predicted label sets for x_i, respectively. The intersection $Y_i \cap \hat{Y}_i$ corresponds to the correctly predicted labels (true positives) whereas the union $Y_i \cup \hat{Y}_i$ represents all labels relevant to either true or predicted labels, and $|\cdot|$ denotes the cardinality of a set.

Ranking Loss counts the ratio of incorrectly ranked label pairs:

$$\mathrm{RL} = \frac{1}{n} \sum_{i=1}^{n} \frac{\{(j, k) \mid Y_{ij} = 1, Y_{ik} = 0, \tilde{Y}_{ij} \leq \tilde{Y}_{ik}\}}{|Y_i|(c - |Y_i|)} \tag{58}$$

where $|Y_i|, |(c - |Y_i|$ denotes the number of positive and negative labels in x_i, and (p, q) is the index of reversed label pair.

Average Precision counts the number of positive labels that rank higher than positive labels and calculates the proportion:

$$\mathrm{AP} = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{|Y_i|} \sum_{j, Y_{ij}=1} \frac{|\{\tilde{Y}_{ij} < \tilde{Y}_{ik}, Y_{ik} = 1\}|}{\mathrm{rk}_{ij}} \tag{59}$$

where $\mathrm{rk}_{ij} \in [1, c]$ is the rank of $\tilde{Y}_{ij}$ out of c predictions in $\tilde{Y}_i$.

C.4 COMPARISON RESULTS ANALYSIS

According to the comparison results in Table. 1 to Table. 4 and Table. 7 to Table. 10, it can be concluded that:

[2] The source code will be released on GitHub upon acceptance.

- BSCPMLr achieves the best average ranking, followed by BSCPMLn, for all four metrics under both noise settings, demonstrating the effectiveness of BSCPML in multi-label classification and label ranking.
- Under 30% noise, BSCPMLr achieves the best average value and ranking score for each metric. Specifically, in 48 cases across 12 datasets and 4 metrics, BSCPMLr and BSCPMLn achieve first place 14 and 9 times, respectively, surpassing all compared algorithms. The corresponding counts are 0 for NATAL, 1 for PAMB, 1 for NLR, 4 for LSNRLS, 4 for PMLND, 1 for LENFN, 1 for PMLFS, 8 for FSMIR, 5 for PMLPLR, and 0 for Schirn. Furthermore, in the 9 cases where BSCPMLn ranks first, BSCPMLr achieves second place in 7 cases, demonstrating BSCPMLr's overall advantage.
- Under 5% noise, BSCPMLn and BSCPMLr exhibit comparable performance, with BSCPMLr showing a slight overall advantage. The numbers of first-place results are 0 for NATAL, 1 for PAMB, 1 for NLR, 9 for LSNRLS, 0 for PMLND, 0 for LENFN, 9 for PMLFS, 4 for FSMIR, 2 for PMLPLR, 4 for Schirn, 7 for BSCPMLn, and 11 for BSCPMLr.
- BSCPMLr shows a more consistent advantage in mean performance under 30% noise, supporting its robustness. In low-noise scenarios, BSCPMLn remains highly competitive, indicating the effectiveness of leveraging clean negative class labels.
- Based on the overall comparison, the results suggest that BSCPMLn is a preferable choice for datasets with relatively high label quality, due to its simpler design and fewer hyperparameters. BSCPMLr is more suitable for the higher-noise setting considered here.

C.5 Comparison results in 5% noise

Table 7: Accuracy ↑ under 5% noise.

Dataset	NATAL	PAMB	NLR	LSNRLS	PMLND	LENFN	PMLFS	FSMIR	PMLPLR	Schirn	BSCPMLn	BSCPMLr
20NG	0.0701	0.0218	0.3217	0.5405	0.5034	0.4066	**0.5510**	0.5307	0.2555	0.4207	0.5381	0.5384
Birds	0.1066	0.0677	0.3941	**0.5799**	0.3870	0.3783	0.4939	0.4665	0.3635	0.3468	0.5153	0.5180
Emotions	0.4463	0.1987	0.2801	0.3632	0.2854	0.4931	0.5215	0.2781	0.4844	**0.5392**	0.5246	0.5277
Foodtruck	0.1816	0.0808	0.2567	**0.4300**	0.3072	0.2464	0.3990	0.4264	0.3556	0.4158	0.4233	0.4142
GpositiveGO	0.6043	0.6715	0.8968	0.6835	0.7787	0.9053	0.4963	0.9095	0.8517	0.8655	0.9177	**0.9183**
Langlog	0.3189	0.0228	0.2066	0.4213	0.2084	0.2807	0.3248	**0.4563**	0.2100	0.3328	0.4312	0.4321
Ohsumed	0.0355	0.1550	0.2944	0.5039	0.4739	0.2696	**0.5315**	0.4911	0.1680	0.2722	0.4880	0.4880
PlantGO	0.1257	0.1747	**0.6399**	0.4650	0.2844	0.4024	0.2321	0.6112	0.4875	0.5274	0.3942	0.3999
VirusGO	0.3987	0.3224	0.8757	0.8318	0.5076	0.8254	0.5162	0.8614	0.8114	0.8005	**0.9000**	0.8928
Water_quality	0.3268	0.1285	0.1378	0.2860	0.1971	0.3541	0.3400	0.3296	0.2465	**0.4225**	0.3352	0.3385
Yeast	0.3644	0.2350	0.2777	0.4659	0.2785	0.3830	0.5061	0.3809	0.4078	**0.5218**	0.5075	0.5085
Yelp	0.3277	0.1832	0.4256	0.4472	0.3781	0.5416	**0.5823**	0.3940	0.2750	0.5608	0.5217	0.5255
AVG	0.2755	0.1885	0.4173	0.5015	0.3825	0.4572	0.4579	0.5113	0.4097	0.5022	0.5414	**0.5418**
RANK	9.92	11.67	7.5	4.67	8.5	6.33	5.08	4.83	7.92	4.67	3.71	**3.21**

Table 8: Hamming Loss ↓ under 5% noise.

Dataset	NATAL	PAMB	NLR	LSNRLS	PMLND	LENFN	PMLFS	FSMIR	PMLPLR	Schirn	BSCPMLn	BSCPMLr
20NG	0.0701	0.1466	0.0913	0.0510	0.0535	0.0882	0.0504	0.0520	0.0793	0.0753	0.0503	**0.0502**
Birds	0.4466	0.2535	0.0963	0.0505	0.0847	0.3181	0.1103	0.0533	0.0948	0.1488	**0.0496**	0.0498
Emotions	0.3870	0.3434	0.3158	0.2205	0.3480	0.3123	0.2168	0.2652	0.2079	0.2306	0.2093	**0.2072**
Foodtruck	0.4023	0.3207	0.2529	0.1564	0.1801	0.3544	0.1588	**0.1552**	0.1597	0.1789	0.1625	0.1657
GpositiveGO	0.2735	0.1032	0.0533	0.1270	0.0820	0.0491	0.3187	0.0461	0.0494	0.0638	0.0395	**0.0391**
Langlog	0.4411	0.2009	0.1973	**0.1445**	0.2087	0.2191	0.3418	0.1491	0.2040	0.2984	0.1530	0.1535
Ohsumed	0.0355	0.0715	0.0523	0.0338	0.0355	0.0623	**0.0332**	0.0348	0.0769	0.0826	0.0345	0.0345
PlantGO	0.1655	0.0803	0.0692	0.0652	0.0932	0.0967	0.1433	0.0636	0.0596	0.1194	0.0599	**0.0583**
VirusGO	0.4768	0.1820	0.0439	0.0443	0.1607	0.0643	0.1303	0.0388	0.0482	0.0739	**0.0305**	0.0333
Water_quality	0.4298	0.4082	0.4096	0.3226	0.3960	0.4462	0.3085	0.3721	0.3099	0.3735	0.3076	**0.3068**
Yeast	0.4298	0.3028	0.4636	**0.2035**	0.2936	0.3663	0.2145	0.2264	0.2120	0.2249	0.2041	0.2048
Yelp	0.3277	0.3741	0.2691	0.2528	0.2585	0.2185	**0.1894**	0.2814	0.3882	0.2225	0.2103	0.2098
AVG	0.3238	0.2323	0.1929	0.1393	0.1829	0.2163	0.1847	0.1448	0.1575	0.1744	**0.1259**	0.1261
RANK	10.67	9.5	7.92	4.08	7.83	8.67	5.92	4.67	6	7.92	2.54	**2.29**

Table 9: Ranking Loss ↓ under 5% noise.

Dataset	NATAL	PAMB	NLR	LSNRLS	PMLND	LENFN	PMLFS	FSMIR	PMLPLR	Schirn	BSCPMLn	BSCPMLr
20NG	0.1511	0.9654	0.2888	0.1440	0.1507	0.2169	0.1580	0.1303	0.2357	0.1685	0.1280	**0.1184**
Birds	0.2146	0.8187	0.2245	0.1760	0.3666	0.3179	0.2551	0.2650	0.1865	0.2219	0.1656	**0.1586**
Emotions	0.2657	0.7908	0.3426	0.1837	0.3900	0.1936	0.1809	0.2375	0.1849	0.1768	**0.1682**	0.1706
Foodtruck	0.3471	0.8980	0.2819	**0.1548**	0.2212	0.2394	0.1640	0.1655	0.1739	0.1802	0.1647	0.1652
GpositiveGO	0.1216	0.3285	0.0466	0.0528	0.1008	0.0328	0.1663	0.0322	**0.0310**	0.0397	0.0349	0.0345
Langlog	0.4418	**0.1246**	0.3584	0.1814	0.3919	0.3991	0.3749	0.1803	0.4228	0.3615	0.2068	0.2068
Ohsumed	0.2590	0.9863	0.3392	0.1991	0.2537	0.3170	0.2025	0.1786	0.2765	0.2100	0.1447	**0.1446**
PlantGO	0.3655	0.8253	0.0980	0.0763	0.2600	0.0650	0.2888	**0.0466**	0.1447	0.1089	0.0645	0.0627
VirusGO	0.3539	0.6776	0.0239	0.0304	0.0824	0.0183	0.1826	0.0204	0.0164	0.0335	**0.0129**	0.0138
Water_quality	0.3753	0.8591	0.3854	0.2868	0.3181	0.3056	**0.2656**	0.3467	0.2834	0.2747	0.2689	0.2679
Yeast	0.3556	0.7322	0.3485	**0.1754**	0.3371	0.3132	0.1789	0.2004	0.1972	0.1861	0.1777	0.1779
Yelp	0.2439	0.8263	0.2890	0.1875	0.2279	0.1937	**0.1397**	0.2259	0.5119	0.1574	0.1663	0.1669
AVG	0.2913	0.7379	0.2522	0.1540	0.2584	0.2177	0.2131	0.1691	0.2221	0.1766	0.1419	**0.1407**
RANK	9.42	11.08	8.75	4.33	8.58	7.25	5.92	5.08	6.67	5.75	2.63	**2.54**

Table 10: Average Precision ↑ under 5% noise.

Dataset	NATAL	PAMB	NLR	LSNRLS	PMLND	LENFN	PMLFS	FSMIR	PMLPLR	Schirn	BSCPMLn	BSCPMLr
20NG	0.0973	0.1888	0.5814	0.7033	0.6752	0.5792	0.6911	0.6973	0.6040	0.6947	**0.7159**	0.7156
Birds	0.5797	0.2920	0.5762	**0.6240**	0.3274	0.4213	0.4954	0.4435	0.6194	0.5534	0.6128	0.6147
Emotions	0.7071	0.5508	0.6262	0.7752	0.6088	0.7673	0.7792	0.7272	0.7867	**0.7931**	0.7910	0.7877
Foodtruck	0.5431	0.3910	0.5740	**0.7507**	0.6834	0.6426	0.7364	0.7381	0.7100	0.7230	0.7365	0.7391
GpositiveGO	0.8731	0.8973	0.9448	0.9391	0.8965	0.9634	0.8176	0.9579	**0.9680**	0.9503	0.9571	0.9576
Langlog	0.4537	0.1975	0.4507	**0.6412**	0.5257	0.4113	0.4290	0.6377	0.4109	0.4476	0.6190	0.6184
Ohsumed	0.1019	0.1304	0.3629	0.5398	0.4208	0.3829	0.5387	0.5171	0.4336	0.5334	0.5780	**0.5785**
PlantGO	0.5127	0.4047	0.7700	0.8025	0.5560	0.8137	0.4921	**0.8502**	0.7623	0.7567	0.8042	0.7993
VirusGO	0.5868	0.5265	0.9544	0.9530	0.7229	0.9598	0.7547	0.9552	0.9616	0.9395	**0.9711**	0.9694
Water_quality	0.5947	0.4701	0.5796	0.6754	0.5820	0.5583	**0.6761**	0.6053	0.6680	0.6685	0.6747	0.6759
Yeast	0.6120	0.5396	0.5465	0.7487	0.5940	0.6126	0.7555	0.7210	0.7473	0.7530	0.7563	**0.7573**
Yelp	0.3990	0.6691	0.7538	0.7874	0.7681	0.8151	**0.8556**	0.7650	0.5700	0.8475	0.8176	0.8180
AVG	0.5051	0.4381	0.6434	0.7450	0.6134	0.6606	0.6685	0.7180	0.6868	0.7217	**0.7529**	0.7526
RANK	9.67	11.42	8.42	4	8.75	7.25	6.17	5.25	6.08	5.42	2.92	**2.67**

C.6 SIGNIFICANCE TEST

To assess statistical significance, we perform the Friedman test with the Iman-Davenport correction, followed by the Nemenyi test at the 0.05 significance level. Each comparison group contains one BSCPML variant and ten baseline methods, evaluated across 12 datasets. As shown in Table 11, all F_F statistics exceed the critical value of 1.9178, rejecting the null hypothesis of equal performance in every metric and noise setting.

Figs. 5 to 8 present the Nemenyi results with a critical difference of 4.3581. Blue bars connect methods without statistically significant differences. Both BSCPML variants achieve the best average ranks within their respective groups and significantly outperform several baselines, including NATAL.

Table 11: Friedman statistics F_F and critical value at the 0.05 significance level. Each comparison group consists of BSCPMLn or BSCPMLr and ten comparison algorithms.

Noise	Evaluation metric	F_F (BSCPMLn)	F_F (BSCPMLr)	Critical value
30%	Accuracy	12.9012	13.5132	1.9178
	Hamming Loss	4.2334	4.4086	
	Ranking Loss	13.2230	13.0124	
	Average Precision	12.4509	12.8750	
5%	Accuracy	11.5992	11.5174	1.9178
	Hamming Loss	10.5111	10.9335	
	Ranking Loss	10.9833	10.9833	
	Average Precision	12.1272	12.2941	

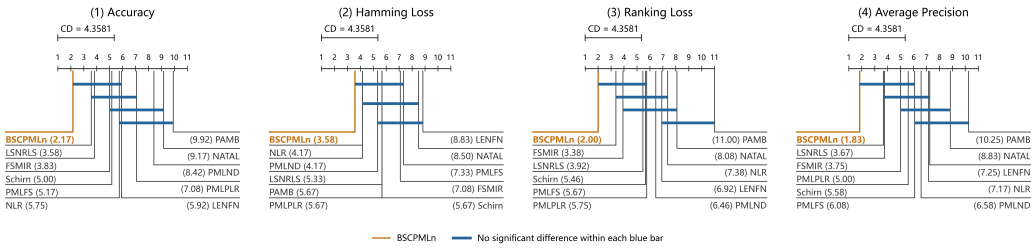

Figure 5: Nemenyi test for BSCPMLn under 30% noise.

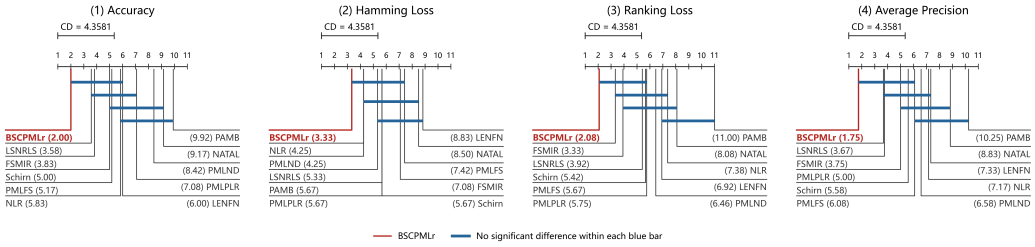

Figure 6: Nemenyi test for BSCPMLr under 30% noise.

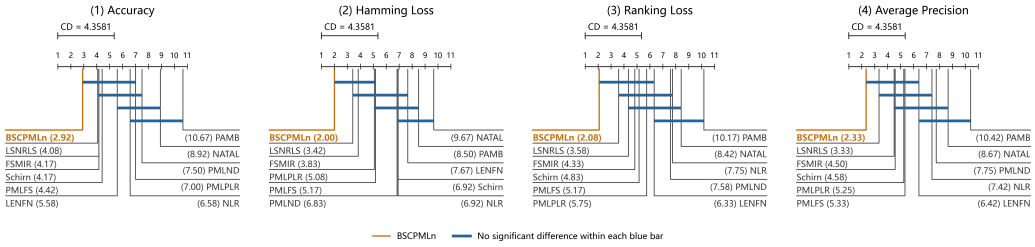

Figure 7: Nemenyi test for BSCPMLn under 5% noise.

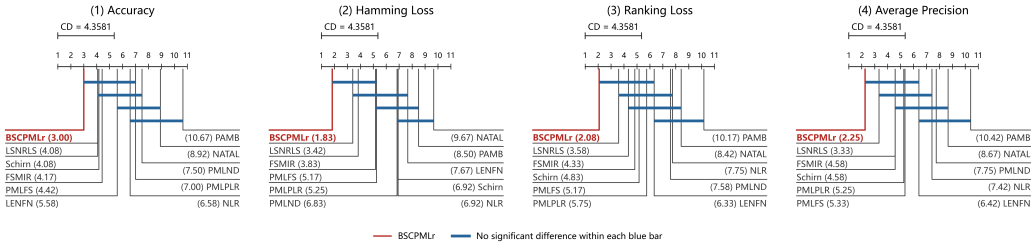

Figure 8: Nemenyi test for BSCPMLr under 5% noise.

C.7 Parameter Sensitivity Analysis

To analyze parameter sensitivity, each hyper-parameter of BSCPML is validated within the range of $10^{\{-4,-3,\cdots,2\}}$. For clarity, we report only the Hamming Loss and Ranking Loss; the corresponding results are presented in Fig. 9. It can be found that BSCPML maintains stable performance across a wide range of parameter settings.

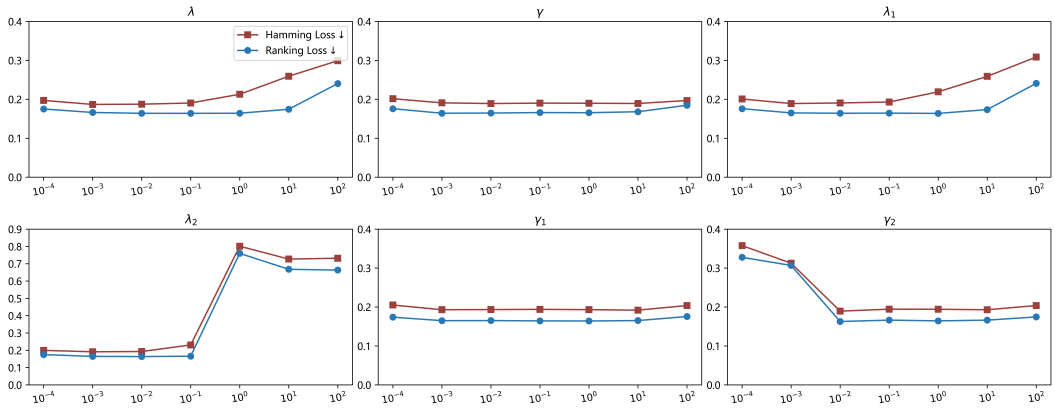

Figure 9: Parameter sensitivity.